SOLUTION OF THE ECE2 LORENTZ FORCE EQUATION: THE RELATIVISTIC BINET EQUATIONS AND APPLICATIONS TO THE VELOCITY CURVE OF A WHIRLPOOL GALAXY AND LIGHT DEFLECTION BY GRAVITATION.

by

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ABSTRACT

The recently derived Lorentz force equation of ECE2 theory is solved by using the relativistic Binet equation for force, and its integral form for the hamiltonian. The relativistic Binet equation is derived from the Sommerfeld hamiltonian and the relativistic orbital velocity calculated straightforwardly. The equation for the orbital velocity is used to derive the observed velocity curve of a whirlpool galaxy and the precisely observed deflection of light or electromagnetic radiation due to gravitation. These are major advances in understanding which overthrow the obsolete Einsteinian relativity.

Keywords: ECE2, relativistic Binet equations, velocity curve of a whirlpool galaxy, light deflection due to gravitation.

UFT 324

1. INTRODUCTION

In recent papers of this series {1 - 12} the Lorentz force equation of ECE2 theory has been derived by Lorentz transformation of the gravitomagnetic field tensor and some calculations made of the gravitomagnetic field. The ECE2 theory is a generally covariant unified field theory that transforms according to the Lorentz transform. This key property is due to the mathematical structure of the ECE2 field equations, a structure that is identical with that of the Maxwell Heaviside (MH) field equations of special relativity. So the ECE2 and MH field tensors transform in the same way - under the Lorentz transformation. However ECE2 is a theory of general relativity with torsion and curvature both non-zero, whereas MH is a theory that is developed in Minkowski spacetime. The concepts of torsion and curvature do not exist in MH theory.

Using this property the well known equations and ideas of special relativity can be used in orbital theory. In this paper it is shown that the Lorentz transform is sufficient to produce the observed velocity curve of a whirlpool galaxy, and the famous result of light deflection due to gravitation. These phenomena are therefore explained by ECE2 theory in a straightforward way.

As usual this paper should be read with its background notes posted with it in the UFT section of www.aias.us. The paper is a brief summary of the detailed material in the notes, especially the later notes in which new ideas have crystallized out. Computer algebra is used to eliminate human error, and to evaluate and graph unavoidably complicated equations. Notes 324(1) and 324(2) begin the evaluation of the relativistic Binet equations, which are derived from the well known lagrangian of special relativity. The relativistic Binet force equation is equivalent to the relativistic ECE2 Lorentz force equation, so a solution of the Binet equation is also a solution of the Lorentz force equation. In Notes 324 (3) and 324(4) the novel and original inference is made of an integral form of the Binet force equation. The integral form allows the evaluation of the hamiltonian for any orbit and the Binet force equation allows the evaluation of the central force and gravitational potential for any orbit. In this paper the orbit is exemplified by use of the plane polar coordinates, i.e. the orbit is a planar orbit. However this method can be applied to non planar orbits {1 - 12} and dynamics in general. In note 324(5) it is exemplified by application to a precessing orbit. Note 324(6) evaluates the relativistic orbital linear velocity, and gives the solution of the Lorentz force equation in terms of the Binet equation. Note 324(7) shows that the velocity curve of a whirlpool galaxy is described precisely and straightforwardly from the relativistic orbital velocity of ECE2, and Note 324(8) shows that light deflection due to gravitation is described precisely by the same relativistic orbital velocity.

Section 2 is a summary of the main developments in the notes, and Section 3 is a numerical and graphical analysis.

2. SOLUTIONS OF THE ECE2 LORENTZ FORCE EQUATION AND APPLICATIONS.

It is shown in this section that the solution of the ECE2 Lorentz force equation for a planar orbit is:

$$\underline{F} = m\left(\gamma\left(\ddot{\underline{r}} + \underline{v}_{\Omega} \times \underline{\Omega}\right) - \frac{\gamma^2}{1+\gamma}\,\frac{\underline{v}_{\Omega}}{c}\left(\frac{\underline{v}_{\Omega}}{c}\cdot\underline{g}\right)\right) = \frac{\partial}{\partial r}\left(\left(\gamma-1\right)mc^2\right)\underline{e}_r \quad -(1)$$

In these equations, a mass m orbits a mass M in an orbit that is in general a relativistic orbit. The relativistic Binet force equation for a planar orbit is:

$$\underline{F} = \frac{\partial}{\partial r}\left(\left(\gamma-1\right)mc^2\right)\underline{e}_r \quad -(2)$$

where the Lorentz factor is defined by:

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \quad -(3)$$

The velocity of the Lorentz factor is defined from the Minkowski metric to be:

$$v^2 = \dot{r}^2 + \dot{\theta}^2 r^2 . \quad -(4)$$

In the Lorentz force equation $\underline{v}_{\underline{\Omega}}$ is the velociy of one frame with respect to another and $\underline{\Omega}$ is the gravitomagnetic field.

The relativistic Binet equations are derived from the well known lagrangian of special relativity:

$$\mathcal{L} = -\frac{mc^2}{\gamma} - U(r) \quad -(5)$$

where U (r) is a central potential. The hamiltonian of special relativity can be derived from this lagrangian and is:

$$H = E + U(r) \quad -(6)$$

where the total relativistic energy is:

$$E = \gamma mc^2 . \quad -(7)$$

The hamiltonian (6) can be rewritten as the Sommerfeld hamiltonian:

$$H(\text{Sommerfeld}) = H - mc^2 = (\gamma - 1)mc^2 + U(r) \quad -(8)$$

where:

$$T = (\gamma - 1)mc^2 \quad -(9)$$

is the relativistic kinetic energy. In the non relativistic limit:

$$T \to \left(1 + \frac{v^2}{2c^2} - 1\right) mc^2 = \frac{1}{2} mv^2 = \frac{p^2}{2m}. \quad -(10)$$

The Euler Lagrange equations of the system are

$$\frac{\partial \mathcal{L}}{\partial \theta} = \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{\theta}} \quad -(11)$$

$$\frac{\partial \mathcal{L}}{\partial r} = \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{r}} \quad -(12)$$

for a central potential that depends only on r, and not on θ , they produce the results:

$$\frac{\partial \mathcal{L}}{\partial \theta} = \gamma m r^2 \dot{\theta} \quad -(13)$$

and:

$$\frac{d}{dt}\left(\gamma m \dot{r}\right) - \gamma m r \dot{\theta}^2 = -\frac{\partial U}{\partial r} = F(r) \quad -(14)$$

Eq. (13) defines the relativistic angular momentum:

$$L = \gamma m r^2 \dot{\theta} \quad -(15)$$

which is a constant of motion:

$$\frac{dL}{dt} = 0. \quad -(16)$$

Eq. (14) defines the relativistic force equation of the orbit:

$$F(r) = \frac{d}{dt}\left(\gamma m \dot{r}\right) - \gamma m r \dot{\theta}^2 \quad -(17)$$

in which:

$$m \frac{d}{dt}\left(\gamma \dot{r}\right) = m\left(\dot{r}\frac{d\gamma}{dt} + \gamma \ddot{r}\right) \quad -(18)$$

Here:

$$\frac{d\gamma}{dt} = \frac{d\gamma}{dv} \cdot \frac{dv}{dt} \quad -(19)$$

therefore:

$$\frac{d}{dt}\left(\gamma m \dot{r}\right) = m\left(\dot{r}\,\frac{\gamma^3 v}{c^2}\,\frac{dv}{dt} + \gamma \ddot{r}\right) \quad -(20)$$

where:

$$v = \left(\dot{r}^2 + \dot{\theta}^2 r^2\right)^{1/2} \quad -(21)$$

In general this is a complicated expression that must be developed with computer algebra.

The Binet equations are derived by making a change of variable {1 - 12}:

$$\frac{d}{d\theta}\left(\frac{1}{r}\right) = -\frac{1}{r^2}\frac{dr}{d\theta} = -\frac{1}{r^2}\frac{dr}{dt}\frac{dt}{d\theta} \quad -(22)$$

where

$$\frac{dt}{d\theta} = \frac{\gamma m r^2}{L} \quad -(23)$$

from Eq. (15). It follows that:

$$\dot{\theta} = \frac{L}{\gamma m r^2} \quad -(24)$$

and

$$\dot{r} = -\frac{L}{\gamma m}\frac{d}{d\theta}\left(\frac{1}{r}\right) \quad -(25)$$

The orbital velocity is therefore:

$$v_N^2 = \dot{r}^2 + \dot{\theta}^2 r^2 = \frac{L^2}{\gamma^2 m^2}\left(\left(\frac{d}{d\theta}\left(\frac{1}{r}\right)\right)^2 + \frac{1}{r^2}\right)$$

$$= \frac{L^2}{m^2}\left(1-\frac{v_N^2}{c^2}\right)\left(\left(\frac{d}{d\theta}\left(\frac{1}{r}\right)\right)^2+\frac{1}{r^2}\right) \quad -(26)$$

and the integral form of the relativistic Binet equation is found directly from the Sommerfeld hamiltonian:

$$H - mc^2 = \left(\left(1-\frac{v_N^2}{c^2}\right)^{-1/2} - 1\right)mc^2 - \frac{mMG}{r} \quad -(27)$$

in which:

$$v_N^2 = \dot{r}^2 + \dot{\theta}^2 r^2, \quad -(28)$$

$$L = \gamma m r^2 \dot{\theta} := \gamma L_0 \quad -(28a)$$

So the relativistic orbital velocity is:

$$v_N^2 = \frac{\frac{L^2}{m^2}\left(\left(\frac{d}{d\theta}\left(\frac{1}{r}\right)\right)^2+\frac{1}{r^2}\right)}{1+\frac{L^2}{m^2c^2}\left(\left(\frac{d}{d\theta}\left(\frac{1}{r}\right)\right)^2+\frac{1}{r^2}\right)} \quad -(29)$$

Note carefully that:

$$H - mc^2 = H(\text{Sommerfeld}) \quad -(30)$$

is a constant of motion, so the relativistic Binet force equation is:

$$F = \frac{\partial}{\partial r}\left((\gamma-1)mc^2\right) \quad -(31)$$

which is the required solution of the Lorentz force equation of ECE2, QED.

In the non relativistic limit, the integral form of the Binet equation is:

$$U = H - \frac{L^2}{2m}\left(\left(\frac{d}{d\theta}\left(\frac{1}{r}\right)\right)^2 + \frac{1}{r^2}\right) \quad -(32)$$

and the Binet force equation is the well known {1 - 12}:

$$F(r) = -\frac{L^2}{mr^2}\left(\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r}\right) \quad -(33)$$

Using these two equations, F, U and H can be found for any planar orbit in the non relativistic limit. Some examples are given in the notes, for example a precessing orbit and a hyperbolic spiral orbit. For a precessing conic section {1 - 12}:

$$r = \frac{\alpha}{1 + \epsilon\cos(x\theta)} \quad -(34)$$

where α is the half right latitude, ϵ the eccentricity and x the precession constant. For this orbit (Note 324(5)), the central force is:

$$F = -\frac{\partial U}{\partial r} = -\frac{x^2 L^2}{mr^2\alpha} + \frac{(x^2 - 1)L^2}{mr^3} \quad -(35)$$

and the gravitational potential is:

$$U = -\frac{x^2 L}{m\alpha r} + \frac{1}{2}(x^2 - 1)\frac{L^2}{mr^2} \quad -(36)$$

The hamiltonian is:

$$H = \frac{x^2 L^2}{2m}\left(\frac{\epsilon^2 - 1}{\alpha^2}\right) \quad -(37)$$

In the Newtonian limit when x is unity, the results become the well known:

$$F = -\frac{mMG}{r^2}, \quad U = -\frac{mMG}{r}, \quad |E| = \frac{mMG}{2a} \qquad -(38)$$

where a is the semi major axis.

It follows that the Einstein theory is not needed to describe a precessing orbit. It can be described classically as above. Note carefully that the Einstein field equation gives an incorrect force law {1 - 12} that is a sum of terms that are inverse squared in r and inverse to the fourth power in r. The correct expression is given in Eq. (35).

As described in detail in Note 324(7), the relativistic orbital velocity (29) gives the correct experimental result for the velocity curve of a spiral galaxy using the hyperbolic spiral orbit of a star moving outwards from the centre of the galaxy:

$$\frac{1}{r} = \frac{\theta}{r_0}. \qquad -(39)$$

From Eqs. (29) and (39) the velocity curve of the spiral galaxy is:

$$v^2 = \frac{L_0^2}{m^2}\left(\frac{1}{r_0^2} + \frac{1}{r^2}\right)\left(1 - \frac{L_0^2}{m^2c^2}\left(\frac{1}{r_0^2} + \frac{1}{r^2}\right)\right)^{-1} \qquad -(40)$$

so goes to the observed constant plateau:

$$v \xrightarrow[r \to \infty]{} \frac{L}{mr_0}\left(1 - \frac{L^2}{m^2c^2r_0^2}\right)^{-1/2}. \qquad -(41)$$

This is a very strong indication that the present theory is both correct and preferred by Ockham's Razor to theories based for example on "dark matter". In contrast the Newtonian velocity curve is well known to be {1 - 12}:

$$v^2(\text{Newton}) = mG\left(\frac{2}{r} - \frac{1}{a}\right) \quad -(42)$$

so:

$$v^2 = \frac{mG}{r}\left(2 + \frac{(\epsilon^2 - 1)}{1 + \epsilon\cos\theta}\right) \xrightarrow[r \to \infty]{} 0 \quad -(43)$$

and the Newtonian theory collapses entirely in a whirlpool galaxy. The Einsteinian orbit is claimed to be:

$$r = \frac{\alpha}{1 + \epsilon\cos(x\theta)} \quad -(44)$$

so the Einsteinian velocity curve is:

$$v^2(\text{Einstein}) = \frac{mG}{r}\left(2 + \frac{(\epsilon^2 - 1)}{1 + \epsilon\cos(x\theta)}\right) \xrightarrow[r \to \infty]{} 0. \quad -(45)$$

and the Einsteinian general relativity also fails completely in a whirlpool galaxy.

So ECE2 is clearly preferred both to Newton and Einstein as a theory of cosmology and a unified field theory. Newton is classical, and Einstein is not a unified field theory.

Finally it is shown that the relativistic orbital velocity (29) gives the precisely correct experimental result for electromagnetic radiation deflected by gravitation. The relativistic orbital velocity from Eq. (29) is:

$$v_N^2 = \frac{v^2}{1 + \frac{v^2}{c^2}} \quad -(46)$$

where v_N^2 is the non relativistic orbital velocity:

$$v_N^2 = \dot{r}^2 + \dot{\theta}^2 r^2 \quad -(47)$$

In light deflection by gravitation the light beam travels very close to c, so:

$$v \sim c \quad (48)$$

It follows that the non-relativistic velocity is defined by:

$$v_N = \frac{c}{2}. \quad (49)$$

The orbit of the light is a hyperbola:

$$r = \frac{\alpha}{1 + \epsilon \cos\theta} \quad (50)$$

in which the eccentricity is very large, so the orbit is deflected by only a few arc seconds from a straight line.

The non relativistic orbital velocity is the Newtonian:

$$v_N^2 = MG\left(\frac{2}{r} - \frac{1}{a}\right) \quad (51)$$

where the semi major axis is:

$$a = \frac{\alpha}{1 - \epsilon^2}. \quad (52)$$

At closest approach:

$$R_0 = \frac{\alpha}{1 + \epsilon}. \quad (53)$$

It follows that:

$$v_N^2 = \frac{MG}{R_0}\left(2 + \frac{\epsilon^2 - 1}{\epsilon + 1}\right) = \frac{MG}{R_0}(1 + \epsilon) \quad (54)$$

and for a very large eccentricity:

$$\epsilon \sim \frac{R_0 v_N^2}{MG} \quad (55)$$

The angle of deflection is:

$$\Delta\theta = \frac{2}{\epsilon} = \frac{2MG}{R_0 v_N^2} \quad -(56)$$

which is the Newtonian result.

However, the non relativistic velocity v for v_0 approaching c is as above:

$$v_N^2 = \frac{c^2}{2} \quad -(57)$$

so the angle of deflection is:

$$\Delta\theta = \frac{4MG}{R_0 c^2} \quad -(58)$$

which is precisely the experimental result, Q. E. D., a result known to great precision. This is overwhelming evidence that ECE2 as solved by the Binet equation is the correct theory of all orbits.

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