# ELLIPSOIDAL AND OTHER THREE DIMENSIONAL ORBITS FROM THE INVERSE SQUARE LAW OF ATTRACTION.

by

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ABSTRACT

Three dimensional orbital theory is applied with the inverse square law of attraction to produce ellipsoidal and other kinds of three dimensional orbits observed in galaxies. The Cartesian representation is used for clarity of representation and complements work in spherical polar representation in recent papers. The three dimensional orbits emerge form a hamiltonian and lagrangian in which the kinetic energy is represented in spherical polar coordinates.

Keywords: ECE theory, three dimensional orbital theory, ellipsoidal and other orbits.

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## 1. INTRODUCTION

In recent papers of this series {1 - 10} three dimensional orbit theory has been developed with the spherical polar coordinate system, which can be regarded as a special case of Cartan geometry. For example the Cartan torsion, curvature and spin connection can be defined by evaluating the Cartan tetrad. The latter is defined by superimposing the spherical polar coordinate unit vectors on the Cartesian unit vectors for a three dimensional mathematical space. The Cartan spin connections resulting from this procedure are the angular velocities that can be calculated from the hamiltonian and lagrangian. For over four hundred years it was thought that orbital theory could be worked out with plane polar coordinates, but recent ECE papers have shown {1 - 10} that the use of spherical polar coordinates produces much more information.

As usual, this paper should be read in conjunction with the background notes published with UFT274 on www.aias.us. The first five notes for this paper define the background of the three dimensional theory. Note 1 derives the Binet equation associated with the beta ellipse and calculates the ratio L / L sub Z, where L is the magnitude of the total angular momentum and where L sub Z is its Z component. This ratio is a constant of motion that can be expressed in terms of the spherical polar coordinates and, in Note 2, in terms of the Cartesian coordinates. Note 3 reduces these results to a precessing ellipse in a plane, and compares this result with experimental data. Note 4 develops the theory of the three dimensional hyperbolic spiral. This note can be read as a transformation from the two dimensional spiral galaxy to its three dimensional equivalent. Note 5 is a correction of an erratum in Note 1. Note 6 gives complete details of the transformation from the polar to Cartesian representation of an elliptical orbit of a mass m around a mass M situated at one focus of the ellipse.

In Section 2 a convenient review is given of the 3D theory, and three equations derived in Cartesian representation from the fundamental theory, in which the force of attraction between m and M is the inverse square law of Hooke and Newton. In two dimensional orbit theory this is well known to produce an ellipse in the plane XY. In three dimensional orbit theory however it produces a far richer variety of orbits, notably the ellipsoidal orbits well known to be observed in galaxies.

In Section 3 the three dimensional orbits are graphed and animated in various ways by co author Horst Eckardt.

## 2. ELLIPSOIDAL AND OTHER TYPES OF ORBIT FROM THE INVERSE SQUARE LAW.

In immediately preceding papers of this series the beta ellipse:

$$r = \frac{\alpha}{1 + \epsilon \cos \beta} \quad -(1)$$

was shown to be equivalent to the hamiltonian:

$$H = \frac{1}{2} m v^2 - \frac{k}{r} \quad -(2)$$

where, in the spherical polar coordinate system ( r, $\theta$ , $\phi$ ):

$$v^2 = \dot{r}^2 + \dot{\beta}^2 r^2 \quad -(3)$$

and

$$\dot{\beta}^2 = \dot{\theta}^2 + \dot{\phi}^2 \sin^2 \theta . \quad -(4)$$

Here $\alpha$ is the half right latitude:

$$\alpha = a(1-\epsilon^2) \quad -(5)$$

where a is the semi major axis of the beta ellipse and where $\epsilon$ is the ellipticity defined by:

$$\epsilon^2 = 1 - \frac{b^2}{a^2} \quad -(6)$$

where b is the semi minor axis of the beta ellipse.

The Cartesian representation of the beta ellipse is derived in all detail in Note 274(6) and is:

$$\frac{X^2}{a^2} + \frac{Y^2}{b^2} = 1 \quad -(7)$$

where:

$$X = a\epsilon + r\cos\beta \quad -(8)$$

and

$$Y = r\sin\beta. \quad -(9)$$

This represents an elliptical orbit as a function of $\beta$ with a mass m orbiting a mass M at one focus of the beta ellipse.

The lagrangian of the beta ellipse is:

$$\mathcal{L} = \frac{1}{2}mv^2 + \frac{k}{r}. \quad -(10)$$

There are four Euler Lagrange equations:

$$\frac{\partial \mathcal{L}}{\partial r} = \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{r}}\right) \quad -(11)$$

$$\frac{\partial \mathcal{L}}{\partial \beta} = \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\beta}}\right) \quad -(12)$$

$$\frac{\partial \mathcal{L}}{\partial \theta} = \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}}\right) \quad -(13)$$

$$\frac{\partial \mathcal{L}}{\partial \phi} = \frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{\phi}}\right) \quad -(14)$$

Eqs. ( 11 ) to ( 14 ) give the four equations of motion:

$$m\ddot{r} = mr\dot{\beta}^2 - \frac{L^2}{mr^3} \quad -(15)$$

$$\frac{d}{dt}\left(mr^2\dot{\beta}\right) = 0 \quad -(16)$$

$$\frac{d}{dt}\left(mr^2\dot{\phi}\sin^2\theta\right) = 0 \quad -(17)$$

$$\frac{d}{dt}\left(mr^2\dot{\theta}\right) = 2\sin\theta\cos\theta\,\dot{\phi}^2 \quad -(18)$$

From the fundamental considerations of UFT269:

$$L_Z = mr^2\dot{\phi}\sin^2\theta \quad -(19)$$

$$L^2 = m^2r^4\left(\dot{\theta}^2 + \dot{\phi}^2\sin^2\theta\right) \quad -(20)$$

so Eq. ( 16 ) is the conservation of the total angular momentum L

$$\frac{dL}{dt} = 0 \quad -(21)$$

and Eq. ( 17 ) is the conservation of the Z component of the total angular momentum:

$$\frac{dL_Z}{dt} = 0. \quad -(22)$$

Therefore L and $L_Z$ are constants of motion, and their ratio is therefore a constant of motion. Eq. ( 18 ) shows that $mr^2\dot{\theta}$ is not a constant of motion.

From Eqs. ( 19 ) and ( 20 ):

$$\frac{d\theta}{dt} = \frac{1}{mr^2}\left(L^2 - \frac{L_Z^2}{\sin^2\theta}\right)^{1/2} \quad -(23)$$

and

$$\frac{d\beta}{dt} = \frac{L}{mr^2} \quad -(24)$$

$$\frac{d\phi}{dt} = \frac{L_Z}{mr^2\sin^2\theta}. \quad -(25)$$

Eqs. ( 23 ) to ( 25 ) follow from the kinetic energy alone:

$$T = \frac{1}{2}m\left(\dot{r}^2 + r^2\left(\dot{\theta}^2 + \dot{\phi}^2\sin^2\theta\right)\right) \quad -(26)$$

and are true therefore for any potential energy. It follows that:

$$\frac{d\beta}{d\theta} = \frac{L}{\left(L^2 - \frac{L_Z^2}{\sin^2\theta}\right)^{1/2}} \quad -(27)$$

and

$$\frac{d\beta}{d\phi} = \frac{L}{L_Z}\sin^2\theta. \quad -(28)$$

Integrating Eq. ( 27 ) with computer algebra {1 - 10} gives:

$$\beta = \int \frac{L\,d\theta}{\left(L^2 - \frac{L_z^2}{\sin^2\theta}\right)^{1/2}} = -\sin^{-1}\left(\frac{L\cos\theta}{(L^2 - L_z^2)^{1/2}}\right) \quad -(29)$$

so:

$$\sin\beta = \frac{-L\cos\theta}{(L^2 - L_z^2)^{1/2}} \quad -(30)$$

From Eq. ( 28 ):

$$\beta = \int \frac{L}{L_z} \sin^2\theta \, d\phi \quad -(31)$$

and:

$$\phi = \frac{L_z}{L} \int \frac{d\beta}{\sin^2\theta} \quad -(32)$$

From Eq. ( 30 ):

$$\left(\frac{L^2}{L^2 - L_z^2}\right) \cos^2\theta = \sin^2\beta \quad -(33)$$

so:

$$\cos^2\theta = \left(\frac{L^2 - L_z^2}{L^2}\right) \sin^2\beta \quad -(34)$$

and

$$\sin^2\theta = 1 - \left(1 - \left(\frac{L_z}{L}\right)^2\right) \sin^2\beta \quad -(35)$$

Therefore:

$$\phi = \frac{L_z}{L} \int \left(1 - \left(1 - \left(\frac{L_z}{L}\right)^2\right) \sin^2\beta\right)^{-1} d\beta$$

$$= \tan^{-1}\left(\frac{L_Z}{L}\tan\beta\right) \quad -(36)$$

by computer algebra. Therefore:

$$\tan\phi = \frac{L_Z}{L}\tan\beta. \quad -(37)$$

It follows that:

$$\tan\beta = \frac{\sin\beta}{\cos\beta} = \frac{(1-\cos^2\beta)^{1/2}}{\cos\beta} = \frac{L}{L_Z}\tan\phi \quad -(38)$$

so:

$$\cos^2\beta = \frac{\cos^2\phi}{\cos^2\phi + \left(\frac{L}{L_Z}\right)^2\sin^2\phi}. \quad -(39)$$

The following equations are deduced in Cartesian representation:

$$X = a\epsilon + \frac{r\cos\phi}{\cos^2\phi + \left(\frac{L}{L_Z}\right)^2\sin^2\phi}. \quad -(40)$$

Therefore Z is proportional to Y:

$$Z = \left(1 - \left(\frac{L_Z}{L}\right)^2\right)^{1/2} Y \quad -(41)$$

Therefore Z vanishes self consistently as the orbit reduces to a planar orbit as follows:

$$L \rightarrow L_Z. \quad -(42)$$

From Eqs. ( 7 ) and ( 40 ):

$$\frac{X^2}{a^2} + \frac{Z^2}{b^2\left(1-\left(\frac{L_Z}{L}\right)^2\right)} = 1 \quad -(43)$$

which is an ellipse in X and Z. The ellipse in X and Y is:

$$\frac{X^2}{a^2} + \frac{Y^2}{b^2} = 1. \quad -(44)$$

Adding Eqs. ( 43 ) and ( 44 ) gives the ellipsoidal orbit:

$$\frac{X^2}{a^2} + \frac{Y^2}{2b^2} + \frac{Z^2}{2b^2\left(1-\left(\frac{L_Z}{L}\right)^2\right)} = 1 \quad -(45)$$

as observed in cigar shaped galaxies. Divide Eq. ( 41 ) by a factor c squared, where c is a semi axis:

$$\frac{Z^2}{c^2} = \frac{1}{c^2}\left(1-\left(\frac{L_Z}{L}\right)^2\right)^{1/2} Y \quad -(46)$$

Subtracting Eq. ( 46 ) from Eq. ( 44 ) gives:

$$\frac{X^2}{a^2} + Y^2\left(\frac{1}{b^2} + \left(\frac{L_Z}{L}\right)\frac{1}{c^2}\right) - \frac{Z^2}{c^2} = 1 \quad -(47)$$

which is a one sheet hyperbeloidal orbit {11} that looks like two mirror imaged cups. This type of result has been graphed in polar representation in immediately preceding papers.

Finally, subtract the root of Eq. ( 46 ) from Eq. ( 44 ) to give the elliptic parabaloidal orbit:

$$\frac{X^2}{a^2} + \frac{Y^2}{b^2} = \frac{Z}{c} \quad -(48)$$

It is clear that many such permutations and combinations are possible, and they all emerge form the inverse square law of attraction. However, this theory is valid for any law of attraction and this inference will be developed in future papers.

ACKNOWLEDGMENTS

The British Government is thanked for a Civil List Pension and the staff of AIAS and others for many interesting discussions. Dave Burleigh is thanked for posting, Alex Hill for translation and broadcasting and Robert Cheshire for broadcasting.

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