THE KEPLER LAWS IN THREE DIMENSIONAL ORBIT THEORY.

by

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ABSTRACT

The three Kepler laws of planetary motion are developed using three dimensional orbit theory and the effect on each law described. The three dimensional orbit theory is developed with spherical polar coordinates and a direct comparison of result sis made with two dimensional orbit theory.

Keywords, ECE theory, x theory, Kepler's laws of planetary motion.

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1. INTRODUCTION

In recent papers of this series {1 - 10} orbit theory has been developed using the spherical polar coordinates and many novel results obtained. In general, orbits are three dimensional, and three dimensional orbits are observed in galaxies. In a three dimensional orbit the radial parameter r is in general dependent on both θ and ϕ of the spherical polar coordinate system. However the theory can be developed to produce the dependence of r on ϕ and of r on θ. It has been shown that conservation of angular momentum does not imply that orbits are planar, and that conservation of angular momentum is compatible with orbits in three dimensions. In general, space is three dimensional, so orbital theory should always be developed in three dimensions. When this is done correctly, the three Kepler Laws are no longer valid in general.

In Section 2 the effect of 3D orbit theory is defined for each of the three Kepler laws and in Section 3 the results are graphed and analysed.

2. THE KEPLER LAWS IN 3D.

The orbit in 3D is defined by the beta ellipse {1 - 10}:

$$\frac{1}{r} = \frac{1}{\alpha}\left(1 + \epsilon \cos \beta\right) \quad -(1)$$

where:

$$\cos \beta = \frac{\cos \phi}{\left(\cos^2 \phi + \left(\frac{L}{L_Z}\right)^2 \sin^2 \phi\right)^{1/2}} \quad -(2)$$

This reduces to the orbit of conventional theory if and only if

$$L \rightarrow L_Z \quad -(3)$$

in which case:

$$\beta \rightarrow \phi \quad -(4)$$

Here the spherical polar coordinate system is (r, θ , ϕ). The angle β is defined by

$$\dot{\beta}^2 = \dot{\theta}^2 + \dot{\phi}^2 \sin^2\theta . \quad -(5)$$

The half right latitude is:

$$\alpha = \frac{L^2}{mk} \quad -(6)$$

and the eccentricity is:

$$\epsilon^2 = 1 + \frac{2EL^2}{mk^2} . \quad -(7)$$

The mass m orbits a mass M and the constant k is defined by:

$$k = mMG \quad -(8)$$

where G is Newton's constant. The force of attraction between m and M is assumed to be:

$$\underline{F} = -\frac{k}{r^2}\,\underline{e}_r \quad -(9)$$

where the radial unit vector in the spherical polar coordinate system {11, 12} is:

$$\underline{e}_r = \sin\theta\cos\phi\,\underline{i} + \sin\theta\sin\phi\,\underline{j} + \cos\theta\,\underline{k} \quad -(10)$$

This entire theory is also applicable to electrodynamics if:

$$k = \frac{e^2}{4\pi\epsilon_0} \quad -(11)$$

where e is the charge on the proton and where ϵ_0 is the vacuum permittivity.

The linear velocity is defined by:

$$\underline{v} = \dot{r}\,\underline{e}_r + r\dot{\theta}\,\underline{e}_\theta + r\dot{\phi}\sin\theta\,\underline{e}_\phi \quad -(12)$$

where:

$$\underline{e}_\theta = \cos\theta\cos\phi\,\underline{i} + \cos\theta\sin\phi\,\underline{j} - \sin\theta\,\underline{k} \quad -(13)$$

$$\underline{e}_\phi = -\sin\phi\,\underline{i} + \cos\phi\,\underline{j} \quad -(14)$$

so the square of velocity is:

$$v^2 = \dot{r}^2 + r^2\left(\dot{\theta}^2 + \dot{\phi}^2\sin^2\theta\right) \quad -(15)$$

The hamiltonian is:

$$H = \frac{1}{2}mv^2 - \frac{k}{r} \quad -(16)$$

and the lagrangian is:

$$\mathcal{L} = \frac{1}{2}mv^2 + \frac{k}{r}. \quad -(17)$$

These may be written in terms of β as:

$$H = \frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\beta}^2\right) - \frac{k}{r} \quad -(18)$$

The hamiltonian is equal to the total energy:

$$H = E \quad -(19)$$

so:

$$E = \frac{1}{2}m\left(\left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\beta}{dt}\right)^2\right) - \frac{k}{r} \quad -(20)$$

Eq. (20) may be rewritten as the beta ellipse:

$$r = \frac{\alpha}{1+\epsilon\cos\beta} . \quad -(21)$$

In order to prove this write:

$$\frac{dr}{dt} = \frac{dr}{d\beta}\frac{d\beta}{dt} \quad -(22)$$

then:

$$E = \frac{1}{2} m \left(\frac{d\beta}{dt}\right)^2 \left(\left(\frac{dr}{d\beta}\right)^2 + r^2\right) - \frac{k}{r} . \quad -(23)$$

From the Euler Lagrange equation:

$$\frac{\partial \mathcal{L}}{\partial \beta} = \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\beta}}\right) \quad -(24)$$

with lagrangian (17) it follows that:

$$\frac{d\beta}{dt} = \frac{L}{mr^2} \quad -(25)$$

From Eqs. (23) and (25):

$$E = \frac{1}{2}\frac{L^2}{mr^4}\left(\left(\frac{dr}{d\beta}\right)^2 + r^2\right) - \frac{k}{r} \quad -(26)$$

so:

$$\left(\frac{dr}{d\beta}\right)^2 = \left(\frac{2mE}{L^2}\right) r^4 - r^2 + \left(\frac{2mk}{L^2}\right) r^3 \quad -(27)$$

From Eq. (21):

$$\frac{dr}{d\beta} = \frac{\epsilon}{\alpha} r^2 \sin\beta \quad -(28)$$

and

$$\cos^2\beta = 1 - \sin^2\beta = \frac{1}{\epsilon^2}\left(\frac{\alpha}{r} - 1\right)^2 \quad -(29)$$

so:

$$\left(\frac{dr}{d\beta}\right)^2 = \frac{r^4}{\alpha^2}\left(\epsilon^2 - 1\right) - r^2 + \frac{2r^3}{\alpha}. \quad -(30)$$

From Eq. (7):

$$\frac{\epsilon^2 - 1}{\alpha^2} = \frac{2mE}{L^2} \quad -(31)$$

and from Eq. (6):

$$\alpha = \frac{L^2}{mk}. \quad -(32)$$

So Eqs. (27) and (30) are the same, QED. The hamiltonian (20) is the same as the beta ellipse (21).

It follows that:

$$m\ddot{r} = -\frac{k}{r^2} + \frac{L^2}{mr^3} = -\frac{L^2}{mr^2}\frac{d^2}{d\beta^2}\left(\frac{1}{r}\right) \quad -(33)$$

from the Binet equation:

$$F(r) = -\frac{L^2}{mr^2}\left(\frac{d^2}{d\beta^2}\left(\frac{1}{r}\right) + \frac{1}{r}\right). \quad -(34)$$

From Eqs. (1) and (2) the orbit is no longer an ellipse in ϕ. It can become completely different from an ellipse as observed in three dimensional galaxies. Keplre's First Law states that r as a function of ϕ is an ellipse, but clearly this is no longer true in 3D. It is true only in 2D, where plane polar coordinates can be used.

Since r is a function of ϕ it follows that:

$$dA = \frac{1}{2} r^2 d\phi \quad -(35)$$

where A is the area swept out by the curve in time t. So the areal velocity is:

$$\frac{dA}{dt} = \frac{1}{2} r^2 \frac{d\phi}{dt} \quad -(36)$$

In the conventional theory this is a constant because:

$$\frac{d\phi}{dt} = \frac{L_z}{mr^2} \quad -(37)$$

where $L_z\underline{k}$ is the Z component of the total angular momentum $\underline{L}$. This is Kepler's Second Law, the areal velocity is constant. However in 3D:

$$\frac{dA}{dt} = \frac{1}{2} r^2 \frac{d\phi}{dt} = \frac{1}{2} r^2 \frac{d\beta}{dt}\frac{d\phi}{d\beta} \quad -(38)$$

where from previous work {1- 10}:

$$\frac{d\phi}{d\beta} = \frac{L_{z}}{L \sin^2\theta} \quad -(39)$$

Therefore in 3D, the areal velocity is no longer constant and Keplre's Second Law is no longer true.

From previous work:

$$\sin^2\theta = \left(\frac{L_z}{L}\right)^2 + \left(1 - \left(\frac{L_z}{L}\right)^2\right)\left(\frac{\cos^2\phi}{\cos^2\phi + \left(\frac{L}{L_z}\right)^2 \sin^2\phi}\right) \quad -(40)$$

so the 3D areal velocity is:

$$\frac{dA}{dt} = \frac{L_z}{2m}\left(\left(\frac{L_z}{L}\right)^2 + \left(1 - \frac{L_z}{L}\right)^2\right)\left(\frac{\cos^2\phi}{\cos^2\phi + \left(\frac{L}{L_z}\right)^2 \sin^2\phi}\right)^{-1} \quad -(41)$$

This becomes the 2D result if an only if:

$$L \rightarrow L_Z \quad -(42)$$

and

$$\theta = \frac{\pi}{2}, \quad \frac{d\theta}{dt} = 0. \quad -(43)$$

The areal velocity develops a ϕ dependence and is graphed in Section 3.

Kepler's Third Law is derived conventionally in 2D orbital theory using:

$$\frac{dA}{dt} = \frac{L_Z}{2m} \quad -(44)$$

so:

$$dt = \left(\frac{2m}{L_Z}\right) dA. \quad -(45)$$

This equation is integrated to give:

$$\tau = \int_0^\tau dt = \frac{2m}{L_Z}\int_0^A dA = \frac{2m}{L_Z} A \quad -(46)$$

where τ is the time taken for one complete orbit. This is proportional to the area A of the orbit. In 2D theory the orbit is the ellipse:

$$r = \frac{\alpha}{1 + \epsilon \cos\phi} \quad -(47)$$

whose area is:

$$A = \pi a b \quad -(48)$$

where a and b are the major and minor semi axes. Therefore:

$$\tau = \frac{2m}{L_Z} \pi a b \quad -(49)$$

and

$$\tau^2 = \frac{4m^2\pi^2}{L_Z^2} a^2 b^2 \quad -(50)$$

However {12}:

$$b^2 = \alpha a \quad -(51)$$

where the half right latitude in 2D theory is:

$$\alpha = \frac{L_Z}{mk} \quad -(52)$$

so

$$\tau^2 = \left(\frac{4m\pi^2}{k}\right) a^3 \quad -(53)$$

The square of the time taken for one orbit is proportional to the cube of semi major axis. This is the 2D version of Kepler's Third Law.

In 3D theory as above:

$$\frac{dA}{dt} = \frac{1}{2} r^2 \frac{d\beta}{dt} \frac{d\phi}{d\beta} \quad -(54)$$

so the areal velocity is:

$$\frac{dA}{dt} = \frac{L_Z}{2m \sin^2\theta} \quad -(55)$$

and

$$dt = \left(\frac{2m \sin^2\theta}{L_Z}\right) dA \quad -(56)$$

In order to integrate this equation it must be determined whether or not A has any dependence on θ. Note that the perihelion of the beta ellipse (α\ ) is defined by the distance of closest approach:

$$r_{min} = \frac{d}{1+\epsilon} = a(1-\epsilon). \quad -(57)$$

At the perihelion:

$$\cos\beta = 1, \quad \beta = 0 \quad -(58)$$

From Eq. (2):

$$\phi = 0. \quad -(59)$$

The angle θ is defined by:

$$\sin^2\theta = \left(\frac{L_Z}{L}\right)^2 + \left(1-\left(\frac{L_Z}{L}\right)^2\right)\left(\frac{\cos^2\phi}{\cos^2\phi + \left(\frac{L}{L_Z}\right)^2 \sin^2\phi}\right). \quad -(60)$$

So at the perihelion:

$$\sin^2\theta = 1, \quad \theta = \pi/2. \quad -(61)$$

Therefore the perihelion of the beta ellipse is defined by:

$$\beta = \phi = 0, \quad \theta = \frac{\pi}{2}. \quad -(62)$$

It follows that its area:

$$A = \pi a b \quad -(63)$$

is defined by the fixed angles in Eq. (62) so A has no dependence on the variable θ .

Similarly at the aphelion:

$$r = r_{max} = \frac{\alpha}{1-\epsilon} = a(1+\epsilon) \quad -(64)$$

$$\phi = \beta = \pi, \quad \theta = \frac{\pi}{2}, \quad -(65)$$

and a and b are again determined by fixed angles.

Therefore Eq. (56) can be integrated as follows:

$$\tau = \int_0^\tau dt = \frac{2m\sin^2\theta}{L_Z}\int dA = \frac{2mA}{L_Z}\sin^2\theta \quad -(66)$$

which can be developed as:

$$\tau^2 = \left(\frac{4m^2\pi^2\alpha\sin^4\theta}{L_Z^2}\right)a^3. \quad -(67)$$

In 3D the half right latitude {1 - 10} is:

$$\alpha = \frac{L^2}{mk} \quad -(68)$$

so: Kepler's Third Law in 3D is:

$$\tau^2 = \left(\frac{4m\pi^2}{k}\left(\frac{L}{L_Z}\right)^2\sin^4\theta\right)a^3 \quad -(69)$$

$$-(70)$$

where:

$$\sin^4\theta = \left[\left(\frac{L_Z}{L}\right)^2 + \left(1-\frac{L_Z}{L}\right)^2\left(\frac{\cos^2\phi}{\cos^2\phi + \left(\frac{L}{L_Z}\right)^2\sin^2\phi}\right)\right]^2$$

Eq. (69) is graphed and discussed in Section 3.

The orbital linear velocity (15) is also changed in 3D theory as follows. In 2D theory {12} the orbital linear velocity is:

$$v^2 = \left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\phi}{dt}\right)^2 \quad -(71)$$

where the angular velocity is:

$$\omega = \frac{d\phi}{dt} = \frac{L_Z}{mr^2} \quad -(72)$$

and

$$\frac{dr}{dt} = \frac{dr}{d\phi}\frac{d\phi}{dt} \quad -(73)$$

So

$$v^2 = \frac{L_Z^2}{m^2 r^2}\left(1 + \frac{1}{r^2}\left(\frac{dr}{d\phi}\right)^2\right) \quad -(74)$$

In this expression:

$$\left(\frac{dr}{d\phi}\right)^2 = \frac{\epsilon^2 r^4}{\alpha^2}\left(1 - \cos^2\phi\right) \quad -(75)$$

where:

$$\cos^2\phi = \frac{1}{\epsilon^2}\left(\frac{\alpha}{r} - 1\right)^2 \quad -(76)$$

So the square of the velocity is:

$$v^2 = \frac{L_Z^2}{m^2\alpha}\left(\frac{\epsilon^2 - 1}{\alpha} + \frac{2}{r}\right)$$

$$= MG\left(\frac{2}{r} - \frac{1}{a}\right) \quad -(77)$$

In 3D theory however:

$$v^2 = \left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\beta}{dt}\right)^2 \quad -(78)$$

and the angular velocity is changed to:

$$\omega = \frac{d\beta}{dt} = \frac{L}{mr^2}. \quad -(79)$$

A direct comparison of Eqs. (71) and (78) can be made as follows. The results are graphed in Section 3. In 2D:

$$\dot{r} = \left(\frac{2}{m}\left(E - \frac{L_Z^2}{2mr^2} + \frac{k}{r}\right)\right)^{1/2} \quad -(80)$$

where:

$$r = \frac{\alpha}{1 + \epsilon\cos\phi} \quad -(81)$$

and

$$\dot{\phi} = \frac{L_Z}{mr^2} \quad -(82)$$

so v can be graphed as a function of ϕ .

In 3D the relevant chain of equations {1 - 10} is:

$$v^2 = \dot{r}^2 + r^2\left(\dot{\theta}^2 + \dot{\phi}^2\sin^2\theta\right) \quad -(83)$$

where:

$$\dot{r} = \left(\frac{2}{m}\left(E - \frac{L^2}{2mr^2} + \frac{k}{r}\right)\right)^{1/2} \quad -(84)$$

$$\dot{\phi} = \frac{L_Z}{mr^2\sin^2\theta} \quad -(85)$$

$$\dot{\theta} = \frac{1}{mr^2}\left(L^2 - \frac{L_Z^2}{\sin^2\theta}\right)^{1/2} \quad -(86)$$

$$r = \frac{\alpha}{1+\epsilon\cos\beta} \quad -(87)$$

$$\cos\beta = \cos\phi\left(\cos^2\phi + \left(\frac{L}{L_Z}\right)^2\sin^2\phi\right)^{-1/2} \quad -(88)$$

$$\sin^2\theta = \left(\frac{L_Z}{L}\right)^2 + \left(1-\left(\frac{L_Z}{L}\right)^2\right)\left(\frac{\cos^2\phi}{\cos^2\phi + \left(\frac{L}{L_Z}\right)^2\sin^2\phi}\right) \quad -(89)$$

So v can be graphed as a function of ϕ in 3D, and the result compared directly with 2D theory.

In these equations:

$$\cos\beta = \cos\phi\left(\cos^2\phi + \left(\frac{L}{L_Z}\right)^2\sin^2\phi\right)^{-1/2} \quad -(90)$$

and

$$\sin\beta = -\frac{L\cos\theta}{(L^2-L_Z^2)^{1/2}} \quad -(91)$$

$$-(92)$$

so it follows that:

$$-\beta = \frac{1}{2}\left[\cos^{-1}\left(\frac{\cos\phi}{\left(\cos^2\phi + \left(\frac{L}{L_Z}\right)^2\sin^2\phi\right)^{1/2}}\right) - \sin^{-1}\left(\frac{L\cos\theta}{(L^2-L_Z^2)^{1/2}}\right)\right]$$

so graphics of this type can be constructed.

3. GRAPHICS AND ANALYSIS

Section by Dr. Horst Eckardt

The Kepler laws in three dimensional orbit theory

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3 Graphics and analysis

First we analyse the range of angles β and θ. The problem with the angular range of inverse trigonometric functions has been cleared. β as well as ϕ range from 0 to 2π. If $\phi > \pi$, one has to use

$$\beta = 2\pi - \beta(\phi) \tag{93}$$

where $\beta(\phi)$ is defined by the original Eq.(2) in section 2. For θ holds similarly:

$$\theta = \pi - \theta(\phi) \tag{94}$$

with $\theta(\phi)$ given by Eq.(40). This leads to smooth, differentiable curves, see Figs 1-3. Parameters were $L = 3,\ L_Z = 1,\ m = k = 1,\ E = -0.04$ from which results $\alpha = 9,\ \epsilon = 0.529$. In particular, aphelion and perihelion come out clearly in Fig. 1.

The dependence of β on θ can be understood as follows. Besides the probelm of continuation of ϕ dependence there is the problem that, for a full turn of the orbit, θ varies from 0 to π and back to 0 again. This means that there are two points (ϕ or β values) per θ value. Therefore the dependence $\phi(\theta)$ or $\beta(\theta)$ is not unique, it has to be extended to two values, see Fig. 4. For graphs it is easier (and probably more instructive) to concentrate on the ϕ dependence.

The 3D surfaces $r(\theta, \phi)$ for the conic orbits

$$r = \frac{\alpha}{1 + \epsilon \cos(\beta)} \tag{95}$$

are shonw in Figs. 5-9. The ϕ dependence is given by Eq.(90) and the θ dependence by Eq.(91). Both can be added to obtain a mixed ϕ/θ dependence of Eq.(92). In adition, the upper or lower branch of the function $\beta(\theta)$ in Fig. 4 can be used. As a result, both branches lead to toroidal surfaces (Figs. 6-7), in total this gives an ellipsoid (Fig. 8). Combination of the ϕ/θ dependence according to Eq.(92) gives a superposition of both surfaces (Fig. 9).

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The other figures show examples of a hyperbolic spiral orbit

$$r = \frac{r_0}{\beta} \tag{96}$$

with a constant r_0. The same classifications as for the ellipsoidal surfaces have been applied (Figs. 10-14). In addition to the 2D plane that we already know from earlier calculations, there is a jet in the Z axis, see Fig. 11. In combination with θ and ϕ dependence, this leads to a disk with finite thickness and a jet, quite similar to some galaxies (Fig. 13). So we are able to describe spiral galaxies with "thickness".

In Fig. 15 we graphed an example of ellipsoidal/conic 3D orbits. These have to be thought as lines on the surfaces previously shown. For $L_Z << L$, the orbit is different from an ellipse. For L_Z approaching L this is a transition to the 2D case and the ellipse rotates itself into the XY plane. All curves go through the points $\theta = \pi/2,\ \phi = 0$ and $\theta = \pi/2,\ \phi = \pi$ and move in the upper or lower half-space, respectively. The length scales have been adopted for the graphs.

The areal time tau (Eq.(69)) varies with ϕ in 3D theory and becomes constant in the limit $L_Z \to L$. (Fig. 16). The linear velocity (Fig. 17) is a quite complicated expression in three dimensions (Eqs.(83-89)) but can be calculated and approaches the 2D limit too. In the 3D orbit there is a broader minimum range, i.e. motion is slower in aphelion and faster in perihelion compared to elliptic orbits in 2D.

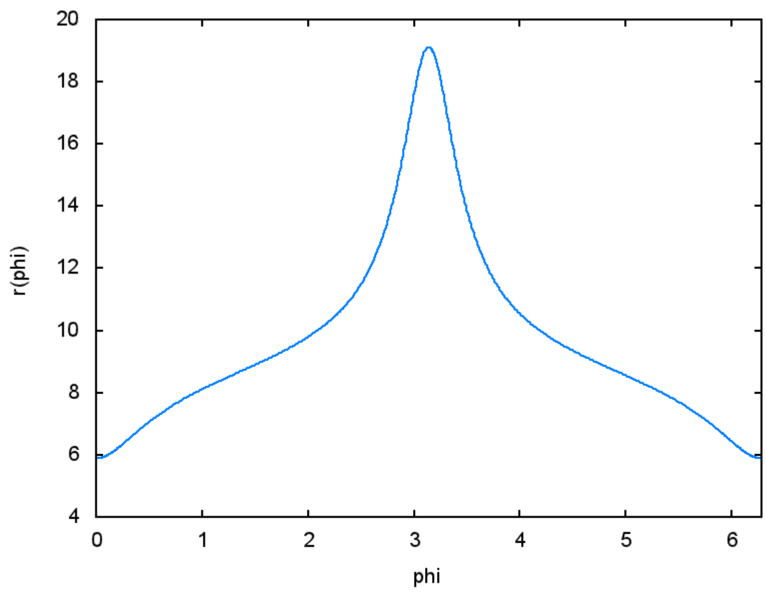

Figure 1: Elliptic orbit $r(\phi)$ for $L = 3,\ L_Z = 1$.

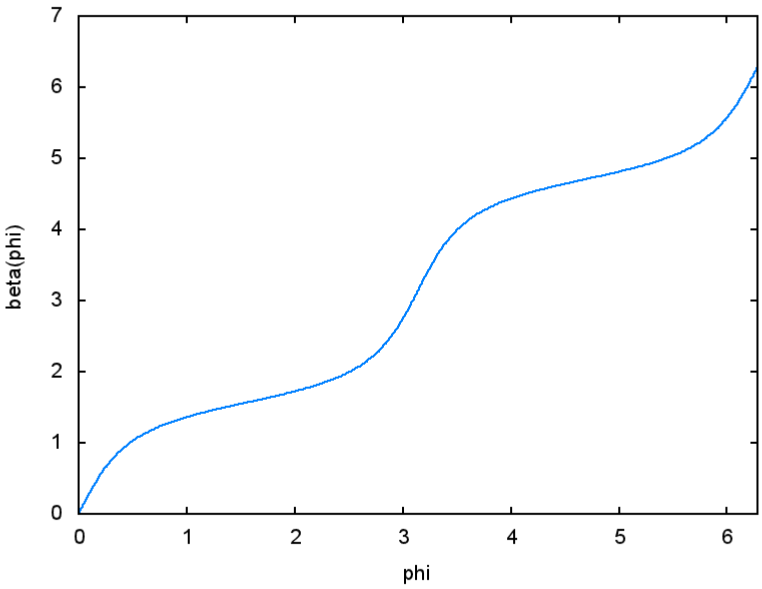

Figure 2: Angular dependence $\beta(\phi)$.

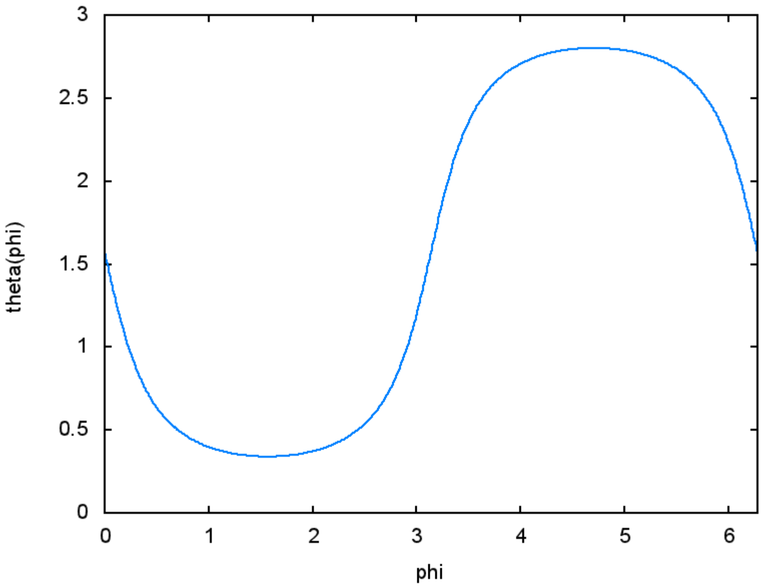

Figure 3: Angular dependence $\theta(\phi)$.

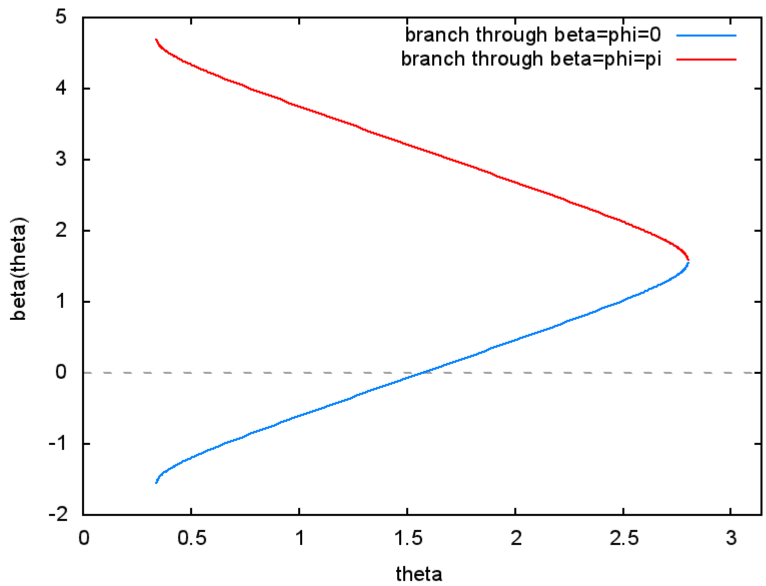

Figure 4: Angular dependence $\beta(\theta)$ with two branches.

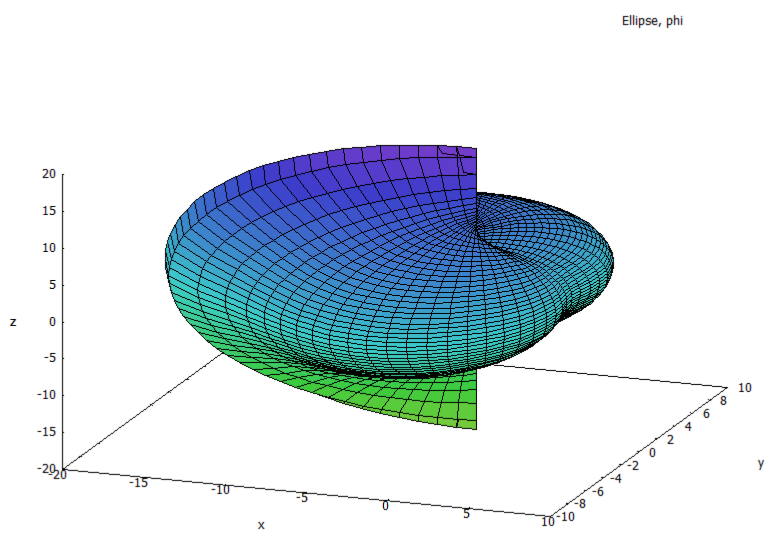

Figure 5: Elliptic orbital surface for ϕ.

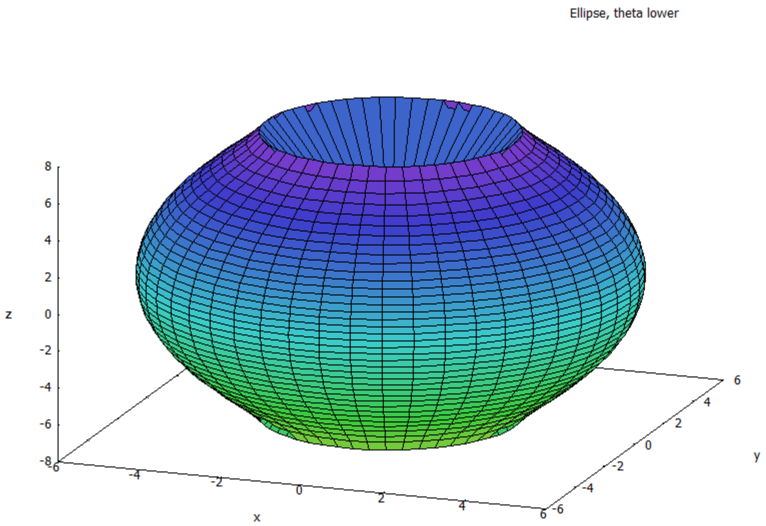

Figure 6: Elliptic orbital surface for θ, lower branch.

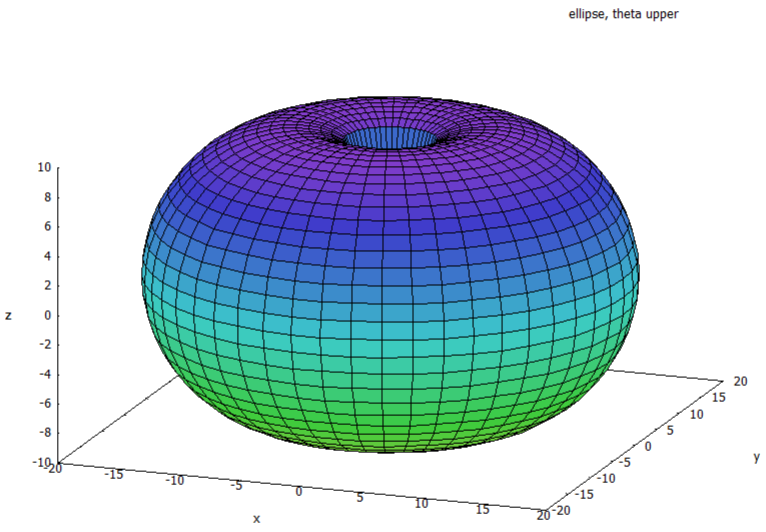

Figure 7: Elliptic orbital surface for θ, upper branch.

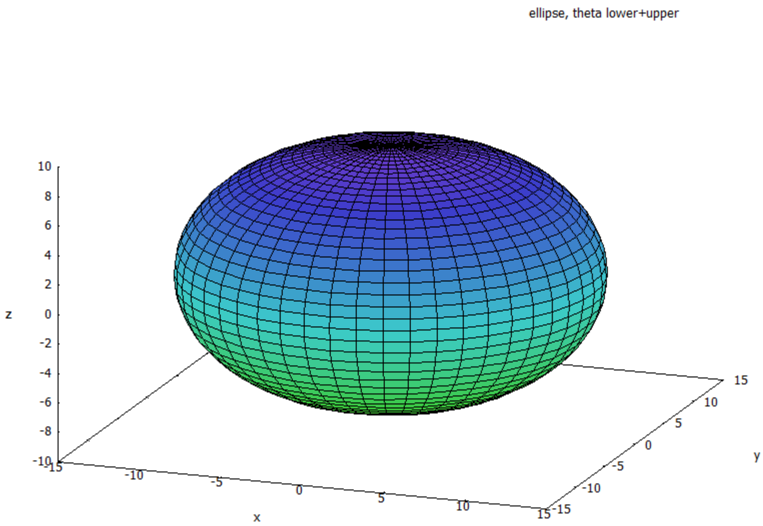

Figure 8: Elliptic orbital surface for θ, lower and upper branch combined.

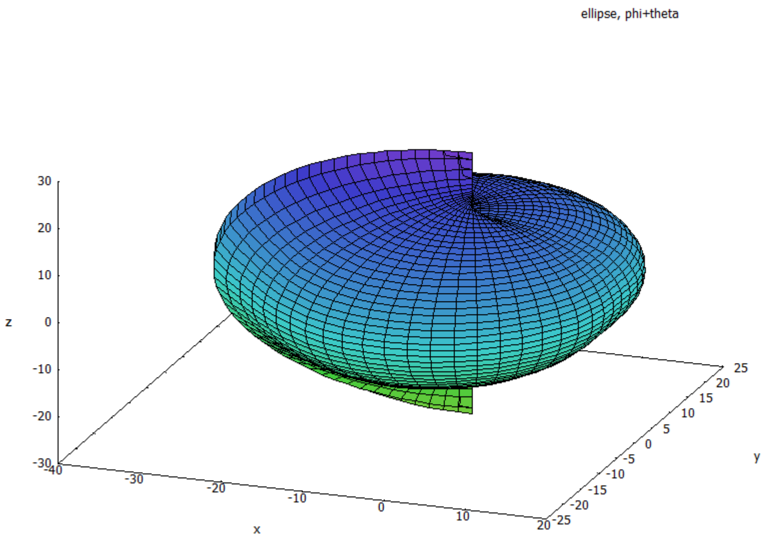

Figure 9: Elliptic orbital surface for ϕ and θ, lower and upper branch combined.

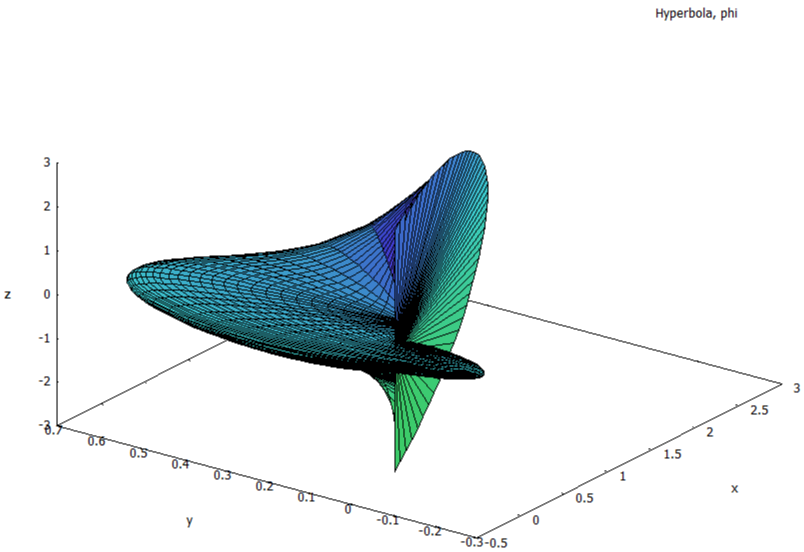

Figure 10: Hyperbolic orbital surface for ϕ.

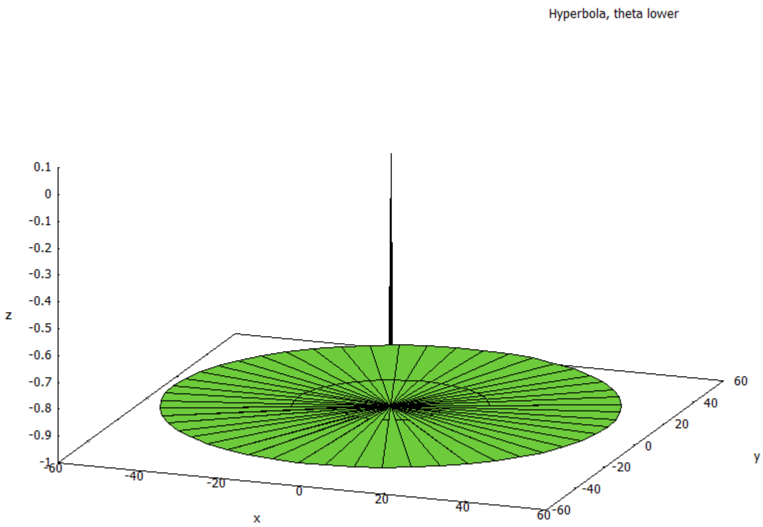

Figure 11: Hyperbolic orbital surface for θ, lower branch.

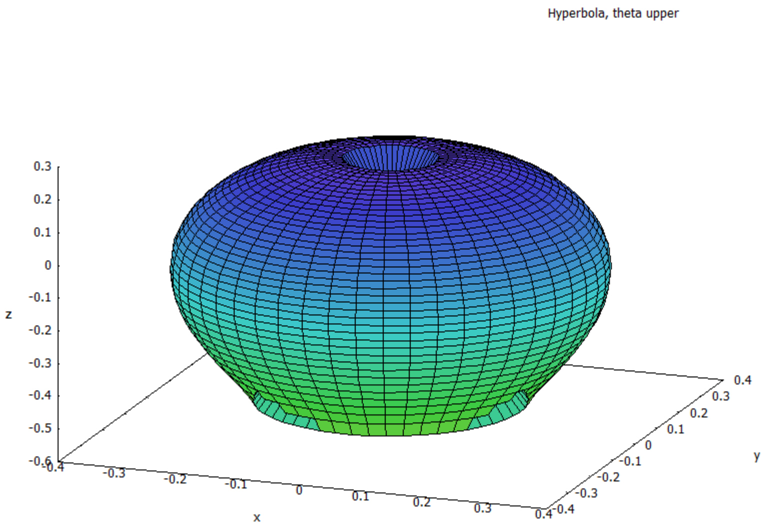

Figure 12: Hyperbolic orbital surface for θ, upper branch.

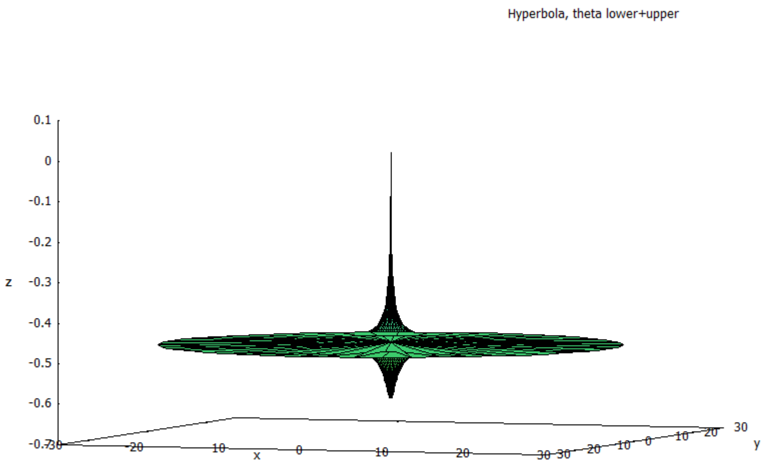

Figure 13: Hyperbolic orbital surface for θ, lower and upper branch combined.

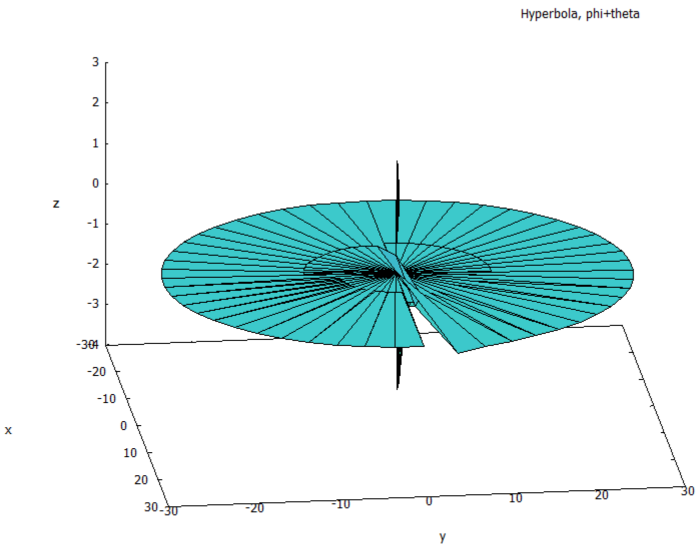

Figure 14: Hyperbolic orbital surface for ϕ and θ, lower and upper branch combined.

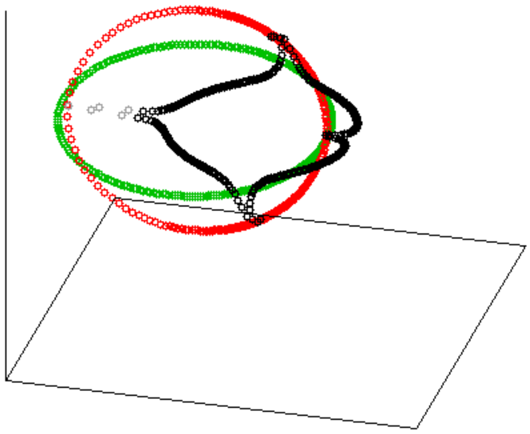

Figure 15: Elliptic orbits with $L = 3$ for $L_Z = 0.1$ (black), $L_Z = 1$ (red), $L_Z = 2.99$ (green).

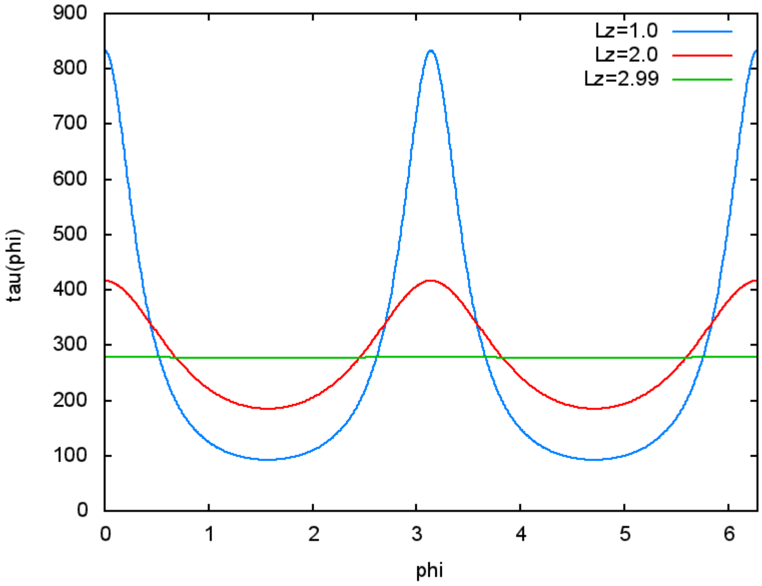

Figure 16: Area times $\tau(\phi)$ with $L = 3$ and varying L_Z.

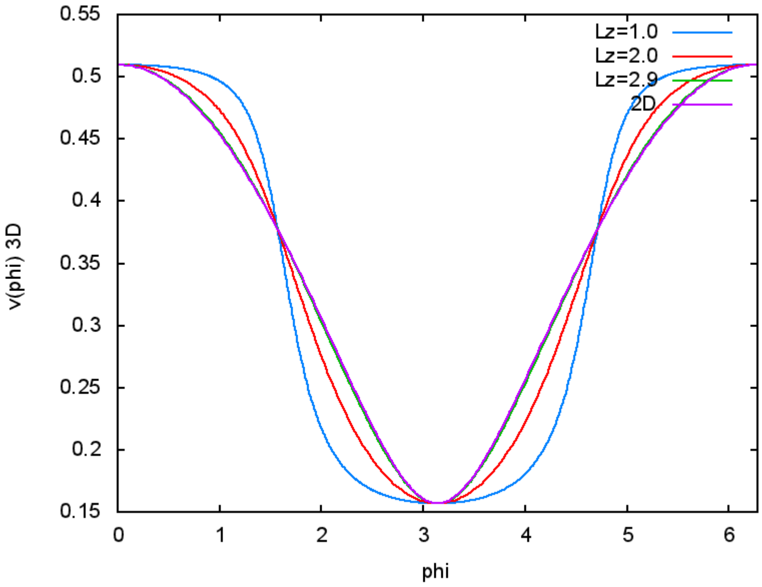

Figure 17: Linear velocities $v(\phi)$ for 2D case and 3D cases with $L = 3$ and varying L_Z.

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