ECE2 : DERIVATION OF THE GAUSS LAW OF MAGNETISM AND THE FARADAY LAW OF INDUCTION.

by

M. W. Evans, H. Eckardt and D. W. Lindstrom,

Civil List, AIAS and UPITEC

(www.webarchive.org.uk, www.aias.us, www.upitec.org, www.atomicprecision.com, www.et3m.net)

ABSTRACT

The second era of the development of ECE theory is given the appellation ECE2, and extends the original 2003 field definitions in terms of torsion to definitions in terms of curvature. Both torsion and curvature are always non zero. The ECE2 theory relies on the discovery in UFT313 of the Jacobi Cartan Evans (JCE) identity which corrects the second Bianchi identity for torsion. A new and straightforward method of removing the Cartan tangent indices is developed, and the Gauss law of magnetism and Faraday law of induction derived in EEC2 theory. ECE2 predicts a non zero magnetic charge current density in general, and a non zero magnetic monopole.

Keywords: ECE2 theory, Gauss law of magnetism and Faraday law of induction, magnetic charge current density.

UFT 316

1. INTRODUCTION

In recent papers of this series {1 - 10} the second Bianchi identity of 1902 has been developed in UFT313 into the Jacobi Cartan Evans (JCE) identity with torsion. It has been shown in several papers of the series that torsion can never be arbitrarily equated to zero, a null torsion means a null curvature and the disappearance of Einsteinian gravitation. For this reason alone it is clear that torsionless Einsteinian gravitational theory is meaningless, and indeed it has been replaced to international acclaim {11} by ECE theory. Eleven years of detailed and accurate scientometrics show this beyond doubt. In UFT314 and UFT315 the tensor notation of UFT313 was developed into vector notation and several new vector equations devised. In Section 2 the methods of UFT313 to UFT315 are used to infer the Gauss law of magnetism and Faraday law of induction. In general it is shown that the magnetic charge current density is non zero. The appellation ECE2 is given to a new era of ECE theory in which the original hypotheses of 2003 are extended to include potential and field definitions based on curvature. These augment the original 2003 definitions, and together with a new and straightforward method of removing tangent indices, lead to a simpler and more powerful theory. Examples of this theory were given in UFT314 and UFT316 in vector notation and these methods are extended to the homogeneous field equations in Section 2. In electrodynamics these are the Gauss law of magnetism and the Faraday law of induction. The resulting equations of ECE2 are the same in format as the original Maxwell Heaviside (MH) equations of the nineteenth century. However, there are profound differences of philosophy, the MH equations are not part of a unified field theory and contain no torsion and no curvature. They are equations of special relativity. The new equations of this paper are part of a generally covariant unified field theory (ECE2) in which both torsion and curvature are always non zero.

As usual this paper should be read with its background notes accompanying UFT316 on www.aias.us, the notes are an intrinsic part of each UFT paper. In Note 316(1) a method is introduced of removing the tangent indices. In Note 316(2) the Cartan and JCE identities are written out in tensor notation and the Cartan identity reduced to vector notation following work initiated in UFT254 and UFT255. In note 316(3) detailed definitions are given of the complex circular basis and notation, while Section 2 is a summary of Notes 316(4) to 316(7).

2. DEFINITIONS OF ECE2 AND THE HOMOGENEOUS FIELD EQUATIONS.

Consider the Cartan identity:

$$D_\mu T^a_{\nu\rho} + D_\rho T^a_{\mu\nu} + D_\nu T^a_{\rho\mu} := R^a_{\mu\nu\rho} + R^a_{\rho\mu\nu} + R^a_{\nu\rho\mu} \quad -(1)$$

in which D_μ is the covariant derivative, $T^a_{\nu\rho}$ is the torsion and $R^a_{\mu\nu\rho}$ the curvature. With the ECE2 definitions:

$$F^a_{\nu\rho} = A^{(0)} T^a_{\nu\rho} \quad -(2)$$

and

$$F^a_{\mu\nu\rho} = W^{(0)} R^a_{\mu\nu\rho} \quad -(3)$$

Eq. (1) becomes:

$$D_\mu F^a_{\nu\rho} + D_\rho F^a_{\mu\nu} + D_\nu F^a_{\rho\mu} := \frac{A^{(0)}}{W^{(0)}} \left(F^a_{\mu\nu\rho} + F^a_{\rho\mu\nu} + F^a_{\nu\rho\mu} \right). \quad -(4)$$

Here $F^a_{\mu\nu}$ is a vector valued two form, the electromagnetic field tensor with the units of magnetic flux density, tesla. $F^a_{\mu\nu\rho}$ is a vector valued three form, a new format of the electromagnetic field tensor. The scalar $W^{(0)}$ has the units of magnetic flux (weber, or tesla metres squared). The scalar $A^{(0)}$ has the units of tesla metres. So

$$\frac{W^{(0)}}{A^{(0)}} = r^{(0)} \quad -(5)$$

where $r^{(0)}$ is a scalar with the units of metres.

In vector notation (UFT254 and UFT255), Eq. (1) splits into two equations, depending on an index:

$$\nu = 0, 1, 2, 3. \quad -(6)$$

The index:

$$\nu = 0 \quad -(7)$$

gives the first vector equation:

$$\underline{\nabla} \cdot \underline{T}^a(\text{spin}) + \underline{\omega}^a{}_b \cdot \underline{T}^b(\text{spin}) = \underline{q}^b \cdot \underline{R}^a{}_b(\text{spin}) \quad -(8)$$

where $\underline{T}^a(\text{spin})$ is the spin torsion vector, $\underline{\omega}^a{}_b$ is the spin connection vector, $\underline{q}^b$ is the tetrad vector, and $R^a{}_b(\text{spin})$ the spin curvature vector. In the original ECE theory the magnetic flux density vector in tesla was defined as:

$$\underline{B}^a = A^{(0)} \underline{T}^a(\text{spin}). \quad -(9)$$

This definition is also used in ECE2 but is augmented by:

$$\underline{B}^a{}_b = W^{(0)} \underline{R}^a{}_b(\text{spin}) \quad -(10)$$

where $W^{(0)}$ is a scalar with the units of magnetic flux (weber). The units of the spin curvature vector $R^a{}_b$ are inverse square metres.

Therefore Eq. (8) of geometry becomes:

$$\underline{\nabla}\cdot\underline{B}^a+\underline{\omega}^a{}_b\cdot\underline{B}^b=\left(\frac{A^{(0)}}{W^{(0)}}\right)\underline{q}^b\cdot\underline{B}^a{}_b$$

$$=\frac{1}{W^{(0)}}\underline{A}^b\cdot\underline{B}^a{}_b=\frac{1}{r^{(0)}}\underline{q}^b\cdot\underline{B}^a{}_b \quad -(11)$$

of electromagnetism. The electromagnetic potential is defined as in the original ECE theory:

$$\underline{A}^b=A^{(0)}\underline{q}^b \quad -(12)$$

so Eq. (11) becomes:

$$\underline{\nabla}\cdot\underline{B}^a=\frac{1}{W^{(0)}}\underline{A}^b\cdot\underline{B}^a{}_b-\underline{\omega}^a{}_b\cdot\underline{B}^b \quad -(13)$$

which is the Gauss law of magnetism, Q. E. D.. The magnetic monopole is defined by:

$$J^0_M=\frac{1}{W^{(0)}}\underline{A}^b\cdot\underline{B}^a{}_b-\underline{\omega}^a{}_b\cdot\underline{B}^b. \quad -(14)$$

ECE2 introduces a procedure for removing the tangent indices a and b in order to simplify these equations for applications in the natural sciences and engineering. The self consistency of this method is tested fully in Note 316(3). For example:

$$\underline{B}=-e_a\underline{B}^a \quad -(15)$$

where e_a is the unit vector in the tangent space. In the complex circular basis:

$$e_{(a)}=\left(1,-\frac{1}{\sqrt{2}}(1+i),-\frac{1}{\sqrt{2}}(1-i),-1\right) \quad -(16)$$

and in the Cartesian basis:

$$e_a=(1,-1,-1,-1). \quad -(17)$$

Therefore in the complex circular basis:

$$\underline{B} = e_{(0)}\underline{B}^{(0)} - e_{(1)}\underline{B}^{(1)} - e_{(2)}\underline{B}^{(2)} - \underline{e}_{(3)}\underline{B}^{(3)}. \quad -(18)$$

By definition

$$\underline{B}^{(0)} = \underline{0} \quad -(19)$$

because the spacelike vector $\bar{B}$ has no timelike component. In general:

$$\underline{B}^{(1)} = \frac{1}{\sqrt{2}}\left(B_X\,\underline{i} - iB_Y\,\underline{j}\right) \quad -(20)$$

and:

$$\underline{B}^{(2)} = \frac{1}{\sqrt{2}}\left(B_X\,\underline{i} + iB_Y\,\underline{j}\right) \quad -(21)$$

so in Eq. (18):

$$\underline{B} = \frac{1}{\sqrt{2}}(1+i)\frac{1}{\sqrt{2}}\left(B_X\underline{i} - iB_Y\underline{j}\right) + \frac{1}{\sqrt{2}}(1-i)\frac{1}{\sqrt{2}}\left(B_X\underline{i} + iB_Y\underline{j}\right) + B_Z\underline{k}$$

$$= B_X\,\underline{i} + B_Y\,\underline{j} + B_Z\,\underline{k} \quad -(22)$$

where we have used:

$$\underline{B}^{(3)} = B_Z\,\underline{k}. \quad -(23)$$

Now multiply both sides of Eq. (13) by $-e_a$ to obtain:

$$\underline{\nabla}\cdot\underline{B} = \frac{1}{W^{(0)}}\underline{A}^{b}\cdot\underline{B}_{b} - \underline{\omega}_{b}\cdot\underline{B}^{b} \quad -(24)$$

in which:

$$\underline{A}^b \cdot \underline{B}_b = e^b e_b \underline{A} \cdot \underline{B} \quad -(25)$$

$$\underline{\omega}_b \cdot \underline{B}^b = e_b e^b \underline{\omega} \cdot \underline{B}. \quad -(26)$$

In the Cartesian basis:

$$e_b e^b = e^b e_b = -2 \quad -(27)$$

and in the complex circular basis:

$$e_b e^{b*} = e^b e_b^* = -2 \quad -(28)$$

where * denotes complex conjugate. So:

$$\underline{\nabla} \cdot \underline{B} = 2\underline{B} \cdot \left(\underline{\omega} - \frac{\underline{A}}{W^{(0)}} \right) \quad -(29)$$

and the magnetic monopole can be defined by:

$$J_m^0 = 2\underline{B} \cdot \left(\underline{\omega} - \frac{\underline{A}}{W^{(0)}} \right) \quad -(30)$$

It vanishes if and only if:

$$\underline{A} = W^{(0)} \underline{\omega}. \quad -(31)$$

As shown in detail in Note 316(5), following UFT254 and UFT255, and the Engineering Model UFT303, the spin torsion and spin curvature are defined in vector notation by:

$$\underline{T}^b(\text{Spin}) = \underline{\nabla} \times \underline{q}^b - \underline{\omega}^b{}_c \times \underline{q}^c \quad -(32)$$

and:

$$\underline{R}^a{}_b(\text{Spin}) = \underline{\nabla} \times \underline{\omega}^a{}_b - \underline{\omega}^a{}_c \times \underline{\omega}^c{}_b \quad -(33)$$

It follows after some vector algebra that:

$$\underline{\nabla} \cdot \underline{q}^b \times \underline{\omega}^a{}_b = 0 \quad -(34)$$

which was first derived in UFT254 and UFT255 and which is the most succinct format of the vector equation (8).

In ECE2, define the magnetic flux potential by:

$$\underline{W}^a{}_b = W^{(0)} \underline{\omega}^a{}_b \quad -(35)$$

in tesla metres. The ECE2 hypothesis (35) augments the original ECE hypothesis:

$$\underline{A}^a = A^{(0)} \underline{q}^a \quad -(36)$$

in tesla metres. Therefore:

$$\underline{\nabla} \cdot \underline{A}^b \times \underline{W}^a{}_b = 0. \quad -(37)$$

The indices can be removed using the new method of this paper to give:

$$\underline{\nabla} \cdot \underline{A} \times \underline{W} = 0. \quad -(38)$$

This is a fundamental relation in ECE2 between $\underline{A}$ and $\underline{W}$.

In order to derive the Faraday law of induction consider as in UFT254 and UFT255 the second vector format of the Cartan identity. This is derived from indices:

$$\sim = 1, 2, 3 \qquad -(39)$$

and is:

$$\frac{1}{c}\frac{\partial \underline{T}^a(\text{spin})}{\partial t} + \underline{\nabla} \times \underset{\sim}{T}^a(\text{orb}) = q^b_{\ 0}\underline{R}^a_{\ b}(\text{spin}) + \underline{q}^b \times \underline{R}^a_{\ b}(\text{orb}) - \left(\omega^a_{\ 0b}\underset{\sim}{T}^b(\text{spin}) + \underline{\omega}^a_{\ b} \times \underset{\sim}{T}^b(\text{orb})\right) \qquad -(39)$$

in which $T^a(\text{orb})$ is the orbital torsion:

$$\underline{T}^a(\text{orb}) = -\underline{\nabla} q^a_{\ 0} - \frac{1}{c}\frac{\partial \underline{q}^a}{\partial t} - \omega^a_{\ 0b}\underline{q}^b + q^b_{\ 0}\underline{\omega}^a_{\ b} \qquad -(40)$$

and $R^a_{\ b}(\text{orb})$ the orbital curvature

$$\underline{R}^a_{\ b}(\text{orb}) = -\underline{\nabla}\omega^a_{\ 0b} - \frac{1}{c}\frac{\partial \underline{\omega}^a_{\ b}}{\partial t} - \omega^a_{\ 0c}\underline{\omega}^c_{\ b} + \omega^c_{\ 0b}\underline{\omega}^a_{\ c}. \qquad -(41)$$

Notes 216(6) and 216(7) translate these equations of geometry into equations of electrodynamics using:

$$\underline{B}^a = A^{(0)}\underline{T}^a(\text{spin}) \qquad -(42)$$

$$\underline{B}^a_{\ b} = W^{(0)}\underline{R}^a_{\ b}(\text{spin}) \qquad -(43)$$

$$\underline{E}^a = cA^{(0)}\underline{T}^a(\text{orb}) \qquad -(44)$$

$$\underline{E}^a_{\ b} = cW^{(0)}\underline{R}^a_{\ b}(\text{orb}) \qquad -(45)$$

After some vector algebra written out in full in Note 216(7), it is deduced that the Faraday law of induction is:

$$\frac{\partial \underline{B}}{\partial t} + \underline{\nabla} \times \underline{E} = \underline{J}_m \quad -(46)$$

$$-(47)$$

where the magnetic current density is:

$$\underline{J}_m = 2 \left(c \left(\omega_0 - \frac{v_0}{r^{(0)}} \right) \underline{B} + \left(\underline{\omega} - \frac{\underline{v}}{r^{(0)}} \right) \times \underline{E} \right)$$

This is zero if and only if:

$$v_0 = r^{(0)} \omega_0 \quad -(48)$$

and

$$\underline{v} = r^{(0)} \underline{\omega}. \quad -(49)$$

ACKNOWLEDGMENTS

The British Government is thanked for a Civil List Pension and the staff of AIAS and others for many interesting discussions. Dave Burleigh is thanked for posting, site maintenance and scientometrics programs, Alex Hill is thanked for translation and broadcasting, and Robert Cheshire for broadcasting.

REFERENCES

{1} M .W. Evans, H. Eckardt, D. W. Lindstrom and S. J. Crothers, "Principles of ECE Theory" (UFT281 to UFT288 and New Generation Publishing, in prep.)

{2} M. W. Evans, Ed., J. Found. Phys. Chem., (open source on www.aias.us and Cambridge International Sceince Publishing, CISP, www.cisp-publishing.com, 2011)

{3} M . W. Evans, Ed., "Definitive Refutations of the Einsteinian General Relativity" (CISP 2012 and open source on www.aias.us)

{4} M .W. Evans, S. J. Crothers, H. Eckardt and K. Pendergast, "Criticisms of the Einstein Field Equation" (UFT301 and CISP 2010).

{5} L. Felker, "The Evans Equations of Unified Field Theory" (UFT302 and Abramis 2007), Spanish translation by Alex Hill on www.aias.us.

{6} H. Eckardt, "The ECE Engineering Model" (UFT303).

{7} M .W. Evans, H. Eckardt and D. W. Lindstrom, "Generally Covariant Unified Field Theory (open source UFT papers and Abramis 2005 to 2011 in seven volumes).

{8} M .W. Evans and L. B. Crowell, "Classical and Quantum Electrodynamics and the B(3) Field" (World Scientific 2001 open source Omnia Opera of www.aias.us).

{9} M. W. Evans and S. Kielich, Eds., "Modern Nonlinear Optics" (Wiley Interscience, New York, 1992, 1993, 1997, 2001) in two editions and six volumes.

{10} M. W. Evans and J.-P. Vigier, "The Enigmatic Photon" (Kluwer, Dordrecht 1994 to 2002 and open souce Omnia Opera Section of www.aias.us in five volumes).

{11} M. W. Evans, "Collected Scientometrics" (UFT307 and New Generation Publishing, London 2015).

{12} M. W. Evans and A. A. Hasanein, "The Photomagneton in Quantum Field Theory" (World Scientific 1994).