

SELF CONSISTENT CALCULATION OF PERIHELION PRECESSION AND  
LIGHT DEFLECTION DUE TO GRAVITATION WITH THE R and x  
THEORIES OF CARTAN GEOMETRY.

by

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ABSTRACT

It is shown that Cartan geometry produces a self consistent and generally applicable and simple orbital theory which is able to describe the properties of orbits in the solar system and also the main properties of whirlpool galaxies. The fundamental structure of the orbital theory is one of Cartan geometry, which reduces to it under well defined conditions. The two new theories are called R and x theories, and are inter related simply. It is shown that they are equivalent to the old Schwarzschild metric (SM) of the obsolete standard physics. It is shown that R and x theories succeed in describing the hyperbolic spiral orbits and velocity curves of stars in a whirlpool galaxy whereas both Newton and Einstein fail qualitatively as is well known.

Keywords: ECE theory, R and x theories, Cartan geometry, planar orbital theory.

4FT 263



## 1. INTRODUCTION

In UFT262 of this series {1 - 10} it was shown that the precession of an object of mass  $m$  in a planar orbit around mass  $M$  can be described precisely by considering a well defined turning point of the orbit. If the orbit is a Kepler / Hooke / Newton ellipse this turning point defines the half right latitude of the ellipse,  $\alpha$ . This is the distance from a focus to the ellipse perpendicular to the major axis. The precession of all objects  $m$  orbiting  $M$  is described by the same turning point analysis, in which the distance from the focus to the ellipse becomes  $\alpha - r_0$ , where  $r_0$  is defined as  $\frac{3}{2}MG / c^2$ . Here  $G$  is Newton's constant and  $c$  is the speed of light in vacuo. The half right latitude is therefore tilted a little due to the precession of the orbit per radian of orbital rotation. It was shown in UFT262 that the precession is described by R theory, equivalent to x theory. It is shown in Section 2 that the same value of turning point  $\alpha - r_0$  is given by the so called "Schwarzschild metric" (SM) of the obsolete standard physics. It must be noted {11} that the SM was never derived by Schwarzschild, who heavily criticised Einstein's theory of orbital precession in a letter of Dec. 1915 translated by Vankov {11}, who adds several more conclusive refutations of the Einstein theory. In ECE theory, a simple form of Cartan geometry is used to calculate all planar orbits, including those in whirlpool galaxies. Both Newton and Einstein fail catastrophically in whirlpool galaxies. This failure should be well known, but is summarized briefly in Section 2 for ease of reference. So the scientific truth about the Einstein theory is quite different from the dogma. In Section 2 it is shown that R theory can be developed into a general kinematic theory based on the well known {12} rotation of the axes of the plane polar coordinates, in which the angular velocity has been shown {1 - 10} to be a Cartan spin connection. It is also shown that R theory is equivalent to x theory for small precessions, where  $x$  is defined by  $1 + \frac{r_0}{\alpha}$  and multiplies the angle  $\theta$  in any equation of the conical section, notably the ellipse and hyperbola. The fundamental explanation for orbital

precession, light deflection and whirlpool galaxies is that they are all due to a well defined Cartan spin connection. They have nothing to do with the incorrect Einstein equation of which the true SM of Dec. 1915 was the first known solution. The reason {1 - 10} is that the Einstein equation is based on an incorrect geometry in which the Cartan torsion did not appear. In Section 2 an infinitesimal line element is developed for R theory, which has the structure of special relativity modified to replace  $r$  by  $r + r_0$  wherever it occurs. It is shown by use of R and x theory that the claim by Einstein to have produced the light deflection due to gravitation cannot be true. The light deflection can be calculated only up to  $\alpha$ , the half right latitude of a hyperbola. This is logically consistent with the fact that planetary precession can be found only up to the same  $\alpha$ . In a precessing planet  $\alpha$  can be found experimentally, but in light deflection it cannot, because the orbit is a hyperbola that is very nearly a straight line. In Section 3, careful numerical integration is used to find from the experimentally observed light deflection due to gravitation. This light deflection is due to  $r_0$ , which is a universal constant for a given M. It is not due to the Einstein theory, which gives only an illusion of accuracy in the solar system, and fails totally in whirlpool galaxies. In ECE theory the SM is a mathematical function which happens to give the same precessional result as R and x theory, the correct and much simpler theories of all orbits.

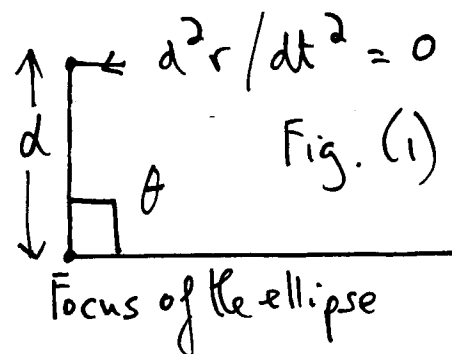
## 2. DEVELOPMENT OF R and x THEORIES

With reference to Fig (1) the turning point (UFT262):

$$\frac{d^2 r}{dt^2} = 0 \quad \text{--- (1)}$$

of a static elliptical orbit occurs at

$$r = \alpha \quad \text{--- (2)}$$



In this case:

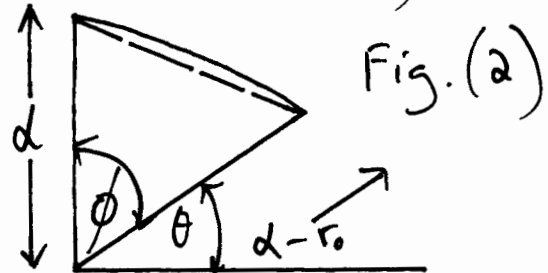
$$\cos \theta = 0, \theta = \frac{\pi}{2} \quad - (3)$$

and is the right angle illustrated in Fig. (1). However in a precessing ellipse the turning point:

$$\frac{d^2 R}{dt^2} = 0, R = r + r_0 \quad - (4)$$

occurs at

$$r = \alpha - r_0 \quad - (4a)$$



and so the half right latitude is tilted as in Fig. (2). For small  $\phi$ :

$$\cos \phi = \frac{\alpha - r_0}{\alpha} \quad - (5)$$

to an excellent approximation in all observed orbital precessions. Defining the angle:

$$\phi = \frac{\pi}{2} - \theta \quad - (6)$$

the Newtonian result is given by:

$$\phi = 0 \quad - (7)$$

and precession changes this to:

$$\cos \phi = 1 - \frac{r_0}{\alpha} \quad - (8)$$

from:

$$\cos \phi = 1 \quad - (9)$$

For a clockwise precession:

$$\Delta \cos \phi = - \frac{r_0}{\alpha} \quad - (10)$$

and for an anticlockwise precession:

$$\Delta \cos \phi = \frac{r_0}{\alpha} \quad - (11)$$

Using the Maclaurin series for small  $\phi$  for an anticlockwise precession:

$$\Delta \phi = \frac{\pi}{2} - \frac{r_0}{\alpha} + \dots \quad - (12)$$

and for a clockwise precession:

$$\Delta \phi = \frac{\pi}{2} + \frac{r_0}{\alpha} + \dots \quad (13)$$

The change in angle due to precession of the ellipse can be deduced from the fact that in the Newtonian theory the angle  $\theta$  is initially  $\frac{\pi}{2}$  as in Fig. ( 1 ), and for a clockwise precession it is changed by  $r_0/\alpha$ . Note that:

$$\theta = \frac{\pi}{2} - \phi \quad - (14)$$

where the plane polar coordinate system is  $(r, \theta)$ . So if  $\phi$  is initially  $\pi/2$ ,  $\theta$  is initially 0. Therefore for a clockwise precession:

$$\Delta \theta = \frac{r_0}{\alpha} \quad - (15)$$

This is exactly the claimed experimental result per radian of orbital precession, so for a complete orbit of  $2\pi$  radians the angle of precession is {12}:

$$\Delta \theta = 2\pi \frac{r_0}{\alpha} \quad - (16)$$

Note carefully that it can be deduced only up to  $\alpha$ . In planetary orbits of the solar system  $\alpha$  can be measured accurately. There is also a major problem in the standard model claim made for the experimentally observed precession (UFT240). This is because the

great majority of the planetary precession is due to the gravitational effect of other planets, and is calculated with Newton. This procedure gives a very tiny anomaly which is attributed to Einstein. It is obvious that the entire precession should be calculated self consistently. This problem is not addressed in the usual dogma. UFT240 is an attempt to remedy this parlous state of planetary physics. The result ( 15 ) has been obtained in this section with the R equation of a precessing orbit:

$$R = \frac{\alpha}{1 + \epsilon \cos \theta} \quad - (17)$$

in which the turning point is defined by:

$$\frac{d^2 R}{dt^2} = 0. \quad - (18)$$

The force law for Eq. ( 17 ) is:

$$F(R) = -\frac{L^2}{mR^2} \left( \frac{d^2}{d\theta^2} \left( \frac{1}{R} \right) + \frac{1}{R} \right) = -\frac{mMG}{R^2} \quad - (19)$$

and:

$$m\ddot{R} = -\frac{mMG}{R^2} + \frac{L^2}{mR^3} \quad - (20)$$

The force law ( 19 ) can be expanded as:

$$F = \frac{-mMG}{(r+r_0)^2} = -\frac{mMG}{r^2} \left( 1 + \frac{r_0}{r} \right)^{-2} = -\frac{mMG}{r^2} \left( 1 - 2\frac{r_0}{r} \right)$$

for:

$$r_0 \ll r. \quad - (22)$$

Note carefully that the law ( 21 ) is non Einsteinian and non Newtonian. The Einstein force law is a sum of terms inverse in r squared and r fourth, whereas Newton is the inverse square law with r.

This R theory is equivalent to x theory {1 - 10}, which has been extensively developed in previous UFT papers. In x theory a precessing orbital ellipse is described by:

$$r = \frac{d}{1 + \epsilon \cos(x\theta)} \quad - (23)$$

whose force law is:

$$F(r) = -\frac{L^2}{mr^2} \left( \frac{d^2}{d\theta^2} \left( \frac{1}{r} \right) + \frac{1}{r} \right) = (x^2 - 1) \frac{L^2}{mr^3} - \frac{x^2 L^2}{d m r^2} \quad - (24)$$

For small angles of precession:

$$d = \frac{L^2}{m^2 M G} \quad , \quad x \sim 1 \quad - (25)$$

and the force law (24) is:

$$F(r) = -\frac{m M G x^2}{r^2} + (x^2 - 1) \frac{L^2}{m r^3} \quad - (26)$$

For all observed precessions in the universe:

$$x \sim 1 \quad - (27)$$

and to an excellent approximation:

$$F(r) \sim -\frac{m M G}{r^2} + (x^2 - 1) \frac{L^2}{m r^3} \quad - (28)$$

Comparing Eqs. (21) and (28) gives:

$$x^2 = 1 + 2 \frac{r_0}{d} \quad - (29)$$

so:

$$x = \left( 1 + 2 \frac{r_0}{d} \right)^{1/2} \quad - (30)$$

If:

$$r_0 \ll \alpha \quad - (31)$$

as in the solar system then:

$$x \sim 1 + \frac{r_0}{\alpha} \quad - (32)$$

This is precisely the correct result, because the precession per radian of rotation is which is of course the same as for R theory. It is seen that the condition of x being almost unity is fulfilled to an excellent approximation. In all observable precessions it seems that x is always close to unity. If x becomes large then the familiar conical sections develop into the subject of fractal conical sections, giving intricate results {1 - 10} of interest to mathematics.

Referring to note 263(7) for details it can be shown that the R theory corresponds to the infinitesimal line element:

$$c^2 d\tau^2 = c^2 dt^2 - dR^2 - R^2 d\theta^2 \quad - (33)$$

which is the line element of special relativity:

$$c^2 d\tau^2 = c^2 dt^2 - dr^2 - r^2 d\theta^2 \quad - (34)$$

with r replaced by R. It is concluded that R theory can be developed from the textbook theory of dynamics by replacing r by R wherever r occurs. Some details and examples of this procedure are given in Note 263(1). The line element ( 33 ) has the structure of special relativity but it is shown next that it gives the same turning point ( 4a ) as the old SM of general relativity:

$$c^2 d\tau^2 = c^2 dt^2 \left(1 - \frac{r_s}{r}\right) - \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 - r^2 d\theta^2 \quad - (35)$$



where the so called "Schwarzschild radius" is defined by:

$$r_s = \frac{2MG}{c^2} \quad - (36)$$

So Eq. ( 35 ) is a very complicated way of rewriting Eq. ( 33 ) to give the same result, Eq. ( 4a ). In ECE theory the SM has no significance physically, it is a mathematical function that happens to work. We either throw away Cartan geometry or the Einstein field equation, both cannot be right. Cartan geometry is the correct geometry, so the SM has no physical significance as a solution of an incorrect Einstein equation. Its significance is that it gives the same turning point as R and x theories as follows.

Define {1 - 10}:

$$E = mc^2 \left( 1 - \frac{r_s}{r} \right) \frac{dt}{d\tau}, \quad L = mr^2 \frac{d\theta}{d\tau} \quad - (37)$$

where E and L are constants of motion, the total energy and angular momentum, here  $\tau$  is the proper time. Eq. ( 35 ) gives:

$$m \left( \frac{dr}{d\tau} \right)^2 = \frac{E^2}{mc^2} - \left( 1 - \frac{r_s}{r} \right) \left( mc^2 + \frac{L^2}{mr^2} \right) \quad - (38)$$

and the proper time can be eliminated with:

$$\frac{dr}{d\tau} = \frac{dr}{d\theta} \frac{d\theta}{d\tau} = \frac{L}{mr^2} \frac{dr}{d\theta} \quad - (39)$$

to give the orbital equation:

$$\left( \frac{dr}{d\theta} \right)^2 = r^4 \left( \frac{1}{b^2} - \left( 1 - \frac{r_s}{r} \right) \left( \frac{1}{a^2} + \frac{1}{r^2} \right) \right) \quad - (40)$$

where

$$a = \frac{L}{mc}, \quad b = \frac{cL}{E} \quad - (41)$$

The light deflection due to gravitation is given by:

$$\Delta\theta = 2 \int_{R_0}^{\infty} \frac{1}{r^2} \left( \frac{1}{b^2} - \left(1 - \frac{r_0}{r}\right) \left( \frac{1}{a^2} + \frac{1}{r^2} \right) \right)^{-1/2} dr - \pi \quad - (42)$$

and was evaluated by accurate numerical analysis in UFT150B and UFT155, which severely criticised Einstein's procedures, also criticised by Schwarzschild, Bauer, Schroedinger, Levi-Civita, Dirac, and Vankov {11}, to name but a few.

Now denote:

$$f = \left( \frac{dr}{d\tau} \right)^2 \quad - (43)$$

then:

$$\frac{df}{d\tau} = \frac{df}{dr} \frac{dr}{d\tau} \quad - (44)$$

so:

$$\frac{d}{d\tau} \left( \frac{dr}{d\tau} \right)^2 = 2 \frac{d^2 r}{d\tau^2} \frac{dr}{d\tau} \quad - (45)$$

The turning point for Eq. (40) occurs at:

$$\frac{d}{d\tau} \left( \frac{dr}{d\tau} \right)^2 = 2 \frac{d^2 r}{d\tau^2} \frac{dr}{d\tau} = 0 \quad - (46)$$

i.e.

$$\frac{d^2 r}{d\tau^2} = 0 \quad - (47)$$

because in general:

$$\frac{dr}{d\tau} \neq 0 \quad - (48)$$

So the turning point occurs at:

$$\frac{d}{d\tau} \left( \frac{E^2}{mc^2} - \left(1 - \frac{r_0}{r}\right) \left( mc^2 + \frac{L^2}{mr^2} \right) \right) = 0 \quad - (49)$$

Using:

$$\frac{d}{d\tau} \left( \frac{E^2}{mc^2} - mc^2 \right) = 0 \quad - (50)$$

Eq. (49) reduces to:

$$\frac{d}{d\tau} \left( mc^2 \frac{r_s}{r} - \frac{L^2}{mr^2} + \frac{L^2}{mr^2} r_s \right) = 0 \quad - (51)$$

Now divide by  $mc^2$  and use:

$$\frac{df}{d\tau} = \frac{df}{dr} \frac{dr}{d\tau} \quad - (52)$$

$$- (53)$$

and Eq. (48). Therefore Eq. (51) reduces to:

$$\frac{d}{dr} \left( \frac{r_s}{r} - \frac{L^2}{m^2 c^2 r^2} + \frac{L^2 r_s}{m^2 c^2 r^3} \right) = 0$$

i.e.

$$-\frac{mG}{r^2} + \frac{L^2}{m^2 r^3} - \frac{3MGL^2}{m^2 c^2 r^4} = 0 \quad - (54)$$

which is the turning point of the Einstein theory, Q. E. D.

This turning point is developed in UFT262, and is for example Eq. (16) of Note

262(6). It is given by:

$$r^2 - dr + r_0 d = 0 \quad - (55)$$

whose roots are:

$$r = d - r_0 \quad - (56)$$

and

$$r = r_0 \quad - (57)$$

Eq. (56) is the same result as that of R and x theory, Q. E. D. The root (57) in the Einstein theory is an obscenity of that mathematically incorrect theory. This root does not occur in the mathematically correct R and x theories.

For ease of reference the following demonstrates the catastrophic or qualitative failure of both Newton and Einstein in whirlpool galaxies.

Consider the position vector  $\underline{r}$  in plane polar coordinates {1 - 10, 12}:

$$\underline{r} = r \underline{e}_r \quad - (58)$$

The linear velocity is:

$$\underline{v} = \frac{d\underline{r}}{dt} = \dot{r} \underline{e}_r + r \dot{\theta} \underline{e}_\theta \quad - (59)$$

The second term on the right hand side is due to the rotation of the plane polar coordinates.

This is an example of Cartan geometry as shown for example UFT262. The Cartan spin connection of the plane polar coordinates is the angular velocity:

$$\underline{\omega} = \frac{d\theta}{dt} \underline{k} = \dot{\theta} \underline{k} \quad - (60)$$

The acceleration is:

$$\underline{a} = \frac{d\underline{v}}{dt} = (\ddot{r} - r\dot{\theta}^2) \underline{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta}) \underline{e}_\theta \quad - (61)$$

The Coriolis acceleration disappears {1 - 10} in all planar orbits:

$$\underline{a}_c = (r\ddot{\theta} + 2\dot{r}\dot{\theta})\underline{e}_\theta = \underline{0} \quad - (62)$$

so the force law of the orbit is:

$$\underline{F} = m\underline{a} = (\ddot{r} - r\dot{\theta}^2)\underline{e}_r \quad - (63)$$

From Eq. (24):

$$\underline{F} = (\ddot{r} - r\dot{\theta}^2)\underline{e}_r = -\frac{L^2}{mr^3} \left( \frac{d^2}{d\theta^2} \left( \frac{1}{r} \right) + \frac{1}{r} \right) \underline{e}_r \quad - (64)$$

where:

$$v^2 = \left( \frac{dr}{dt} \right)^2 + r^2 \left( \frac{d\theta}{dt} \right)^2 \quad - (65)$$

The total angular momentum of the system is a constant of motion:

$$L = mr^2\omega \quad - (66)$$

and the magnitude of the Cartan spin connection of the plane polar coordinates is:

$$\omega = \frac{L}{mr^2} \quad - (67)$$

The orbital linear velocity from fundamental plane polar geometry is therefore:

$$v^2 = \omega^2 \left( \left( \frac{dr}{d\theta} \right)^2 + r^2 \right) \quad - (68)$$

Note carefully that this is a powerful general result of Cartan geometry and has been obtained without assuming a force law. It is therefore more general than Newton and Einstein. ECE theory is more general than the Newtonian and Einsteinian theories.

In a whirlpool galaxy it is observed that:

$$v \xrightarrow{r \rightarrow \infty} v_{\infty} = \text{constant} \quad - (69)$$

for a star orbiting the central mass M. Here  $v_{\infty}$  is a constant. From Eq. (68):

$$v^2 = \left( \frac{L}{mr^2} \right)^2 \left( r^2 + \left( \frac{dr}{d\theta} \right)^2 \right) \quad - (70)$$

so:

$$v \xrightarrow{r \rightarrow \infty} \frac{L}{mr^2} \frac{dr}{d\theta} = v_{\infty} \quad - (71)$$

and:

$$\frac{d\theta}{dr} \xrightarrow{r \rightarrow \infty} \left( \frac{L}{m v_{\infty}} \right) \frac{1}{r^2} \quad - (72)$$

so:

$$\theta \xrightarrow{r \rightarrow \infty} - \left( \frac{L}{m v_{\infty}} \right) \frac{1}{r} \quad - (73)$$

which is a hyperbolic spiral as observed experimentally in whirlpool galaxies in the large r limit, where r is the distance between a star and the central mass of a whirlpool galaxy.

The experimental result has been explained from the pure geometry of plane polar coordinates and the concept of force has not been used. This means that geometry is more fundamental than force, the stars travel in an orbit defined by geometry and not by force. This is a concept of general relativity and an example of Cartan geometry.

Force is defined from the geometry using the equation:

$$\underline{F} = m \underline{\ddot{a}} \quad - (74)$$

For any orbit:

$$\underline{F} = m \left( \ddot{r} - r \dot{\theta}^2 \right) \underline{e}_r \quad - (75)$$

an equation that can be rewritten {1 - 10, 12} as:

$$\underline{F} = -\frac{L^2}{mr^3} \left( \frac{d^2}{dt^2} \left( \frac{1}{r} \right) + \frac{1}{r} \right) \underline{e}_r \quad - (76)$$

In the seventeenth century the force was defined by Newton as:

$$\underline{F} = m \underline{a} = m \ddot{r} \underline{e}_r \quad - (77)$$

so part of Eq. (75) was missing. This is the centrifugal force:

$$\underline{F}_c = -mr \dot{\theta}^2 \underline{e}_r = -\frac{L^2}{mr^3} \underline{e}_r \quad - (78)$$

The concept of orbit is best understood using:

$$m \ddot{r} \underline{e}_r = \underline{F}(r) + \frac{L^2}{mr^3} \underline{e}_r \quad - (79)$$

i. e.

$$m \ddot{r} = F(r) + \frac{L^2}{mr^3} \quad - (80)$$

It appears that Eq. (80) was first written down by Leibniz in 1689, but in an empirical manner. The word "centrifugal" was first used by Huygens in the mid seventeenth century, and the word "centripetal" was first used by Newton. However the centrifugal force was never used by Newton in publications such as the Principia of 1687. Only a few copies of the first edition were printed. Of course the centrifugal force has been sensed back to the stone age, but it was not really understood until Coriolis in the eighteen thirties. The analysis of this paper shows very clearly that it is a spin connection of Cartan geometry and general relativity. It is due to the spinning axes of the plane polar system of coordinates.

From Eq. (76) the force law given by Eq. (73) is:

$$\underline{F} = -\frac{L^2}{mr^3} \underline{e}_r \quad - (81)$$

so:

$$mr'' = -\frac{L^2}{mr^3} + \frac{L^2}{mr^3} = 0 \quad (82)$$

in the large  $r$  limit of a whirlpool galaxy. This means that:

$$\frac{d^2 r}{dt^2} = \frac{d}{dt} \left( \frac{dr}{dt} \right) = 0 \quad - (83)$$

and:

$$\frac{dr}{dt} = \text{constant} \quad - (84)$$

This implies self consistently that:

$$v_\omega = \frac{dr}{dt} = \text{constant} \quad - (85)$$

as observed experimentally. The force law (81) was first derived by Roger Coats in the late seventeenth and early eighteenth centuries, long before whirlpool galaxies were known. Coats helped Newton to produce the second edition of the Principia.

In order to prove the qualitative failure of the Newton theory in whirlpool galaxies note that the orbit of planets in the solar system is an ellipse to an excellent approximation:

$$\frac{1}{r} = \frac{1}{a} (1 + \epsilon \cos \theta) \quad - (86)$$

The force law from Eqs. (76) and (86) is:

$$\underline{F} = -\frac{mMG}{r^2} \underline{e}_r \quad - (87)$$



with

$$\alpha = \frac{L^2}{m^2 M G} \quad - (88)$$

so

$$m \ddot{r} = - \frac{m M G}{r^2} + \frac{L^2}{m r^3} \quad - (89)$$

and this equation, first written down by Leibniz in 1689, is the result of Cartan geometry. It is always taught as the attractive force being counterbalanced by the centrifugal force. However that is an anthropomorphic explanation. It is really an equation of geometry. Obviously the Coats and Newton force laws and orbits are completely different. The Newton orbit was discovered by Kepler in the early seventeenth century, the velocity curve of the whirlpool galaxy in the late nineteen fifties. It has taken until ECE theory to realize that the whirlpool galaxy is a vivid manifestation of the Coats orbit, and that both are manifestations of Cartan geometry.

From Eq. ( 86 ):

$$\frac{dr}{d\theta} = \epsilon r^2 \sin \theta \quad - (90)$$

in which:

$$\sin^2 \theta = 1 - \cos^2 \theta = 1 - \frac{1}{\epsilon^2} \left( \frac{dr}{r} - 1 \right)^2 \quad - (91)$$

so the Newtonian linear velocity is:

$$v^2 = \frac{M G}{r} \left( \frac{2}{r} - \frac{1}{a} \right) \quad - (92)$$

The details of the calculation are given in note 263(5). Here  $a$  is the semi major axis of the

ellipse:

$$a = \frac{\alpha}{1 - \epsilon^2} \quad - (93)$$

Note that:

$$\frac{1}{a} = \frac{1 - \epsilon^2}{\alpha} = \frac{1}{r} (1 + \epsilon \cos \theta) (1 - \epsilon^2) \quad - (94)$$

so:

$$v^2(\text{Newton}) = \frac{M G}{r} (2 - (1 - \epsilon^2) (1 + \epsilon \cos \theta)) \quad - (95)$$

It follows that:

$$v(\text{Newton}) \xrightarrow{r \rightarrow \infty} 0 \quad - (96)$$

because:

$$-1 \leq \cos \theta \leq 1 \quad - (97)$$

and

$$0 < \epsilon < 1. \quad - (98)$$

So the Newton theory fails completely to describe the velocity curve of a whirlpool galaxy because experimentally:

$$v \xrightarrow{r \rightarrow \infty} v_{\infty} = \text{constant} \quad - (99)$$

It is of great importance to note that the Einstein theory does no better. The Einstein theory was designed to produce a tiny precession of the elliptical orbit of Newton, because at the time of development of Einsteinian general relativity (1905 - 1915) it was thought that

Newton was adequate for all astronomy with tiny anomalies, the precession being one of them. In the decade 1905 to 1915 whirlpool galaxies were unknown, galaxies looked like amorphous objects with the telescopes of the time. Cartan geometry was unknown, spacetime torsion was unknown, and the flaws in Riemann geometry also unknown. As x theory shows, the Einstein theory produces:

$$r = \frac{\alpha}{1 + \epsilon \cos \alpha \theta}, \quad \alpha = 1 + \frac{r_0}{\alpha} \quad - (100)$$

with:

$$r_0 \ll \alpha. \quad - (101)$$

However, Einstein hit on the right result with entirely the wrong geometry. Using Eq. (100)

in Eq. (65) gives {1 - 10}:

$$v^2(\text{Einstein}) = \left( \frac{L}{mr^2} \right) \left( 1 + \left( \frac{\alpha \epsilon \sin(\alpha \theta)}{1 + \epsilon \cos(\alpha \theta)} \right)^2 \right) \quad - (102)$$

so:

$$v(\text{Einstein}) \xrightarrow{r \rightarrow \infty} 0 \quad - (103)$$

which is completely wrong, not just wrong. Eq. (103) follows from:

$$\begin{aligned} -1 &\leq \sin(\alpha \theta) \leq 1 \\ -1 &\leq \cos(\alpha \theta) \leq 1 \\ 0 &< \epsilon < 1 \end{aligned} \quad - (104)$$

The right result is given by the force law (81) of the hyperbolic Coats spiral. This is completely different from the Einstein force law:

$$\underline{F} = \left( -\frac{mM\epsilon}{r^2} - \frac{3M\epsilon L^2}{mc^2 r^4} \right) \underline{e}_r \quad - (105)$$

and completely different from the Newton force law:

$$\underline{F} = -\frac{mM_G}{r^2} \underline{e}_r \quad - (106)$$

Only ECE theory can describe the solar system through R and x theory, and also the main features of whirlpool galaxies from fundamental plane polar geometry, a Cartan geometry with the angular velocity being the spin connection.

### 3. NUMERICAL ANALYSIS OF LIGHT DEFLECTION DUE TO GRAVITATION

Section by Dr. Horst Eckardt.

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# Self consistent calculation of perihelion precession and light deflection due to gravitation with the R and x theories of Cartan geometry

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[www.atomicprecision.com](http://www.atomicprecision.com), [www.upitec.org](http://www.upitec.org))

## 3 Numerical analysis of light deflection due to gravitation

First we investigate the derivatives of  $R$  in some more detail. The acceleration of the radius coordinate  $R$  is given by

$$\frac{d^2 R}{dt^2} = F(R) + m R \omega^2. \quad (107)$$

The left hand side can be rewritten by

$$\frac{d^2 R}{dt^2} = \frac{d^2 R}{d\theta^2} \frac{d^2 \theta}{dt^2} = \frac{d^2 R}{d\theta^2} \frac{d\omega}{dt} = \frac{d^2 R}{d\theta^2} \frac{d\omega}{d\theta} \frac{d\theta}{dt} = \frac{d^2 R}{d\theta^2} \frac{d\omega}{d\theta} \omega \quad (108)$$

and the force is

$$F(R) = -\frac{mMG}{R^2} = -\frac{L^2}{\alpha m R^2} \quad (109)$$

with

$$\alpha = \frac{L^2}{m^2 GM} \quad (110)$$

so that in total

$$\frac{d^2 R}{d\theta^2} \frac{d\omega}{d\theta} \omega = -\frac{L^2}{\alpha m R^2} + m R \omega^2. \quad (111)$$

Using

$$\omega = \frac{L}{mR^2}, \quad \frac{d\omega}{d\theta} = -\frac{2L}{mR^3} \frac{dR}{d\theta} \quad (112)$$

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we obtain

$$-2 \frac{L^2}{m^2 R^3} \frac{d^2 R}{d\theta^2} \frac{dR}{d\theta} = \frac{L^2}{mR^3} - \frac{L^2}{\alpha m R^2} \quad (113)$$

which can be simplified to

$$\frac{d^2 R}{d\theta^2} \frac{dR}{d\theta} = \frac{mR^2}{2\alpha} (R - \alpha). \quad (114)$$

The right hand side of Eq.(113) represents the second time derivative of  $R$  or radial acceleration component. This is plotted for an ellipse

$$R(\theta) = \frac{\alpha}{1 + \epsilon \cos(\theta)} \quad (115)$$

in Fig. 3. The zero crossings (turning points) can be seen to appear for  $\theta = \pi/2$  and  $\theta = 3\pi/2$  as predicted. These zero crossings (where  $R = \alpha$ ) can also directly be seen from Eq.(114).

Computing the derivative  $d^2 R/d\theta^2$  alone does not make much sense since the first derivative of  $R$  has zeros at  $\theta = 0, \pi$ . These are the radii for perihelion and aphelion. If the first derivative is moved to the right hand side of (114),  $d^2 R/d\theta^2$  goes to infinity at these points. Exactly this can be seen from Fig. 4.

Now we apply the theory of light deflection to the solar system. Compared to UFT 155, the integral for light deflection, Eq.(42), can be solved analytically. The result is

$$\Delta\theta = \frac{2}{\alpha} (r_0 - \alpha) \left( \text{asin} \left( \frac{R_0 - \alpha}{\epsilon R_0} \right) - \text{asin} \left( \frac{1}{\epsilon} \right) \right) \quad (116)$$

The Integral has been evaluated in dependence of  $\alpha$  for the case of light deflection by the sun. The upper and lower bounds of the integral (called  $I1$  and  $I0$ ) are plotted in Fig. 5, together with their difference which appears in the angle of deflection. Obviously the integral does not exist for the upper bound below a certain  $\alpha$  as already obtained numerically in paper 155. For higher values of  $\alpha$  there is no significant dependence any more.

In Fig. 6 the angle of deflection is compared with the experimental value. The corresponding  $\alpha$  was computed by a numerical root finding method. The result is

$$\alpha = 1.639992 \cdot 10^{14} \text{ m} \quad (117)$$

while the radius of the earth orbit is only about  $10^{11}$  m. This is a quite large value of  $\alpha$ . The corresponding eccentricity of the hyperbola is

$$\epsilon = 235735.06 \quad (118)$$

which is quite huge too. However this is a consistent result because the light orbit is nearly a straight line which would correspond to an infinite  $\epsilon$ . The angle of light deflection is directly connected by  $\epsilon$  via

$$\Delta\theta = \frac{2}{\epsilon} = 8.4840 \cdot 10^{-6} \quad (119)$$

which is exactly the experimental value again, q.e.d.

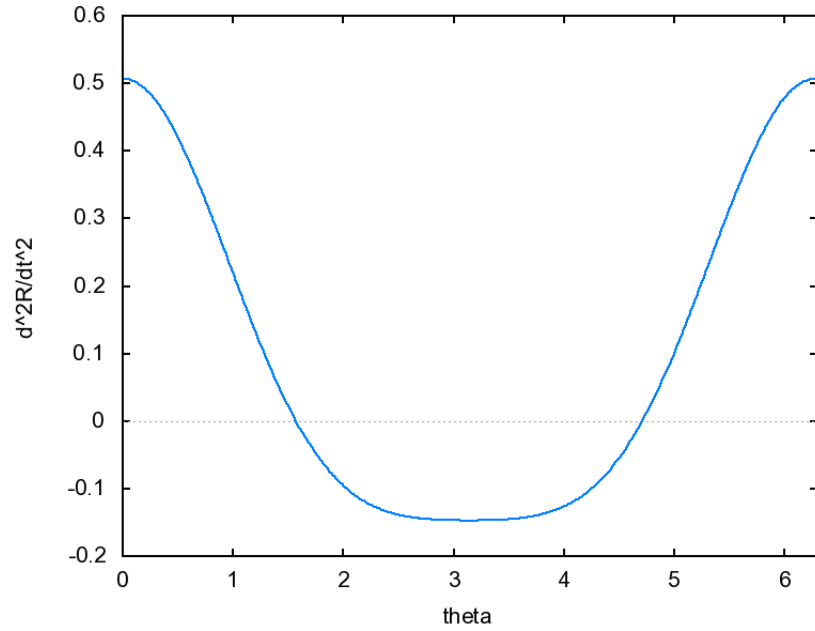


Figure 3: Second derivative of  $R(t)$  for  $m = \alpha = L = 1$ .

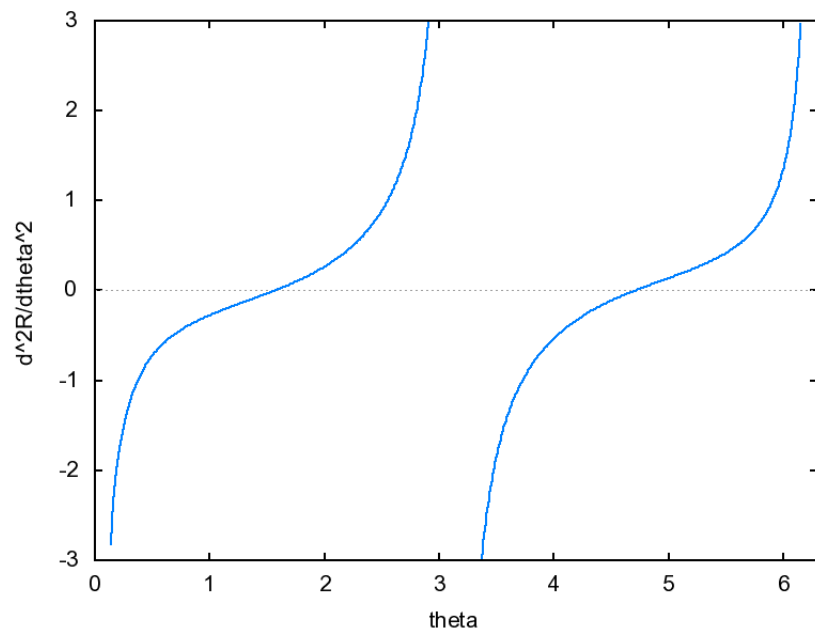


Figure 4: Second derivative of  $R(\theta)$  for  $m = \alpha = L = 1$ .



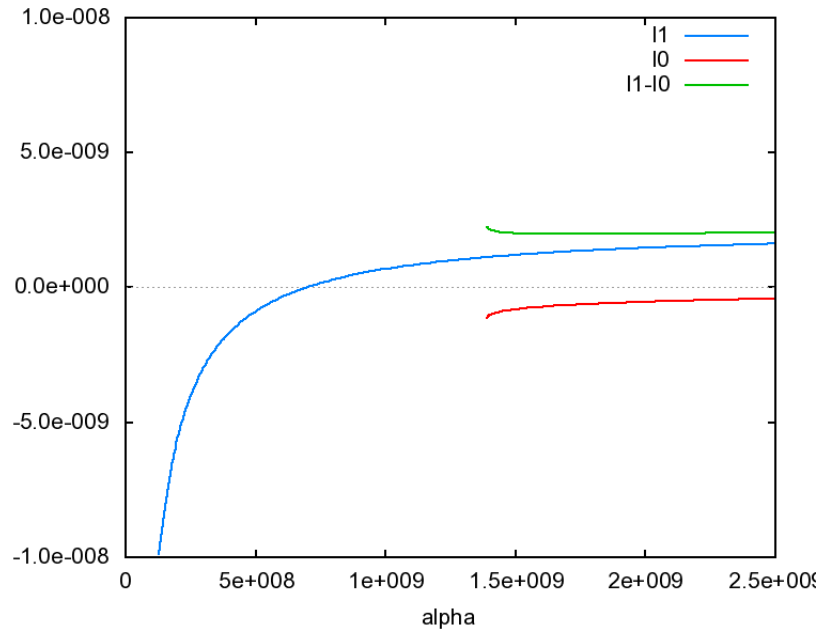


Figure 5: Integral values of Eq.(42) for two radius values and their difference (parameters for the solar system).

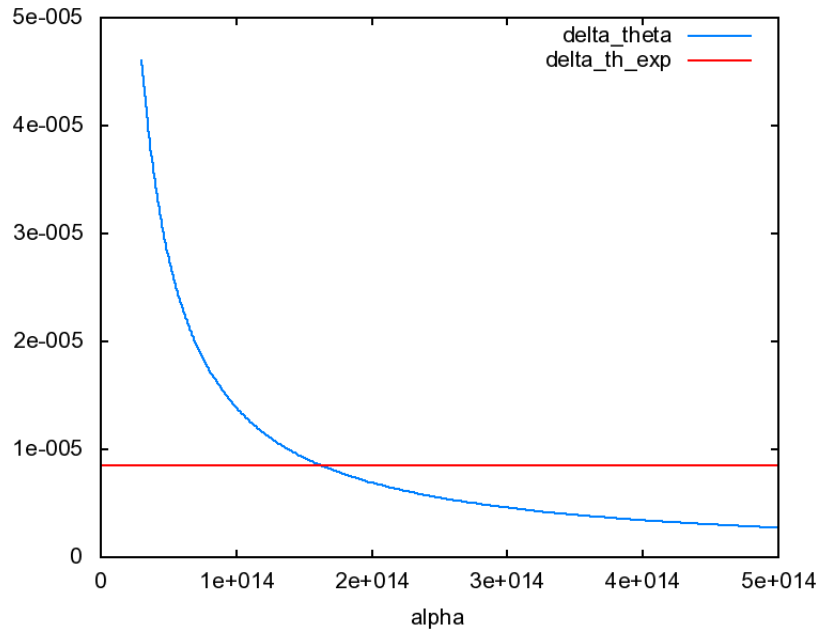


Figure 6:  $\alpha$  dependence of Eq.(116) for two radius values and their difference (parameters for the solar system).