CURVATURE BASED ECE FIELD EQUATIONS FROM THE JACOBI CARTAN EVANS (JCE) IDENTITY.

by

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ABSTRACT

A new era of ECE theory is developed from the Jacobi Cartan Evans (JCE) identity inferred in UFT313. Unified field equations are defined directly from the JCE identity using a new definition of curvature. The latter is transformed into the field with a new ECE hypothesis. The result is illustrated for the electromagnetic field, the new curvature based ECE theory gives the Maxwell Heaviside equations in a space with non zero torsion and curvature with geometrically defined magnetic and electric charge current densities.

Keywords: New era of ECE theory, curvature based field equations, Jacobi Cartan Evans identity.

UFT315

1. INTRODUCTION

The Jacobi Cartan Evans (JCE) identity corrects the original 1902 second Bianchi identity for torsion, and was inferred in UFT313 of this series {1-10}. It was inferred from the fundamental Jacobi identity and in so doing also produced the Evans torsion identity of UFT112. A new vector identity was inferred in UFT314 from the tensorial JCE identity. In this paper a new era of ECE theory is initiated from the JCE identity by defining a new type of curvature which is transformed directly into fields using a new ECE hypothesis. The JCE identity gives field equations for the four fundamental fields: electromagnetism, gravitation , weak and strong nuclear. The field equations are exemplified in this paper with electromagnetism, and it is shown that the JCE identity produces the Maxwell Heaviside (MH) field equations in a space with non zero torsion and curvature with geometrically well defined magnetic and electric charge / current densities. In this new era of ECE theory there are no upper indices in the field equations, so for electromagnetism for example their format is the same as the MH equations. However, the magnetic and electric charge current densities are defined geometrically, and the equations are those of a generally covariant unified field theory (ECE theory) and not special relativity. The field equations of gravitation and of the weak and strong nuclear fields have the same format precisely as the field equations of electromagnetism, and field equations for the interaction of the fundamental fields can be developed.

This paper is accompanied as usual by detailed background notes which must be read with the paper. The notes accompany UFT315 on www.aias.us. Note 315(1) begins the development of the vector format of the tensorial JCE identity. Notes 315(2) to 315(5) give comprehensive detail on the development of the Evans torsion identity of UFT112 into identities of electromagnetism in vector format. Note 315(5) is useful because it gives all the complicated detail needed to reduce the tensorial to the vectorial field equations, including details of Hodge duality and the totally antisymmetric unit tensor in four dimensions, details of field definitions and so on. These details are almost always missing from textbooks but are fundamentally important. Note 315(6) begins the development of the new field equations of ECE, and note 315(7) gives the development in final form. Section 2 is based on Note 315(7).

2. NEW ECE FIELD EQUATIONS FROM THE JCE IDENTITY

Consider the JCE identity in a space of any dimensionality:

$$D_\rho R^a{}_{\lambda\mu\nu} + D_\nu R^a{}_{\lambda\rho\mu} + D_\mu R^a{}_{\lambda\nu\rho} := R^a{}_{\lambda\rho\alpha} T^\alpha{}_{\mu\nu} + R^a{}_{\lambda\nu\alpha} T^\alpha{}_{\rho\mu} + R^a{}_{\lambda\mu\alpha} T^\alpha{}_{\nu\rho}. \quad -(1)$$

This is a cyclic sum of covariant derivatives of curvature tensors. In Eq. (1) the index a of the Cartan tangent space has been used. In the original second Bianchi identity of 1902 this cyclic sum is incorrectly zero. The correct identity (1) includes torsion terms on the right hand side. In a space of four dimensions {1 - 12} a second JCE identity can be defined:

$$D_\rho \tilde{R}^a{}_{\lambda\mu\nu} + D_\nu \tilde{R}^a{}_{\lambda\rho\mu} + D_\mu \tilde{R}^a{}_{\lambda\nu\rho} := R^a{}_{\lambda\rho\alpha} \tilde{T}^\alpha{}_{\mu\nu} + R^a{}_{\lambda\nu\alpha} \tilde{T}^\alpha{}_{\rho\mu} + R^a{}_{\lambda\mu\alpha} \tilde{T}^\alpha{}_{\nu\rho} \quad -(2)$$

where the tilde denotes Hodge duality. The reason for this is that the Hodge dual of an antisymmetric tensor in four dimensions is another antisymmetric tensor {1 - 12}. Eqs. (1) and (2) are equivalent to:

$$D_\mu \tilde{R}^a{}_\lambda{}^{\mu\nu} := R^a{}_{\lambda\mu\alpha} \tilde{T}^{\alpha\mu\nu} \quad -(3)$$

and

$$D_\mu R^a{}_\lambda{}^{\mu\nu} := R^a{}_{\lambda\mu\alpha} T^{\alpha\mu\nu} \quad -(4)$$

The JCE identity can be expressed as Eq. (3) in a space of any dimensions.

Now define a new curvature tensor $R^{\mu\nu}$ as follows:

$$R^{\mu\nu} := q^\lambda{}_a R^a{}_\lambda{}^{\mu\nu} \quad -(5)$$

Its Hodge dual is:

$$\tilde{R}^{\mu\nu} := q^\lambda{}_a \tilde{R}^a{}_\lambda{}^{\mu\nu} \quad -(6)$$

Note carefully that these are fundamental new definitions. The curvature $R^{\mu\nu}$ and its Hodge dual $\tilde{R}^{\mu\nu}$ lead to new field equations. In electromagnetism for example they have the same format as the MH equations but within the context of a generally covariant unified field theory (ECE theory). The definition (5) also allows the magnetic and electric charge / current densities to be defined geometrically, so the fundamental nature of the magnetic and electric charges or monopoles and the magnetic and electric currents is understood in terms of geometry. This is of course the fundamental meaning of ECE theory, all physics is geometry within scaling factors.

Using the tetrad postulate it follows as in Note 315(7) that:

$$D_\mu \tilde{R}^{\mu\nu} = R_{\mu\alpha} \tilde{T}^{\alpha\mu\nu} \quad -(7)$$

and

$$D_\mu R^{\mu\nu} = R_{\mu\nu} T^{\alpha\mu\nu} \quad -(8)$$

Now use the definition of the covariant derivative of a rank two tensor (1 - 12}:

$$D_\sigma T^{\mu_1 \mu_2} = \partial_\sigma T^{\mu_1 \mu_2} + \Gamma^{\mu_1}_{\sigma\lambda} T^{\lambda \mu_2} + \Gamma^{\mu_2}_{\sigma\lambda} T^{\mu_1 \lambda} \quad -(9)$$

to find that:

$$D_\mu \tilde{R}^{\mu\nu} = \partial_\mu \tilde{R}^{\mu\nu} + \Gamma^{\mu}_{\mu\lambda} \tilde{R}^{\lambda\nu} + \Gamma^{\nu}_{\mu\lambda} \tilde{R}^{\mu\lambda} \quad -(10)$$

and

$$D_\mu R^{\mu\nu} = \partial_\mu R^{\mu\nu} + \Gamma^{\mu}_{\mu\lambda} R^{\lambda\nu} + \Gamma^{\nu}_{\mu\lambda} R^{\mu\lambda} . \quad -(11)$$

It follows that:

$$\partial_\mu \tilde{R}^{\mu\nu} = j^\nu \quad -(12)$$

and

$$\partial_\mu R^{\mu\nu} = J^\nu \quad -(13)$$

where:

$$j^\nu = R_{\mu\alpha} \tilde{T}^{\alpha\mu\nu} - \Gamma^{\mu}_{\mu\lambda} \tilde{R}^{\lambda\nu} - \Gamma^{\nu}_{\mu\lambda} \tilde{R}^{\mu\lambda} \quad -(14)$$

and

$$J^\nu = R_{\mu\alpha} T^{\alpha\mu\nu} - \Gamma^{\mu}_{\mu\lambda} R^{\lambda\nu} - \Gamma^{\nu}_{\mu\lambda} R^{\mu\lambda} . \quad -(15)$$

Now define the electromagnetic field tensor $F^{\mu\nu}$ and its Hodge dual $\tilde{F}^{\mu\nu}$ by

$$F^{\mu\nu} := W^{(0)} R^{\mu\nu} \quad -(16)$$

and

$$\tilde{F}^{\mu\nu} := W^{(0)} \tilde{R}^{\mu\nu} \quad -(17)$$

where $W^{(0)}$ is a scalar with the units of magnetic flux (weber, or tesla metres squared). Eqs (16) and (17) are new ECE hypotheses. It follows that the tensorial equations of electromagnetism are:

$$\partial_\mu \tilde{F}^{\mu\nu} = W^{(0)} j^{\nu} := j^{\nu}_{M} \quad -(18)$$

and

$$\partial_\mu F^{\mu\nu} = W^{(0)} J^{\nu} := J^{\nu}_{E} \quad -(19)$$

where j^{ν}_{M} and J^{ν}_{E} are the magnetic and electric charge current densities. To transform Eqs. (18) and (19) into vector equations define the field tensor and its Hodge dual as:

$$F^{\mu\nu} := \begin{bmatrix} 0 & -E_x/c & -E_y/c & -E_z/c \\ E_x/c & 0 & -B_z & B_y \\ E_y/c & B_z & 0 & -B_x \\ E_z/c & -B_y & B_x & 0 \end{bmatrix} \quad -(20)$$

and

$$\tilde{F}^{\mu\nu} := \begin{bmatrix} 0 & -B_x & -B_y & -B_z \\ B_x & 0 & E_z/c & -E_y/c \\ B_y & -E_z/c & 0 & E_x/c \\ B_z & E_y/c & -E_x/c & 0 \end{bmatrix} \quad -(21)$$

to obtain:

$$\underline{\nabla} \cdot \underline{B} = \tilde{W}^{(0)} j^0 \quad -(22)$$

$$\underline{\nabla} \times \underline{E} + \frac{\partial \underline{B}}{\partial t} = c\tilde{W}^{(0)} \underline{j} \quad -(23)$$

$$\underline{\nabla} \cdot \underline{E} = c\tilde{W}^{(0)} J^0 \quad -(24)$$

$$\underline{\nabla} \times \underline{B} - \frac{1}{c^2}\frac{\partial \underline{E}}{\partial t} = \tilde{W}^{(0)} \underline{J}. \quad -(25)$$

In these equations:

$$\underline{j} = j^1 \underline{i} + j^2 \underline{j} + j^3 \underline{k} = j_X \underline{i} + j_Y \underline{j} + j_Z \underline{k} \quad -(26)$$

and

$$\underline{J} = J^1 \underline{i} + J^2 \underline{j} + J^3 \underline{k} = J_X \underline{i} + J_Y \underline{j} + J_Z \underline{k}. \quad -(27)$$

For:

$$\nu = 0, 1, 2, 3 \quad -(28)$$

Eq. (14) defines j^0, j^1, j^2 and j^3; and Eq. (15) defines J^0, J^1, J^2 and J^3.

Eq. (16) has the correct units because $F^{\mu\nu}$ is defined in tesla (magnetic flux density in weber per square metre), and $W^{(0)}$ has the units of weber (magnetic flux). The units of the new curvature tensor are inverse square metres.

Eqs. (22) to (25) have the same structure as the MH equations of the nineteenth century, but they differ fundamentally because they are written in a space with non zero curvature and torsion. Furthermore they are part of a generally covariant unified field theory and are not equations of special relativity as originally devised by Heaviside, Lorentz and Poincaré. The equations that are almost always referred to in the dogmatic standard literature as "the Maxwell equations" are in fact the Heaviside equations. Finally Eqs. (18) and (19) contain a geometrically defined magnetic and electric charge / current density. The original MH equations contain no magnetic charge current density. In the nineteenth century MH equations charge and current are empirical, and have no internal structure.

The magnetic charge density or magnetic monopole is defined as:

$$j^0_{\underline{M}} = W^{(0)} j^0 \quad -(29)$$

where

$$j^0 = R_{\mu\alpha} \tilde{T}^{\alpha\mu 0} - \Gamma^{\mu}_{\mu\lambda} \tilde{R}^{\lambda 0} - \Gamma^{0}_{\mu\lambda} \tilde{R}^{\mu\lambda} \quad -(30).$$

The magnetic current density is

$$\underline{j}_{\underline{M}} = c W^{(0)} \underline{j} \quad -(31)$$

where $\underline{j}$ is defined by Eq. (26) and where the components of $\underline{j}$ are:

$$j^{\nu} = R_{\mu\alpha} \tilde{T}^{\alpha\mu\nu} - \Gamma^{\mu}_{\mu\lambda} \tilde{R}^{\lambda\nu} - \Gamma^{\nu}_{\mu\lambda} \tilde{R}^{\mu\lambda} \quad -(32)$$

with:

$$\nu = 1, 2, 3. \quad -(33)$$

The electric charge density is defined by:

$$J^0_E = c W^{(0)} J^0 \quad -(34)$$

where:

$$J^{0} = R_{\mu\alpha} T^{\alpha\mu 0} - \Gamma^{\mu}_{\mu\lambda} R^{\lambda 0} - \Gamma^{0}_{\mu\lambda} R^{\mu\lambda} \quad -(35)$$

and the electric current density is:

$$\underline{J}_E = W^{(0)} \underline{J} \quad -(36)$$

where

$$\underline{J} = J^{1}\underline{i} + J^{2}\underline{j} + J^{3}\underline{k} . \quad -(37)$$

For:

$$\nu = 1, 2, 3 \quad -(38)$$

then:

$$J^{\nu} = R_{\mu\alpha} T^{\alpha\mu\nu} - \Gamma^{\mu}_{\mu\lambda} R^{\lambda\nu} - \Gamma^{\nu}_{\mu\lambda} R^{\mu\lambda} . \quad -(39)$$

Obviously Eqs. (22) to (25) have been tested experimentally in the standard literature because they are respectively the Gauss, Faraday, Coulomb and Ampère Maxwell laws, but in ECE theory many new results are possible that go well beyond the standard model {1 - 12}. The entire development of ECE theory from 2003 to present can be repeated with the new theory of this section.

It is also possible to define the electromagnetic field as:

$$F^{a}{}_{\lambda\mu\nu} := W^{(0)} R^{a}{}_{\lambda\mu\nu} . \quad -(40)$$

The original ECE hypothesis of 2003 is:

$$F^{a}_{\mu\nu} := A^{(0)} T^{a}_{\mu\nu} . \quad -(41)$$

Using the Cartan identity:

$$D_\mu T^a_{\nu\rho} + D_\rho T^a_{\mu\nu} + D_\nu T^a_{\rho\mu} := R^a_{\mu\nu\rho} + R^a_{\rho\mu\nu} + R^a_{\nu\rho\mu} \quad -(42)$$

it follows that:

$$D_\mu F^a_{\nu\rho} + D_\rho F^a_{\mu\nu} + D_\nu F^a_{\rho\mu} := \frac{A^{(0)}}{W^{(0)}} \left(F^a_{\mu\nu\rho} + F^a_{\rho\mu\nu} + F^a_{\nu\rho\mu} \right) \quad -(43)$$

so the two field definitions (40) and (41) are fundamentally related.

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