ORBITAL PRECESSION FROM ECE2 AND FROM THE LAGRANGIAN OF SPECIAL RELATIVITY.

by

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ABSTRACT

The solution of the gravitomagnetic Lorentz force equation of ECE2 is expressed in terms of the lagrangian and hamiltonian of special relativity. The lagrangian is the classical Sommerfeld lagrangian and is solved by computer algebra and numerical methods. A scatter plot method is used to how that the true orbit is a precessing ellipse. This is not the Einsteinian result because the Einstein theory is incorrect in many well known ways. Therefore ECE2 gives a precessing ellipse, the exact experimental value for light deflection by gravitation, and the velocity curve of a whirlpool galaxy.

Keywords: ECE2 theory, gravitomagnetic Lorentz force equation, the lagrangian of special relativity, precession of the perihelion.

UFT 325

1. INTRODUCTION

In recent papers of this series {1 - 12}, the original second Bianchi identity of 1902 has been developed into the Jacobi Cartan Evans (JCE) identity of UFT313, and ECE2 theory developed in several ways in terms of vector field equations and a relativistic and gravitomagnetic Lorentz force equation (UFT314 - UFT320, UFT322 - UFT324). In the immediately preceding paper it was shown that ECE2 gives the precisely correct experimental result for the deflection of electromagnetic radiation by gravitation, and also the correct velocity curve of a whirlpool galaxy. Both Einstein and Newton fail completely in a whirlpool galaxy as is well known (UFT288). In this paper the relativistic Lorentz force equation is solved to give the precession of the perihelion by using its equivalence to the well known lagrangian and hamiltonian of special relativity.

As usual this paper should be read along with its background notes and supplementary postings of computer protocol on the blog of www.aias.us. Notes 325(1) to 325(4) discuss various aspects of the solution of the relativistic Binet force equation and its integrated format discovered in UFT324. Note 5 derives the relativistic orbital velocity of special relativity from the lagrangian, Note 6 discusses the true orbit of special relativity, which is the correct orbit of a planet. Notes (1) to (4) contain several refutations of the Einstein method, which is also refuted definitively in Notes 9 and (9a), adding to the multiple refutations of the Einstein method, and refuting graphics, in UFT232. Section 2 of this apper is based on Notes 5, 7 and 8.

In Section 3, the relativistic lagrangian is solved numerically, using computer algebra, numerical integration as in UFT239, and a scatter plot method of producing the true orbit. This is the first time that the true orbit of a precessing planet or any other object has been derived without empiricism. The true orbit is not that of Newton or Einstein, and can only be approximated by x theory, developed extensively in previous UFT papers.

2. DEVELOPMENT OF THE LAGRANGIAN AND HAMILTONIAN

Consider the hamiltonian and lagrangian of special relativity. These are also the hamiltonian and lagrangian of ECE2, equivalent to the ECE2 Lorentz force equation as shown in immediately preceding papers. The hamiltonian of special relativity is:

$$H = \gamma mc^2 + U \quad -(1)$$

and the lagrangian is:

$$\mathcal{L} = -\frac{mc^2}{\gamma} - U \quad -(2)$$

where

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \quad -(3)$$

is the Lorentz factor, and where U is the potential energy. The well known Sommerfeld hamiltonian is:

$$H_S = H - mc^2 = (\gamma - 1)mc^2 + U \quad -(4)$$

where

$$T = (\gamma - 1)mc^2 \quad -(5)$$

is the relativistic kinetic energy. The relativistic total energy is:

$$E = \gamma mc^2. \quad -(6)$$

Assume that the force of attraction between a mass m orbiting a mass M is given by the centrally directed gravitational potential energy:

$$U = -\frac{mMG}{r} \quad - (7)$$

whose force law is the Hooke / Newton inverse square force law:

$$F = -\frac{\partial U}{\partial r} = -\frac{mMG}{r^2} . \quad - (8)$$

The velocity v is defined by the infinitesimal line element of special relativity:

$$c^2 d\tau^2 = (c^2 - v_N^2) dt^2 \quad - (9)$$

where $d\tau$ is the infinitesimal of proper time, the time in a frame moving with the particle, and where dt is the infinitesimal of time in a frame with respect to which the particle moves. It follows that the velocity in Eq. (9) is defined by:

$$v_N^2 = \dot{r}^2 + r^2 \dot{\theta}^2 \quad - (10)$$

where:

$$\dot{r} = \frac{dr}{dt} \quad - (11)$$

and where the angular velocity is:

$$\omega = \dot{\theta} = \frac{d\theta}{dt} . \quad - (12)$$

As in UFT324 the two Euler Lagrange equations of this system are

$$\frac{\partial \mathcal{L}}{\partial \theta} = \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{\theta}} = 0 \quad - (13)$$

and

$$\frac{\partial \mathcal{L}}{\partial r} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{r}}. \quad -(14)$$

Eq. (13) gives the relativistic angular momentum:

$$L = \gamma m r^2 \dot{\theta} \quad -(15)$$
$$:= \gamma L_0$$

and this is a constant of motion such that:

$$\frac{dL}{dt} = 0. \quad -(16)$$

Therefore the angular velocity is defined by:

$$\dot{\theta} = \frac{L}{\gamma m r^2}. \quad -(17)$$

Using the change of variable:

$$\frac{d}{d\theta}\left(\frac{1}{r}\right) = -\frac{1}{r^2}\frac{dr}{dt}\frac{dt}{d\theta} \quad -(18)$$

it is found that:

$$\dot{r} = -\frac{L}{\gamma m}\frac{d}{d\theta}\left(\frac{1}{r}\right). \quad -(19)$$

It follows as in UFT324 and notes to UFT325 that the velocity is defined by:

$$v_N^2 = \dot{r}^2 + \dot{\theta}^2 r^2 = \frac{L^2}{\gamma^2 m^2}\left(\left(\frac{d}{d\theta}\left(\frac{1}{r}\right)\right)^2 + \frac{1}{r^2}\right) \quad -(20)$$

i. e.

$$v_N^2 = \frac{\frac{L^2}{m^2}\left(\left(\frac{d}{d\theta}\left(\frac{1}{r}\right)\right)^2 + \frac{1}{r^2}\right)}{1 + \frac{L^2}{m^2c^2}\left(\left(\frac{d}{d\theta}\left(\frac{1}{r}\right)\right)^2 + \frac{1}{r^2}\right)}$$

$$v_N^2 = \frac{v^2}{1 + \left(\frac{v}{c}\right)^2} \quad ; \quad v = \gamma v_N, \quad \underline{p} = \gamma m \underline{v}_N \quad -(21)$$

where the Newtonian orbital velocity is:

$$v_N^2 = \frac{L_0^2}{m^2}\left(\left(\frac{d}{d\theta}\left(\frac{1}{r}\right)\right)^2 + \frac{1}{r^2}\right). \quad -(22)$$

As shown in UFT324, Eq. (21) gives the precise correct result for light deflection by gravitation using:

$$v \rightarrow c \quad -(23)$$

and

$$v_N^2 = \frac{c^2}{2} \quad -(24)$$

so the well known Newtonian deflection:

$$\Delta\phi = \frac{2mG}{R_0 v_N^2} \quad -(25)$$

becomes the observed deflection:

$$\Delta\phi = \frac{4mG}{R_0 c^2} \quad -(26)$$

where R_0 is the distance of closest approach.

For the hyperbolic orbit of a star in a whirlpool galaxy:

$$\frac{1}{r} = \frac{\theta}{r_0} \quad -(27)$$

Eq. (2) gives the relativistic velocity:

$$v^2 = \frac{L_0^2}{m^2}\,\frac{\left(\frac{1}{r_0^2}+\frac{1}{r^2}\right)}{1-\frac{L_0^2}{m^2c^2}\left(\frac{1}{r_0^2}+\frac{1}{r^2}\right)} \quad -(28)$$

which goes to a plateau for infinite r:

$$v \xrightarrow[r\to\infty]{} \frac{L_0}{m r_0}\left(1-\frac{L_0^2}{m^2c^2r_0^2}\right)^{-1/2} \quad -(29)$$

as observed experimentally. It is well known that the Einstein and Newton theories produce a velocity that goes to zero with infinite r, and fail completely in whirlpool galaxies. The Einstein theory fails completely due to omission of torsion, and as shown in the accompanying Note 9, computer algebra and protocol, gives an exceedingly complicated orbit which diverges. It cannot be a correct description of nature because its geometry is basically incorrect.

The Einsteinian hamiltonian and lagrangian are:

$$H(\text{Einstein}) = \frac{1}{2}m\left(\dot{r}^2+r^2\dot{\theta}^2\left(1+\frac{r_0}{r}\right)\right)+U \quad -(30)$$

and

$$\mathcal{L}(\text{Einstein}) = \frac{1}{2}m\left(\dot{r}^2+r^2\dot{\theta}^2\left(1+\frac{r_0}{r}\right)\right)-U \quad -(31)$$

where:

$$U = -mMG/r \quad -(32)$$

and where:

$$r_0 = \frac{2MG}{c^2}. \quad -(33)$$

The conserved angular momentum of the Einsteinian theory is:

$$L = \frac{\partial \mathcal{L}}{\partial \dot{\theta}} = \left(1 + \frac{r_0}{r}\right) m r^2 \dot{\theta} \quad -(34)$$

and it follows that:

$$\dot{\theta} = \frac{L}{mr^2}\left(1 + \frac{r_0}{r}\right)^{-1} \quad -(35)$$

and

$$\dot{r} = -\frac{L}{m}\left(1 + \frac{r_0}{r}\right)^{-1} \frac{d}{d\theta}\left(\frac{1}{r}\right). \quad -(36)$$

The Einsteinian orbital velocity is:

$$v^2 = v_N^2 \left(1 + \frac{r_0}{r}\right)^{-2}. \quad -(37)$$

As

$$v_N \rightarrow c \quad -(38)$$

and at the distance of closest approach:

$$r = R_0 \quad -(39)$$

the Newtonian light deflection due to gravitation is changed to:

$$\Delta \phi = \frac{2MG}{R_0 c^2}\left(1 + \frac{r_0}{R_0}\right)^2 \quad -(40)$$

and this is not the experimental result (26), Q. E. D. The Einstein theory is clearly incorrect.

The Euler Lagrange equation (14) gives the relativistic Leibnitz orbital equation:

$$\frac{d}{dt}(\gamma m \dot{r}) - \gamma m r \dot{\theta}^2 = -\frac{\partial U}{\partial r} = F(r) \quad -(41)$$

as shown in detail in Note 325(8). In the limit:

$$\gamma \rightarrow 1 \quad -(42)$$

Eq. (41) becomes the 1689 Leibnitz orbital equation:

$$m\frac{d^2 r}{dt^2} = m r \omega^2 - \frac{mMG}{r^2}. \quad -(43)$$

The Newtonian or non relativistic orbital velocity is well known to be:

$$v_N^2 = \frac{L^2}{m^2}\left(\left(\frac{d}{d\theta}\left(\frac{1}{r}\right)\right)^2 + \frac{1}{r^2}\right) = MG\left(\frac{2}{r} - \frac{1}{a}\right) \quad -(44)$$

where the semi major axis is:

$$a = \frac{\alpha}{1-\epsilon^2} \quad -(45)$$

and in which the Newtonian orbit is the conic section:

$$r = \frac{\alpha}{1+\epsilon\cos\theta} \quad -(46)$$

Using Eqs. (21) and (44) a velocity curve can be derived:

$$v^2 = MG\left(\frac{2}{r} - \frac{1}{a}\right)\left(1 - \frac{MG}{c^2}\left(\frac{2}{r} - \frac{1}{a}\right)\right)^{-1} \quad -(47)$$

and this is plotted in Section 3, in which the important conclusion is made that the true orbit of a planet is given by the lagrangian of special relativity. Further work using supercomputers can increase the numerical precision of the present work to the point at which a comparison can be made with experimental data. It is already known that ECE2 gives light deflection in an exactly correct way as described already.

Orbital precession from ECE2 and from the Lagrangian of special relativity

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3 Lagrange theory and numerical solutions

3.1 Sommerfeld Lagrange theory

The Sommerfeld Lagrangian of Eq.(2) is

$$\mathscr{L} = -\frac{m\, c^2}{\gamma} - U \tag{48}$$

with potential energy, γ factor and velocity

$$U = -\frac{m\, M\, G}{r}, \tag{49}$$

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}, \tag{50}$$

$$v^2 = \dot{r}^2 + r^2\, \dot{\theta}^2. \tag{51}$$

The evaluation of Lagrange equations

$$\frac{\partial \mathscr{L}}{\partial r} = \frac{d}{dt}\frac{\partial \mathscr{L}}{\partial \dot{r}}, \tag{52}$$

$$\frac{\partial \mathscr{L}}{\partial \theta} = \frac{d}{dt}\frac{\partial \mathscr{L}}{\partial \dot{\theta}} \tag{53}$$

gives from the Sommerfeld Lagrangian (48):

$$\ddot{r} = -\frac{c^2\, G\, M + \gamma^3\, r^4\, \dot{r}\, \dot{\theta}\, \ddot{\theta} + \left(\gamma^3\, r^3\, \dot{r}^2 - \gamma\, c^2\, r^3\right)\, \dot{\theta}^2}{\gamma^3\, r^2\, \dot{r}^2 + \gamma\, c^2\, r^2}. \tag{54}$$

and

$$\ddot{\theta} = -\frac{\gamma^2\, r^2\, \dot{r}\, \dot{\theta}^3 + \left(\gamma^2\, r\, \dot{r}\, \ddot{r} + 2\, c^2\, \dot{r}\right)\, \dot{\theta}}{\gamma^2\, r^3\, \dot{\theta}^2 + c^2\, r} \tag{55}$$

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Both equations contain the second derivatives of r and θ in linear form. To obtain an equation set useable for numerical integration, both $\ddot{\theta}$ and $\ddot{r}$ have to be separated first. From the two equations with two unknowns (54,55) the solutions are

$$\ddot{r} = \frac{\left(-\gamma^2 v^2 + \gamma^2 \dot{r}^2 - c^2\right) G M + r \left(\gamma^3 v^4 + \gamma c^2 v^2\right) + r \dot{r}^2 \left(-\gamma^3 v^2 - \gamma c^2\right)}{r^2 \left(\gamma^3 v^2 + \gamma c^2\right)}, \tag{56}$$

$$\ddot{\theta} = \frac{\gamma \dot{r} \dot{\theta} G M + r \dot{r} \dot{\theta} \left(-2 \gamma^2 v^2 - 2 c^2\right)}{r^2 \left(\gamma^2 v^2 + c^2\right)}. \tag{57}$$

These are the relativistic Lagrange equations for central motion in a two-dimensional polar coordinate system. The non-relativistic form of them is obtained by assuming $\gamma \approx 1$ and making the transition $c \to \infty$ which leads to

$$\ddot{r} = r \dot{\theta}^2 - \frac{G M}{r^2}, \tag{58}$$

$$\ddot{\theta} = -\frac{2 \dot{r} \dot{\theta}}{r}. \tag{59}$$

These are exactly the non-relativistic equations from Newton theory.

For comparison we also investigate the equations of motion from x theory. The x potential is given by

$$U = \frac{L^2 \left(x^2 - 1\right)}{2 m r^2} - \frac{L^2 x^2}{\alpha m r}. \tag{60}$$

With the half latus rectum

$$\alpha = \frac{L^2}{m^2 M G} \tag{61}$$

the non-relativistic Lagrangian

$$\mathscr{L} = \frac{1}{2} m v^2 - U \tag{62}$$

is

$$\mathscr{L} = \frac{m x^2 G M}{r} - \frac{L^2 \left(x^2 - 1\right)}{2 m r^2} + \frac{m \left(r^2 \dot{\theta}^2 + \dot{r}^2\right)}{2} \tag{63}$$

which leads to the non-relativistic Lagrange equations with x correction:

$$\ddot{r} = -\frac{x^2 G M}{r^2} + \frac{L m^2 \left(x^2 - 1\right)}{m^2 r^3} + r \dot{\theta}^2, \tag{64}$$

$$\ddot{\theta} = -\frac{2 \dot{r} \dot{\theta}}{r}. \tag{65}$$

The equation for $\ddot{\theta}$ is unchanged. For $x = 1$ the non-relativistic equations are obtained. For $x < 1$ a negative contribution of $1/r^3$ is added to the radial force component, leading to a precession of the ellipse in the direction of the orbital motion which is observed for the planet Mercury.

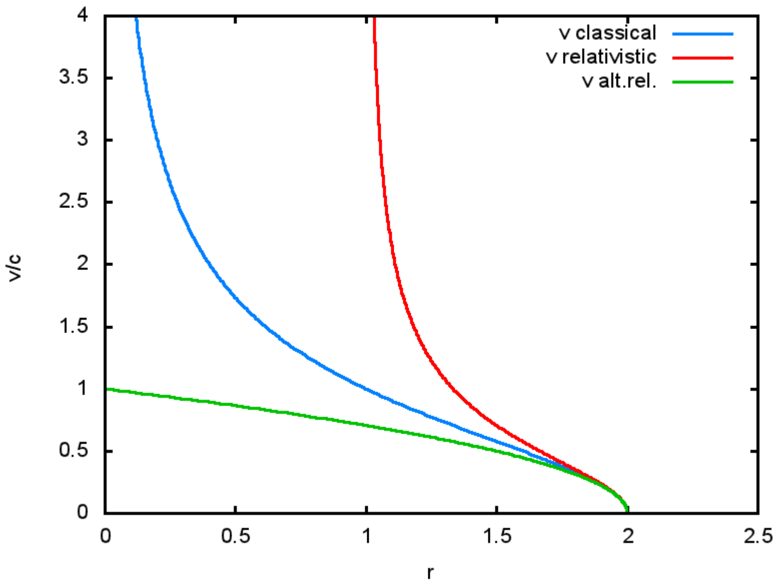

Figure 1: Newtonian and relativistic velocity ratio v/c.

3.2 Velocity comparison

For a graphical examination of the results, we first will examine the graphs of the velocities. As was shown in Eq.(21) of section 2, the non-relativistic velocity is given by

$$v_N^2 = \frac{v^2}{1 + v^2/c^2} \tag{66}$$

where v_N for an ellipse is given according to Eq.(44) by

$$v_N^2 = M\,G\left(\frac{2}{r} - \frac{1}{a}\right) \tag{67}$$

with $a = \alpha/(1 - \epsilon^2)$ being the major axis. Solving Eq.(66) for v gives

$$v^2 = \frac{v_N^2}{1 - v_N^2/c^2}. \tag{68}$$

By inserting (67) into (68), v and v_N can be compared in their radial dependence. The ratios v/c and v_N/c are graphed in Fig. 1. All parameters were set to unity. For $r \to 0$, the classical velocity diverges to an infinite value. For the relativistic velocity, this happens for $v_N = c$ where we have $v_N = 1$ in the actual scaling. This behaviour motivates an alternative definition for v' and v'_N with reversed signs in the denominator:

$$v_N'^2 = \frac{v'^2}{1 - v'^2/c^2}, \tag{69}$$

$$v'^2 = \frac{v_N'^2}{1 + v_N'^2/c^2}. \tag{70}$$

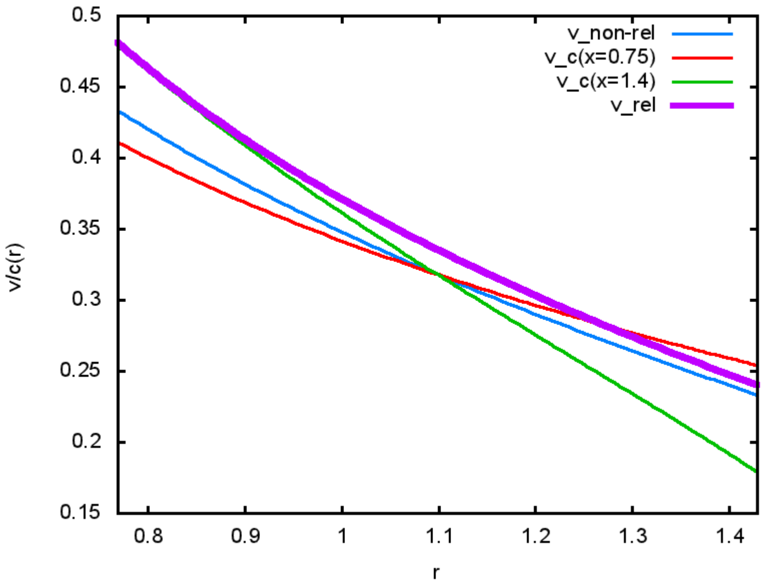

Figure 2: velocity ratio v/c for non-relativistic (Newtonian) and relativistic case, and for two values of x theory.

This effectively leads to an alternative relativistic velocity curve v/c which approaches unity for $r \to 0$ as expected (green curve in Fig. 1). Otherwise the case $v = c$ is reached at a much higher r value. However the non-relativistic formula of the orbit has been used in (67) which may be a source of an error, and the elliptic orbit is defined only in a restricted range of r.

It was shown earlier that the classical velocity in case of x theory correction is

$$v_x^2 = \frac{L^2}{\alpha^2\, m^2} \left(\frac{\alpha\, \left(x^2+1\right)}{r} + \left(\epsilon^2-1\right)\, x^2 \right). \tag{71}$$

To make this comparable with Eq.(67), we replace L^2 by

$$L^2 = m^2\, M\, G\, \alpha \tag{72}$$

and obtain

$$v_x^2 = \frac{\left(\left(\left(\epsilon^2-1\right)\, r+\alpha\right)\, x^2+\alpha\right)\, G\, M}{\alpha\, r}. \tag{73}$$

From (67) and (68) then follows

$$v_N^2 = \frac{\left(\left(\epsilon^2-1\right)\, r+2\,\alpha\right)\, G\, M}{\alpha\, r}, \tag{74}$$

$$v^2 = \frac{\left(\left(\epsilon^2-1\right)\, r+2\,\alpha\right)\, G\, M\, c^2}{\left(\left(\epsilon^2-1\right)\, r+2\,\alpha\right)\, G\, M+\alpha\, c^2\, r}. \tag{75}$$

With parameters set to $\epsilon = 0.3$, $c = 3$, all other parameters unity, we can compare all three velocity expressions. The results are graphed in Fig. 2 for two values of $x = 0.75$ and $x = 1.4$. These are the velocities for a precessing or non-precessing ellipse in the high-relativistic case up to $v/c = 0.5$. The curves $v(r)/c$ are plotted in the range [rmin, rmax] of an ellipse with $\alpha = 1$. It can be noticed that the relativistic curve (thick line) is not covered exactly neither by x theory nor by Newton theory. The latter underestimates velocity at perihelion. Using $x = 0.75$ (red line) fits the velocity of x theory near to the aphelion but underestimates it at perihelion. Values for $x > 1$ cannot remedy this because the slope becomes too large. The true theoretical orbit is the one from the Lagrangian of special relativity, which is also the one from ECE2 theory.

3.3 Numerical solution of relativistic orbits and control parameters

The equations (56-57) have been solved numerically as for example in UFT paper 239 (but we used θ as the integration variable therein, while we use the time in this work). The results $r(t)$ and $\theta(t)$ can be combined in a so-called scatter-plot in a polar coordinate system to obtain the orbit $r(\theta)$. So the orbit is not given by an analytical formula in this case but numerically by "points". The result is graphed in Fig. 3, showing directly the precession of the ellipse. Orbital motion is in positive mathematical angular direction as is the rotation of the elliptic orbit. This is in coincidence with astronomical findings.

There is additional information that can be obtained from the solutions $r(t)$ and $\theta(t)$ and their derivatives. Important checks are the constants of motion: relativistic angular momentum and energy. We validated that the relativistic momentum is conserved as well as the relativistic energy (Hamiltonian). Fig. 4 shows the ratio v/c which is minimal in aphelion as expected. The same holds for the difference of the r component of force and the angular momentum between relativistic and non-relativistic calculation (Fig. 5). The total energy is identical to the non-relativistic case at aphelion (Fig. 6).

We did the same numerical calculation for the potential of x theory. In the Newtonian case ($x = 1$) the well known ellipses follow, for the x theory the precessing ellipses, all numerically, and can be compared with the relativistic solution. It is a bit difficult to define comparable x factors for the relativistic case because there is no analytically given orbit and both theories show quantitatively different behaviour, see discussion of Fig. 2 above. The initial conditions do not reflect expressions like ϵ and α, one has to use r, θ, $\dot{r}$ and $\dot{\theta}$ primarily, where we pre-computed $\dot{\theta}$ from the same value of given non-relativistic angular momentum in all cases. Fig. 7 shows the precessing orbit for $x = 0.98$. The v/c ratio (Fig. 8) looks similar as for the relativistic theory, The force difference changes signs in the orbit and the angular momentum is the same as in the non-relativistic case, i.e. $\Delta L = 0$. The difference of total energy (Fig. 10) shows more variation than in the relativistic case.

The numerical calculations have shown that the solution of relativistic Lagrange equations is a parameter-free, first-principles method of solving the relativistic Kepler problem. It will be difficult however to obtain orbital precession values for real planets because these effects are very small and require very high numerical accuracy.

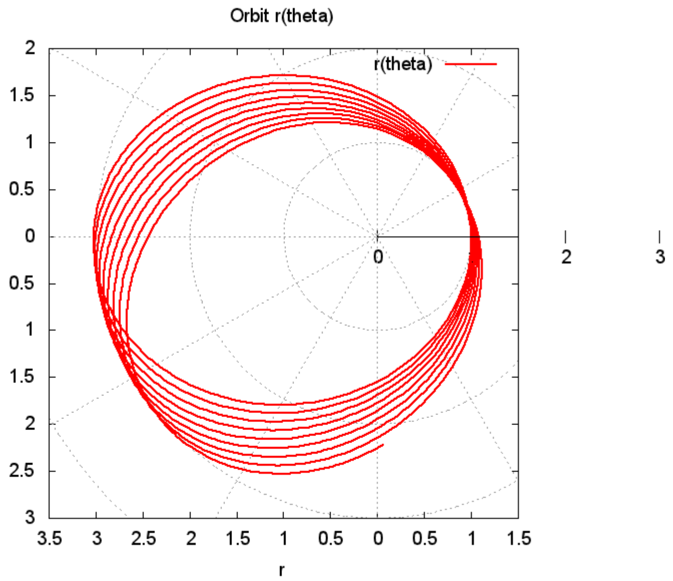

Figure 3: Orbit from relativistic theory.

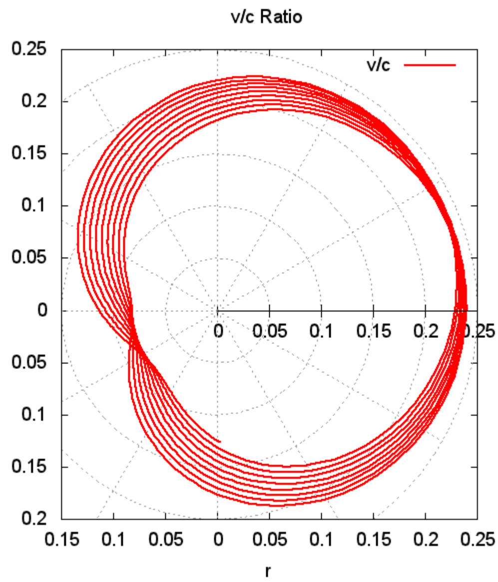

Figure 4: Ratio v/c from relativistic theory.

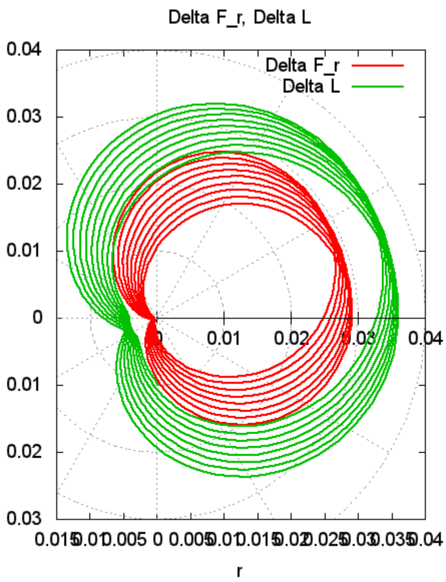

Figure 5: Difference of force component F_r and angular momentum L between relativistic and Newton theory.

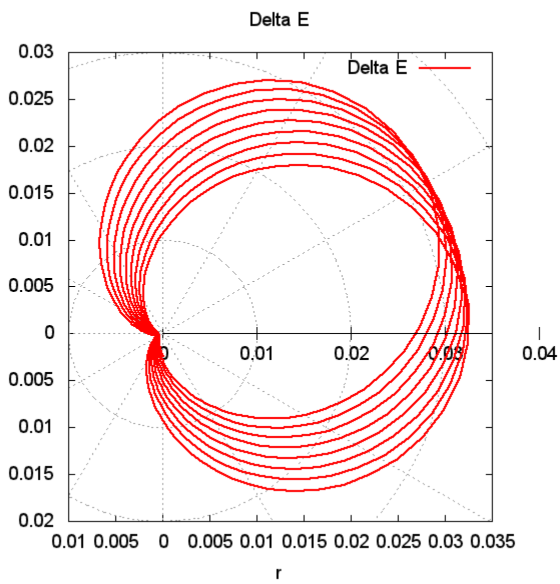

Figure 6: Difference of total energy E between relativistic and Newton theory.

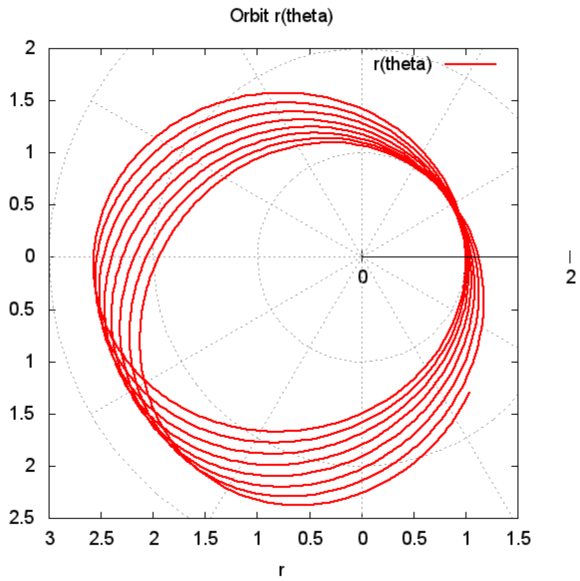

Figure 7: Orbit from non-relativistic x theory, $x = 0.98$.

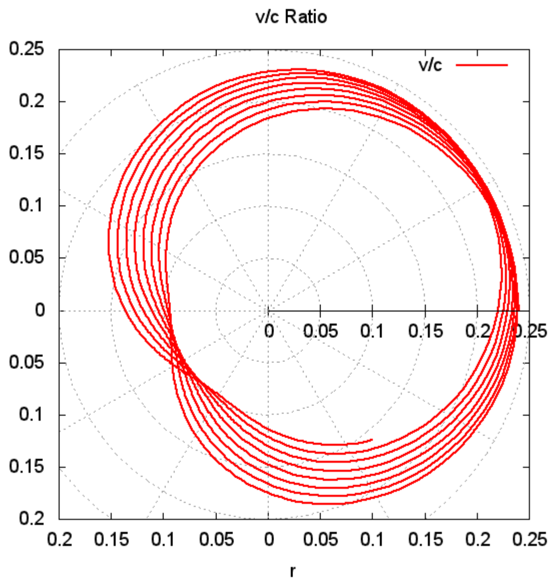

Figure 8: Ratio v/c from non-relativistic x theory.

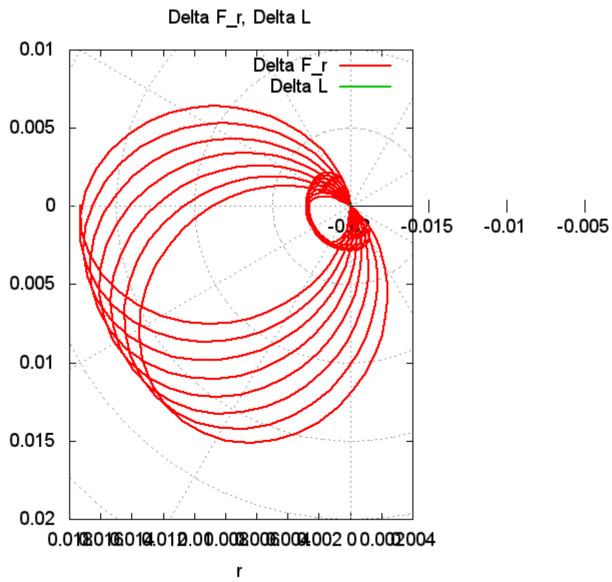

Figure 9: Difference of force component F_r and angular momentum L between x theory and Newton theory.

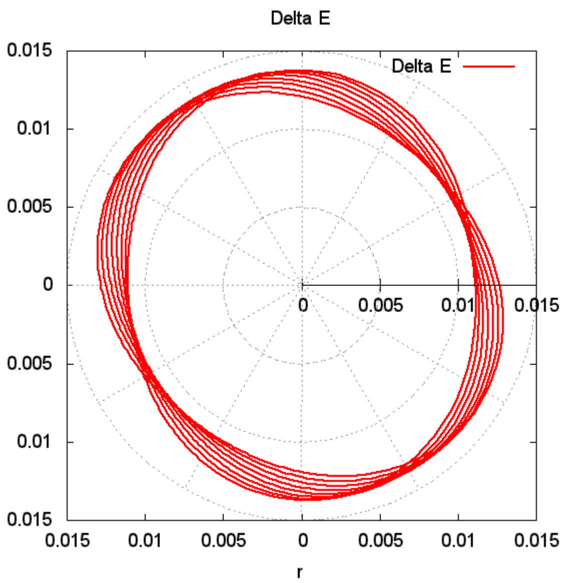

Figure 10: Difference of total energy E between x theory and Newton theory.

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