NON NEWTONIAN VELOCITY AND ACCELERATIONS IN SPHERICAL POLAR COORDINATES: ORBITAL THEORY IN THREE DIMENSIONS.

by

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ABSTRACT

The non Newtonian velocity and accelerations of a three dimensional orbit are calculated in spherical polar coordinates. These are the orbital linear velocity, the centrifugal and Coriolis accelerations, and the acceleration due to the time derivative of angular velocity. Two constraint equations are deduced and orbits of various kinds evaluated from these constraint equations. Some of the properties of the constraint equations show structure similar to atomic orbitals, but structure occurring in macroscopic orbits. If the angular momentum is constrained to be in the Z axis the orbit approaches a planar precessing ellipse.

4FT271

1. INTRODUCTION

In recent papers of this series the theory of three dimensional orbits has been initiated using the spherical polar coordinates {1 - 10}. It has been shown that there is no reason why the orbit from the inverse square law of attraction should be a planar ellipse. In general it is three dimensional and has a rich panoply of properties which can be demonstrated with three dimensional graphics. In Section 2 this development is continued by evaluating the non Newtonian properties of a three dimensional orbit: the orbital linear velocity and the three non Newtonian accelerations. These are the centrifugal and Coriolis accelerations and the acceleration due to the time derivative of angular velocity. Consideration of the non Newtonian accelerations leads to two novel constraint equations which can be solved to give new types of orbits. When graphed in three dimensions these orbits display a macroscopic structure similar to atomic and molecular orbitals on the microscopic scale. Three dimensional orbits exist in general and can be searched for in astronomy. In Section 3 a selection of graphics is given with analysis of the main features. All the calculations of this paper are checked with computer algebra as for all UFT papers.

2. THE NON NEWTONIAN PROPERTIES OF THREE DIMENSIONAL ORBITS.

As usual this paper should be read in conjunction with its background notes accompanying UFT271 on www.aias.us. Notes 271(1) to 271(3) give some background analysis in preparation for Note 271(4), which is a review of the calculation of the non Newtonian velocity and accelerations in plane polar coordinates carried out in previous papers of this series. Note 271(5) gives detail which is summarized in this section.

It ha been shown in the immediately preceding UFT papers that the linear velocity in spherical polar coordinates {11} is:

$$\underline{v} = \dot{r}\,\underline{e}_r + r\dot{\theta}\,\underline{e}_\theta + r\dot{\phi}\sin\theta\,\underline{e}_\phi \quad -(1)$$

where the unit vectors of the spherical polar coordinate system are defined {12} by:

$$\underline{e}_r = \sin\theta\cos\phi\,\underline{i} + \sin\theta\sin\phi\,\underline{j} + \cos\theta\,\underline{k} \quad -(2)$$

$$\underline{e}_\theta = \cos\theta\cos\phi\,\underline{i} + \cos\theta\sin\phi\,\underline{j} - \sin\theta\,\underline{k} \quad -(3)$$

$$\underline{e}_\phi = -\sin\phi\,\underline{i} + \cos\phi\,\underline{j} \quad -(4)$$

forming the cyclically symmetric set of equations:

$$\underline{e}_\phi \times \underline{e}_r = \underline{e}_\theta \quad -(5)$$

$$\underline{e}_\theta \times \underline{e}_\phi = \underline{e}_r \quad -(6)$$

$$\underline{e}_r \times \underline{e}_\theta = \underline{e}_\phi . \quad -(7)$$

Eq. (1) can be written as:

$$\underline{v} = \frac{d\underline{r}}{dt} + \frac{dr}{dt}\,\underline{e}_r + \underline{\omega} \times \underline{r} \quad -(8)$$

where the position vector is:

$$\underline{r} = r\,\underline{e}_r \quad -(9)$$

and where the angular velocity or Cartan spin connection {1 - 10} is

$$\underline{\omega} = \dot{\theta}\,\underline{e}_\phi - \dot{\phi}\sin\theta\,\underline{e}_\theta . \quad -(10)$$

Therefore this theory is part of ECE unified field theory based on Cartan geometry. The linear acceleration is therefore:

$$\underline{a} = \frac{d\underline{v}}{dt} = \frac{d}{dt}\left(\dot{r}\,\underline{e}_r + \underline{\omega} \times \underline{r}\right) \quad -(11)$$

$$= \ddot{r}\,\underline{e}_r + \dot{r}\,\dot{\underline{e}}_r + \dot{\underline{\omega}} \times \underline{r} + \underline{\omega} \times \frac{d\underline{r}}{dt} \quad -(12)$$

Now use {12}:

$$\dot{\underline{e}}_r = \dot{\theta}\,\underline{e}_\theta + \dot{\phi}\sin\theta\,\underline{e}_\phi \quad -(13)$$

From Eqs. (1) and (10):

$$\underline{\omega} \times \frac{d\underline{r}}{dt} = \underline{\omega} \times \underline{v} = \dot{r}\dot{\theta}\,\underline{e}_\theta + \dot{r}\dot{\phi}\sin\theta\,\underline{e}_\phi - \left(r\dot{\phi}^2\sin^2\theta + r\dot{\theta}^2\right)\underline{e}_r \quad -(14)$$

From Eqs. (9) and (10):

$$\underline{\omega} \times (\underline{\omega} \times \underline{r}) = -\left(r\dot{\phi}^2\sin^2\theta + r\dot{\theta}^2\right)\underline{e}_r. \quad -(15)$$

It follows that:

$$\underline{\omega} \times \underline{v} = \underline{\omega} \times (\underline{\omega} \times \underline{r}) + \dot{r}\dot{\theta}\,\underline{e}_\theta + \dot{r}\dot{\phi}\sin\theta\,\underline{e}_\phi. \quad -(16)$$

From Eqs. (12), (13) and (16):

$$\underline{a} = \ddot{r}\,\underline{e}_r + \underline{\omega} \times (\underline{\omega} \times \underline{r}) + \dot{\underline{\omega}} \times \underline{r} + 2\left(\dot{r}\dot{\theta}\,\underline{e}_\theta + \dot{r}\dot{\phi}\sin\theta\,\underline{e}_\phi\right). \quad -(17)$$

Now note that:

$$\underline{\omega} \times \frac{dr}{dt}\,\underline{e}_r = \dot{r}\dot{\theta}\,\underline{e}_\theta + \dot{r}\dot{\phi}\sin\theta\,\underline{e}_\phi \quad -(18)$$

so the linear acceleration in spherical polar coordinates is:

$$\underline{a} = \ddot{r}\,\underline{e}_r + \underline{\omega} \times (\underline{\omega} \times \underline{r}) + \dot{\underline{\omega}} \times \underline{r} + 2\,\underline{\omega} \times \frac{dr}{dt}\,\underline{e}_r \quad -(19)$$

The main results of this analysis are as follows.

1) The angular velocity or Cartan spin connection is:

$$\underline{\omega} = \dot{\theta}\,\underline{e}_\phi - \dot{\phi}\sin\theta\,\underline{e}_\theta$$

$$= \frac{L}{mr^2} \quad -(20)$$

where $\underline{L}$ is the total angular momentum in three dimensions:

$$\underline{L} = L_X \underline{i} + L_Y \underline{j} + L_Z \underline{k} \quad -(21)$$

with three Cartesian components in general.

2) The orbital linear velocity is:

$$\underline{v}_{orbital} = \underline{\omega} \times \underline{r} = \dot{r} \underline{e}_r + r \dot{\theta} \underline{e}_\theta + r \dot{\phi} \sin\theta \, \underline{e}_\phi \quad -(22)$$

and is three dimensional in general. To convert to Cartesian unit vectors use Eqs. (2) to (4).

3) The centrifugal acceleration is:

$$\underline{\omega} \times (\underline{\omega} \times \underline{r}) = -\left(r \dot{\phi}^2 \sin^2\theta + r \dot{\theta}^2 \right) \underline{e}_r \quad -(23)$$

and has three Cartesian coordinates in general.

4) The Coriolis acceleration is:

$$\underline{a}(\text{Coriolis}) = 2 \underline{\omega} \times \frac{dr}{dt} \underline{e}_r = 2\left(\dot{r} \dot{\theta} \underline{e}_\theta + \dot{r} \dot{\phi} \sin\theta \, \underline{e}_\phi \right) \quad -(24)$$

and this is also three dimensional in general.

5) The third type of non - Newtonian acceleration is:

$$\underline{a}_3 = \dot{\underline{\omega}} \times \underline{r} \quad -(25)$$

and can be evaluated using:

$$\dot{\underline{\omega}} = \frac{d}{dt}\left(\dot{\theta}\,\underline{e}_\phi - \dot{\phi}\sin\theta\,\underline{e}_\theta\right) \qquad -(26)$$

$$= \ddot{\theta}\,\underline{e}_\phi + \dot{\theta}\,\dot{\underline{e}}_\phi - \ddot{\phi}\sin\theta\,\underline{e}_\theta - \dot{\phi}\,\dot{\theta}\cos\theta\,\underline{e}_\theta - \dot{\phi}\sin\theta\,\dot{\underline{e}}_\theta$$

Now consider the following fundamental relations given in reference (12):

$$\dot{\underline{e}}_r = \dot{\theta}\,\underline{e}_\theta + \sin\theta\,\dot{\phi}\,\underline{e}_\phi \qquad -(27)$$

$$\dot{\underline{e}}_\theta = -\dot{\theta}\,\underline{e}_r + \dot{\phi}\cos\theta\,\underline{e}_\phi \qquad -(28)$$

$$\dot{\underline{e}}_\phi = -\sin\theta\,\dot{\phi}\,\underline{e}_r - \dot{\phi}\cos\theta\,\underline{e}_\theta \qquad -(29)$$

It follows that the angular acceleration is:

$$\dot{\underline{\omega}} = \left(\ddot{\theta} - \dot{\phi}^2\sin\theta\cos\theta\right)\underline{e}_\phi - \left(2\dot{\phi}\dot{\theta}\cos\theta + \ddot{\phi}\sin\theta\right)\underline{e}_\theta \qquad -(30)$$

and:

$$\dot{\underline{\omega}} \times \underline{r} = r\left(\ddot{\theta} - \dot{\phi}^2\sin\theta\cos\theta\right)\underline{e}_\theta + r\left(\ddot{\phi}\sin\theta + 2\dot{\theta}\dot{\phi}\cos\theta\right)\underline{e}_\phi .$$

So this acceleration is again non Newtonian in general.

The Leibniz orbital equation is therefore:

$$m\ddot{r}\,\underline{e}_r = -m\left(\underline{\omega}\times(\underline{\omega}\times\underline{r}) + 2\underline{\omega}\times\frac{dr}{dt}\underline{e}_r + \dot{\underline{\omega}}\times\underline{r}\right) - \frac{k}{r^2}\underline{e}_r \qquad -(31)$$

and applies equally well to macroscopic orbits of a mass m around a mass M, or to the motion of an electron around a proton. In the former case:

$$k = mMG \qquad -(32)$$

where G is Newton's constant. In the latter case:

$$k = \frac{e^2}{4\pi\epsilon_0} \qquad -(33)$$

where e is the charge on the proton and where ϵ_0 is the vacuum permittivity.

Eq. (31) splits into two equations:

$$m\ddot{r} = r\left(\dot{\theta}^2 + \dot{\phi}^2 \sin^2\theta\right) - \frac{k}{r^2} \quad -(34)$$

and:

$$2\underline{\omega} \times \frac{dr}{dt}\underline{e}_r + \dot{\underline{\omega}} \times \underline{r} = 0. \quad -(35)$$

Eq. (34) is true for each component of $\underline{e}_r$, with:

$$m\ddot{r}\,\underline{e}_r = m\ddot{r}\left(\sin\theta\cos\phi\,\underline{i} + \sin\theta\sin\phi\,\underline{j} + \cos\theta\,\underline{k}\right) \quad -(36)$$

showing in another way that the orbit is three dimensional. The orbit is constrained by Eq. (35), which gives:

$$2\left(\dot{r}\dot{\theta}\,\underline{e}_\theta + \dot{r}\dot{\phi}\sin\theta\,\underline{e}_\phi\right) + r\left(\ddot{\theta} - \dot{\phi}^2\sin\theta\cos\theta\right)\underline{e}_\theta + r\left(\ddot{\phi}\sin\theta + 2\dot{\theta}\dot{\phi}\cos\theta\right)\underline{e}_\phi = \underline{0} \quad -(37)$$

and two constraint equations:

$$r\ddot{\theta} - r\dot{\phi}^2\sin\theta\cos\theta + 2\dot{r}\dot{\theta} = 0 \quad -(38)$$

and:

$$r\ddot{\phi}\sin\theta + 2r\dot{\theta}\dot{\phi}\cos\theta + 2\dot{r}\dot{\phi}\sin\theta = 0. \quad -(39)$$

From the immediately preceding paper UFT270:

$$\dot{\phi} = \frac{L_Z}{mr^2\sin^2\theta} \quad -(40)$$

and:

$$\dot{\theta} = \frac{1}{mr^2}\left(L^2 - \frac{L_Z^2}{\sin^2\theta}\right)^{1/2} \qquad -(41)$$

where L_Z is the Z component of the total angular momentum $\underline{L}$.

The angular momentum in three dimensions is defined by:

$$\underline{L} = \underline{r} \times \underline{p} \qquad -(42)$$

and conservation of angular momentum implies that:

$$\frac{d\underline{L}}{dt} = \underline{0}. \qquad -(43)$$

However:

$$\frac{d\underline{L}}{dt} = \underline{r} \times \underline{p} + \underline{r} \times \dot{\underline{p}} \qquad -(44)$$

so conservation of angular momentum implies that there is no net torque:

$$\underline{Tq} = \frac{d\underline{L}}{dt} = m\,\underline{r} \times \underline{a} = \underline{0}. \qquad -(45)$$

The acceleration is:

$$\underline{a} = \ddot{r}\,\underline{e}_r + \underline{\omega} \times (\underline{\omega} \times \underline{r}) + \dot{\underline{\omega}} \times \underline{r} + 2\underline{\omega} \times \frac{dr}{dt}\underline{e}_r \qquad -(46)$$

with:

$$\dot{\underline{\omega}} \times \underline{r} + 2\underline{\omega} \times \frac{dr}{dt}\underline{e}_r = \underline{0} \qquad -(47)$$

so the acceleration is:

$$\underline{a} = \ddot{r}\,\underline{e}_r + \dot{\underline{\omega}} \times (\underline{\omega} \times \underline{r})$$
$$= \left(\ddot{r} - \left(r\dot{\theta}^2 + r\sin^2\theta\,\dot{\phi}^2\right)\right)\underline{e}_r \quad -(48)$$

implying that:

$$\underline{r} \times \underline{a} = \underline{0} \quad -(49)$$

QED. The analysis is therefore self consistent.

In terms of Cartesian unit vectors {1 - 12}:

$$\underline{r} = r\,\underline{e}_r = r\left(\sin\theta\cos\phi\,\underline{i} + \sin\theta\sin\phi\,\underline{j} + \cos\theta\,\underline{k}\right)$$
$$= X\underline{i} + Y\underline{j} + Z\underline{k} \quad -(50)$$

and

$$\underline{p} = m\Big(\left(\dot{r}\sin\theta\cos\phi + r\cos\theta\cos\phi\,\dot{\theta} - r\sin\theta\sin\phi\,\dot{\phi}\right)\underline{i}$$
$$+ \left(\dot{r}\sin\theta\sin\phi + r\cos\theta\sin\phi\,\dot{\theta} + r\sin\theta\cos\phi\,\dot{\phi}\right)\underline{j}$$
$$+ \left(\dot{r}\cos\theta - r\dot{\theta}\sin\theta\right)\underline{k}\Big) = p_x\underline{i} + p_y\underline{j} + p_z\underline{k} \quad -(51)$$

So the angular momentum is parallel to the angular velocity:

$$\underline{L} = mr^2\underline{\omega} = mr^2\left(\dot{\theta}\,\underline{e}_\phi - \dot{\phi}\,\underline{e}_\theta\right)$$
$$= mr^2\Big(\left(-\dot{\theta}\sin\phi + \dot{\phi}\sin\theta\cos\theta\cos\phi\right)\underline{i} \quad -(52)$$
$$+ \left(\dot{\theta}\cos\phi - \dot{\phi}\sin\theta\cos\theta\sin\phi\right)\underline{j}$$
$$+ \dot{\phi}\sin^2\theta\,\underline{k}\Big) = L_x\underline{i} + L_y\underline{j} + L_z\underline{k}.$$

Therefore neither $\underline{r}$ nor $\underline{p}$ are confined in general to the XY plane, and $\underline{L}$ is not confined to the Z axis perpendicular to this plane. The usual theory of orbits {11} begins with these very restrictive and unjustifiable assumptions. The latter are equivalent to assuming that:

$$\theta = \pi / 2 \quad -(53)$$

$$\dot{\theta} = 0 \quad -(54)$$

when the analysis reduces to:

$$\underline{r} = r\left(\cos\phi\,\underline{i} + \sin\phi\,\underline{j}\right) \quad -(55)$$

$$\underline{p} = m\left(\left(\dot{r}\cos\phi - r\dot{\phi}\sin\phi\right)\underline{i} + \left(\dot{r}\sin\phi + r\dot{\phi}\cos\phi\right)\underline{j}\right) \quad -(56)$$

where:

$$\underline{L} = \underline{r} \times \underline{p} = -mr^2\dot{\phi}\,\underline{e}_\theta \quad -(57)$$

Using Eq. (3) the usual angular momentum of planar orbital theory is obtained:

$$\underline{L} = mr^2\dot{\phi}\,\underline{k} \quad \text{for } \theta = \frac{\pi}{2} \quad -(58)$$

QED. Remarkably, the restrictive assumptions (53, 54) have been the basis of orbital theory for over four hundred years, and so most of the information about orbits has never been analyzed.

Consider the three following equations which determine all three dimensional orbits due to the inverse square law:

$$m\ddot{r} = r\left(\dot{\theta}^2 + \dot{\phi}^2\sin^2\theta\right) - \frac{k}{r^2} \quad -(59)$$

$$r\ddot{\theta} - r\dot{\phi}^2\sin\theta\cos\theta + 2\dot{r}\dot{\theta} = 0 \quad -(60)$$

$$r\ddot{\phi}\sin\theta + 2r\dot{\theta}\dot{\phi}\cos\theta + 2\dot{r}\dot{\phi}\sin\theta = 0 \quad -(61)$$

In two dimensions they become:

$$m\ddot{r} = r\dot{\phi}^2 - k / r^2 \quad -(62)$$

$$0 = 0 \quad -(63)$$

$$r\ddot{\phi} + 2\dot{r}\dot{\phi} = 0 \quad -(64)$$

and Eq. (64) has been deduced in previous UFT papers {1 - 10}. In two dimensions:

$$\dot{\phi} = \frac{L}{mr^2} \quad -(65)$$

so

$$\ddot{\phi} = \frac{d\dot{\phi}}{dt} = \frac{d\dot{\phi}}{dr}\frac{dr}{dt} = -\frac{2L}{mr^3}\dot{r} \quad -(66)$$

and Eq. (64) follows immediately, QED.

Consider now Eq. (61) in three dimensions. Eq. (66) can no longer be used because $\dot{\phi}$ is a function of r and θ, and these are not independent variables. Similarly $\dot{\theta}$ is a function of r and ϕ, which again are not independent variables. So in three dimensions Eqs. (60) and (61) are simultaneous differential equations. As the orbit approaches the planar orbit:

$$\theta \rightarrow \frac{\pi}{2} \quad -(67)$$

and

$$r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 \quad -(68)$$

$$r\ddot{\phi} + 2\dot{r}\dot{\phi} = 0 \quad -(69)$$

and Eqs. (60) and (61) simplify to:

$$\dot{\phi} \rightarrow \frac{L_Z}{mr^2}, \quad \dot{\theta} \rightarrow \frac{\left(L^2 - L_Z^2\right)^{1/2}}{mr^2}. \quad -(70)$$

It is possible to evaluate the angular accelerations $\ddot{\theta}$ and $\ddot{\phi}$ as follows, and these are graphed in Section 3. They show very interesting orbital type structure found in atoms and molecules, but they are macroscopic in nature. They are given by the following set of equations derived in the immediately preceding papers:

$$\ddot{\theta} = \dot{\phi}^2 \sin\theta \cos\theta - \frac{2}{r} \dot{r} \dot{\theta} \quad -(71)$$

$$\ddot{\phi} = -2 \left(\frac{\dot{\theta} \dot{\phi}}{\tan\theta} + \frac{\dot{r} \dot{\phi}}{r} \right) \quad -(72)$$

where the beta orbit is:

$$r = \frac{\alpha}{1 + \epsilon \cos\beta} \quad -(73)$$

with:

$$\sin\beta = - \frac{L}{(L^2 - L_Z^2)^{1/2}} \cos\theta \quad -(74)$$

The half right latitude α and eccentricity ϵ of the beta orbit are:

$$\alpha = \frac{L^2}{mk}, \quad \epsilon^2 \quad 1 + \frac{2EL^2}{mk^2} \quad -(75)$$

where E is the total energy and L is the magnitude of the total angular momentum.

The angular velocities are given by Eqs. (40) and (41) and the quantity $\dot{r}$ is given by the hamiltonian:

$$H = E = \frac{1}{2} m \dot{r}^2 + \frac{L^2}{2mr^2} - \frac{k}{r} \quad -(76)$$

so:

$$\dot{r} = \left(\frac{2}{m} \left(E - \frac{L^2}{2mr^2} + \frac{k}{r} \right) \right)^{1/2} \quad -(77)$$

Using these equations both $\ddot{\phi}$ and $\ddot{\theta}$ can be worked out in terms of ϕ and θ. The graphs of $\ddot{\theta}$ against θ and of $\ddot{\phi}$ against ϕ characterize the three dimensional orbit as do the graphs of r against θ and r against ϕ. For a planar orbit:

$$\ddot{\theta} = 0 \quad -(78)$$

and:

$$\ddot{\phi} = -\frac{2\dot{r}\dot{\phi}}{r} \quad -(79)$$

where $\dot{r}$ is given by Eq. (77) and where:

$$r = \frac{\alpha}{1 + \epsilon\cos\phi} \quad -(80)$$

with:

$$\dot{\phi} = \frac{L_Z}{mr^2} \quad -(81)$$

So $\ddot{\phi}$ can be graphed against ϕ for a planar orbit and compared directly with the result for a three dimensional orbit for the same inverse square law.

The relation between β and ϕ is:

$$\tan\beta = \frac{L}{L_Z}\tan\phi \quad -(82)$$

and the relation between β and θ is:

$$\sin\beta = -\frac{L}{(L^2 - L_Z^2)^{1/2}}\cos\theta \quad -(83)$$

so ϕ can be expressed in terms of θ and vice versa. As the planar limit is approached:

$$L \rightarrow L_Z \quad -(84)$$

and from Eq. (82):

$$\beta \rightarrow \phi \quad -(85)$$

QED. The equation (73) correctly reduces to the planar limit showing in another way that the theory is self consistent. Using the Maclaurin expansion Eq. (82) becomes:

$$\beta + \frac{\beta^3}{3} + \frac{2}{15}\beta^5 + \dots = \frac{L}{L_Z}\left(\phi + \frac{\phi^3}{3} + \frac{2}{15}\phi^5 + \dots\right) \quad -(86)$$

and from Eqs. (84) and (85) the following results are obtained as the planar orbit is approached:

$$\beta \rightarrow \frac{L}{L_Z}\phi \quad -(87)$$

$$\frac{\beta^3}{3} \rightarrow \frac{L}{L_Z}\frac{\phi^3}{3} \quad -(88)$$

and so on within the limits of the Maclaurin expansion:

$$\beta < 1, \quad \phi < 1. \quad -(89)$$

The three dimensional beta ellipse approaches a planar precessing ellipse:

$$r = \frac{\alpha}{1 + \epsilon\cos\left(\frac{L}{L_Z}\phi\right)} \quad -(90)$$

which becomes a static planar ellipse of and only if L_Z becomes L exactly. Planetary orbits are well known to be precessing ellipses, suggesting that they are remnants of three dimensional beta ellipses.

Non-Newtonian velocity and accelerations in spherical polar coordinates: orbital theory in three dimensions

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3 Graphics and analysis

In this section we discuss orbits, radial and angular velocities and accelerations in spherical polar coordinates. The elliptic orbit is given by Eq.(73), the angular accelerations are defined in (71) and (72) and the angular velocities by (40) and (41). The radial acceleration is given by (34) and the radial velocity by (77). The angular quantities are independent from the potential while the radial quantities depend on it. All orbits can be described by θ or ϕ. Both coordinates can be transformed into one another via

$$\phi = -\frac{1}{2}\left(\operatorname{asin}\left(\frac{(\cos(\theta)+1)\,L^2-{L_Z}^2}{|\cos(\theta)+1|\,\sqrt{L^4-{L_Z}^2\,L^2}}\right)\right. \tag{91}$$

$$\left.+\operatorname{asin}\left(\frac{(\cos(\theta)-1)\,L^2+{L_Z}^2}{|\cos(\theta)-1|\,\sqrt{L^4-{L_Z}^2\,L^2}}\right)\right),$$

$$\theta = \pi - \operatorname{acos}\left(\frac{\tan(\phi)\,\sqrt{L^2-{L_Z}^2}}{\sqrt{\tan(\phi)^2\,L^2+{L_Z}^2}}\right). \tag{92}$$

The derived coordinate β can be computed from ϕ and θ by

$$\beta(\phi) = \operatorname{atan}\left(\frac{\tan(\phi)\,L}{L_Z}\right), \tag{93}$$

$$\beta(\theta) = -\operatorname{asin}\left(\frac{\cos(\theta)\,L}{\sqrt{L^2-{L_Z}^2}}\right). \tag{94}$$

The orbit is given by

$$r = \frac{\alpha}{1+\epsilon\cos(\beta)} \tag{95}$$

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with α and ϵ defined by Eq.(75).

We first discuss the orbital surfaces in dependence of ϕ. Figs. 1-3 show the radial orbital surface and its first and second time derivative. The orbital ellipses of the derivatives are carved due to the zero crossings there. The θ derivatives (Figs. 4-6) show an atomic-like orbital structure with lobes of differing size. The ϕ derivatives (Fig. 6-7) show both forms.

The θ surfaces are mostly ellipsoids and tori which were already shown in earlier papers. The surface of $\dot{r}$ is a double ellipsoid (Fig. 8) and $\ddot{\theta}$ is open at one side like a chalice (Fig. 9). This structure is shown as a standard plot $\ddot{\theta}(\theta)$ in Fig. 10 as an example. The kink at $\theta = \pi/2$ may be caused by artifacts of inverse trigonometric functions. For the sufaces shown it should be noted that only positive values can be displayed in the 3D plots. The graphics program partially converts negative to positive values. A more thorough analysis requires looking at the standard diagrams but is not so illustrative.

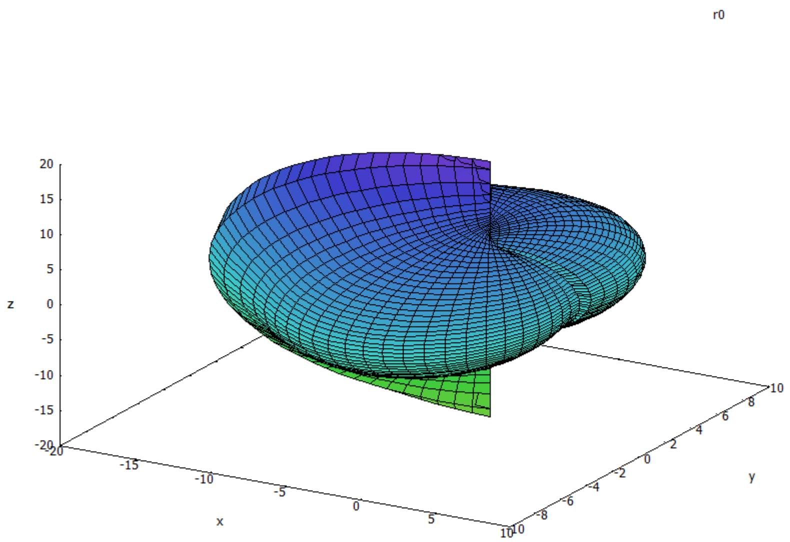

Figure 1: ϕ dependence of r.

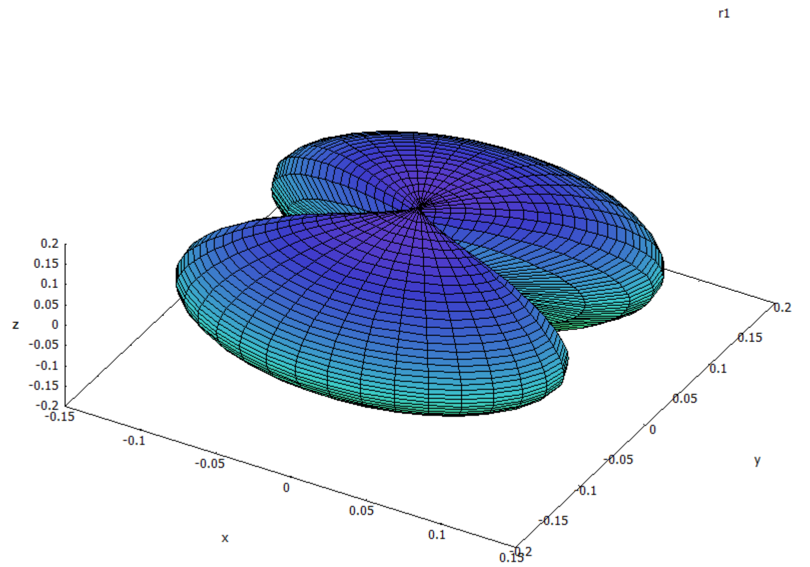

Figure 2: ϕ dependence of $\dot{r}$.

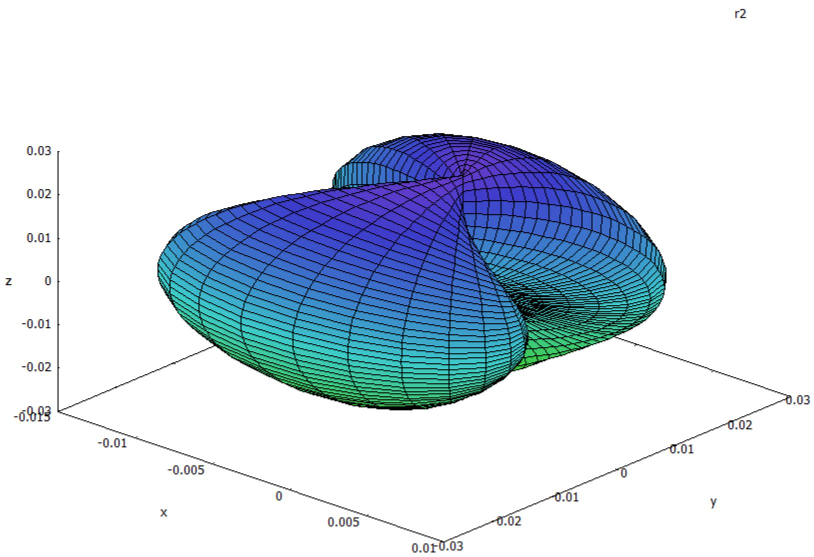

Figure 3: ϕ dependence of $\ddot{r}$.

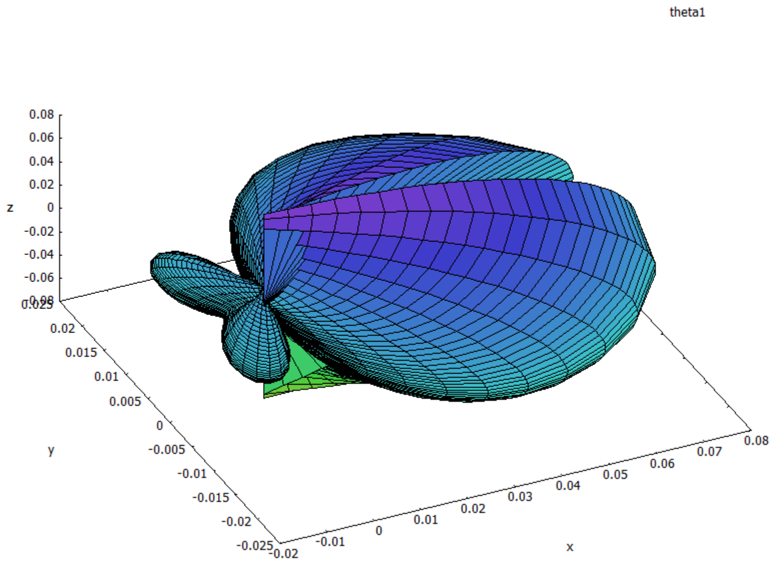

Figure 4: ϕ dependence of $\dot{\theta}$.

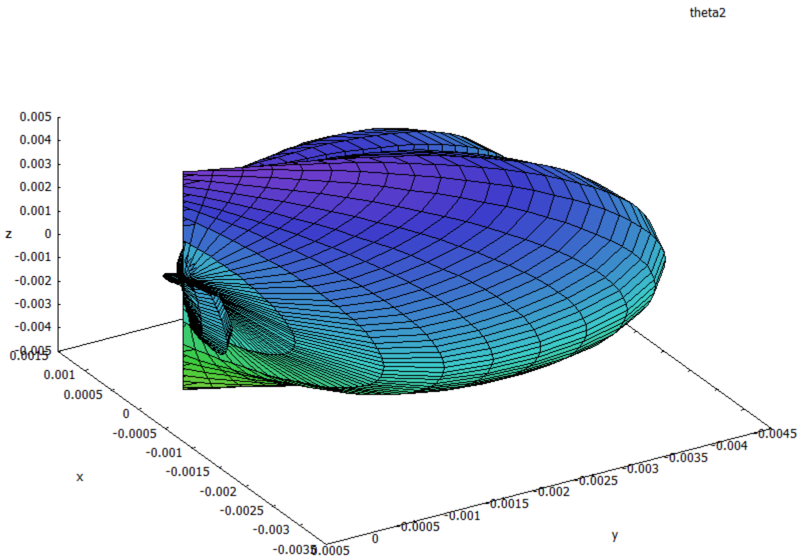

Figure 5: ϕ dependence of $\ddot{\theta}$.

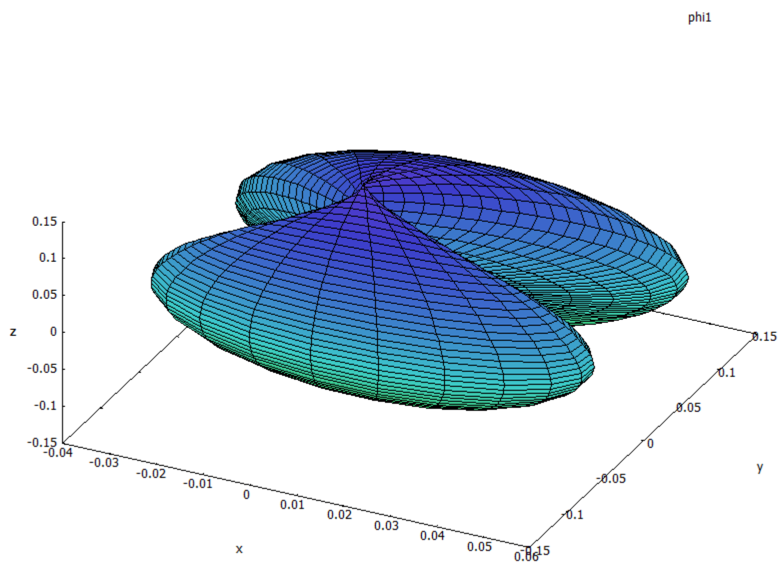

Figure 6: θ dependence of $\dot{\phi}$.

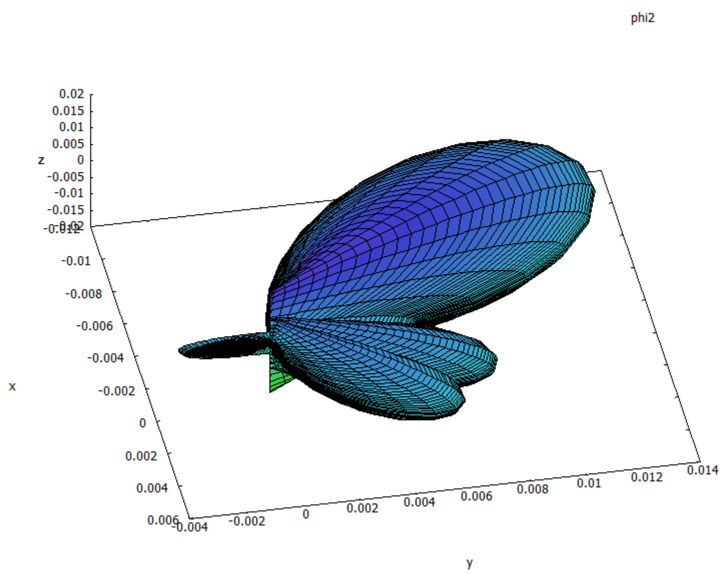

Figure 7: θ dependence of $\ddot{\phi}$.

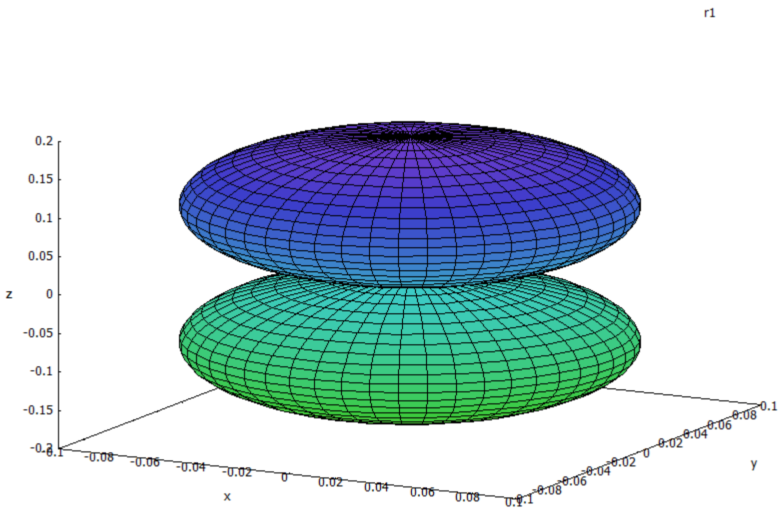

Figure 8: θ dependence of $\dot{r}$.

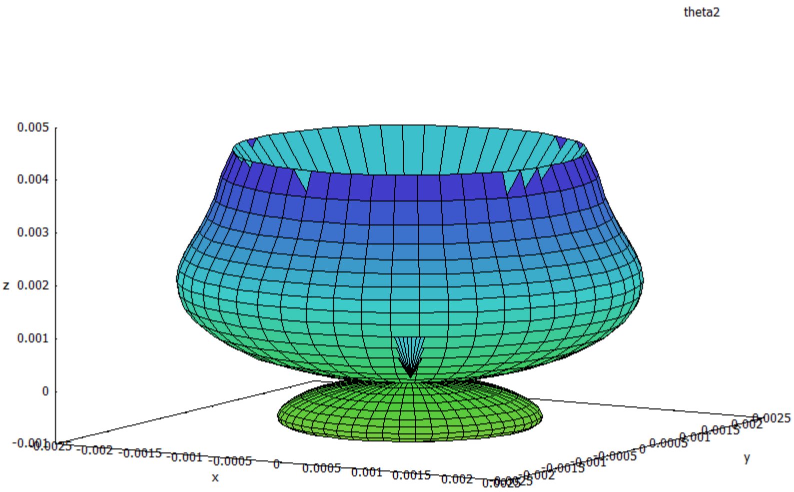

Figure 9: θ dependence of $\ddot{\theta}$.

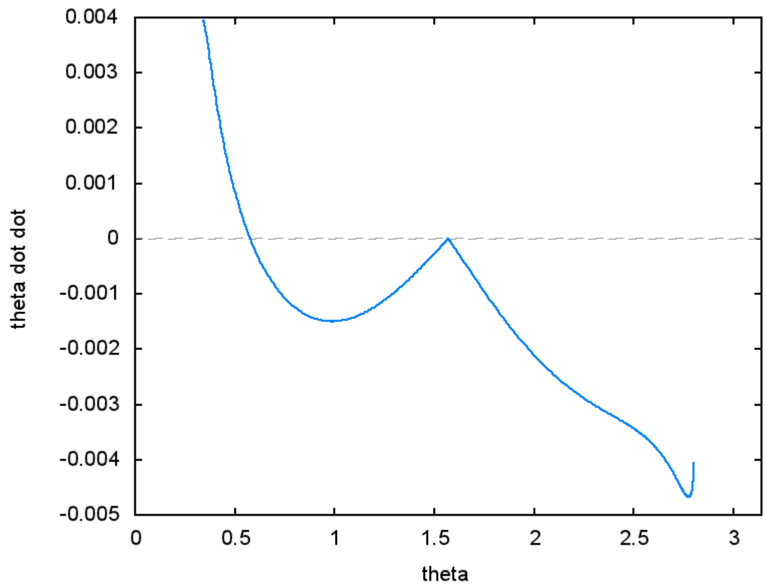

Figure 10: Functional θ dependence of $\ddot{\theta}$.

3. GRAPHICS AND ANALYSIS

Section by Dr. Horst Eckardt

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