PRECESSING ELLIPTICAL ORBITS IN ECE2 SPECIAL RELATIVITY, THEORY AND COMPUTATION.

by

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ABSTRACT

The existence of precessing elliptical orbits from special relativity is confirmed with numerical and theoretical methods used to find the true orbit using equations of special relativity based on the Lorentz covariance of ECE2 theory. The true orbit is given by a numerical solution using the lagrangian and hamiltonian of special relativity. The results are analysed graphically in several ways. The analyitcal methods are based on the hamiltonian, lagrangian and infinitesimal line element of special relativity. The analytical problem is in general intractable, but the numerical solution clearly shows precession.

Keywords: ECE2 theory, precession of the perihelion with ECE2 special relativity.

UFT 328

1. INTRODUCTION

In recent papers of this series of over five hundred papers and books on ECE and ECE2 theory in English and Spanish {1 - 12}, the ECE2 theory has been developed from the Jacobi Cartan Evans (JCE) identity of UF 313. ECE2 is simpler than ECE and makes use both of non zero torsion and non zero curvature. From about UFT324 onwards the Lorentz covariance of ECE2 special relativity has been used to show that the precession of a planar orbit can be described by special relativity. Several further refutations of the Einsteinian general theory have been given, notably in UFT327, so the Einstein theory is thoroughly obsolete. In ECE and ECE2 new explanations have been found for the claims of Einsteinian general relativity (EGR), notably the precession of the perihelion, the subject of this paper.

This paper should be read with its background notes, posted with UFT328 on www.aias.us. In note 328(1) a general analytical method is developed based on the general precessing orbit. In Note 328(2) an initial investigation is made of a key concept in special relativity, the ratio p / L of the linear to the angular momentum. In general this ratio is relativistic, and the numerical results of Section 3 show that it can be computed from the lagrangian and hamiltonian of special relativity to give the true orbit without the intercession of any type of additional hypothesis or modelling. The ratio p / L has been investigated in several previous UFT papers of recent years using theories of the infinitesimal line element. ECE2 special relativity is preferred to all these theories because it is simpler. In note 328(3) it is shown how the orbit is related to the ratio p / L, so if the latter can be found the orbit can be found. In Note 328(4) a simple analytical approximation is used to develop the ECE2 theory of orbits, and in Note 328(5) a hamiltonian method is developed.

These notes are briefly summarized in Section 2, and the notes should be read with Section 2. In Section 3 various graphical results are given which show that the orbit of ECE special relativity is a precessing ellipse.

2. A SUMMARY OF ANALYTICAL AND COMPUTATIONAL METHODS.

`Consider the lagrangian and hamiltonian of special relativity, respectively:

$$\mathcal{L} = -\frac{mc^2}{\gamma} - U \quad -(1)$$

and

$$H = \gamma mc^2 + U. \quad -(2)$$

Here γ is the Lorentz factor:

$$\gamma = \frac{dt}{d\tau} = \left(1 - \frac{v_0^2}{c^2}\right)^{-1/2} \quad -(3)$$

defined directly from the infinitesimal line element of special relativity {1 - 12}:

$$c^2 d\tau^2 = (c^2 - v_0^2)\, dt^2 \quad -(4)$$

In these equations the gravitational potential is:

$$U = -\frac{mMG}{r} \quad -(5)$$

where a mass m orbits a mass M separated by a distance $\underline{r}$ .The orbit is considered to be planar and is described by the plane polar coordinates (r, θ). In the infinitesimal line element τ is the proper time in a frame moving with m, and t the time in the frame of an observer, with respect to whom the mass m moves with a velocity $\underline{v}_0$. The latter is the classical velocity defined by:

$$v_0^2 = \left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\theta}{dt}\right)^2 . \quad -(6)$$

As shown in UFT324 and UFT325 the lagrangian analysis leads to:

$$\ddot{r} = \frac{\left(-\gamma^2 v^2 + \gamma^2 \dot{r}^2 - c^2\right) mG + r\left(\gamma^3 r^4 + \gamma c^2 v^2\right) + r\dot{r}^2\left(-\gamma^3 v^2 - \gamma c^2\right)}{r^2\left(\gamma^3 v^2 + \gamma c^2\right)} \quad -(7)$$

and:

$$\ddot{\theta} = \frac{\gamma \dot{r}\dot{\theta} mG + r\dot{r}\dot{\theta}\left(-2\gamma^2 v^2 - 2c^2\right)}{r^2\left(\gamma^2 v^2 + c^2\right)} \quad -(8)$$

In the classical limit these equations reduce to the Leibnitz equation:

$$\ddot{r} = r\dot{\theta}^2 - \frac{mG}{r^2} \quad -(9)$$

and

$$\ddot{\theta} = -\frac{2\dot{r}\dot{\theta}}{r} . \quad -(10)$$

In order to deduce the relativistic orbit the ratio p / L must be computed, and this can be done using:

$$\dot{r} = \int \ddot{r}\, dt \quad -(11)$$

and

$$\dot{\theta} = \int \ddot{\theta}\, dt \quad -(12)$$

giving various results graphed in Section 3.

The relativistic velocity $\underline{v}$ is defined by

$$\underline{v} = \gamma \underline{v}_0 \quad -(13)$$

and the relativistic momentum is:

$$\underline{p} = m\underline{v} = \gamma m \underline{v}_0 . \quad -(14)$$

The relativistic angular momentum is defined from the lagrangian analysis by:

$$L = \gamma m r^2 \dot{\theta} \quad -(15)$$

and is a constant of motion. The other constant of motion is the relativistic hamiltonian (H).

The relativistic momentum is not a constant of motion and in terms of proper time it is

defined by:

$$p^2 = m^2 \left(\left(\frac{dr}{d\tau} \right)^2 + r^2 \left(\frac{d\theta}{d\tau} \right)^2 \right) . \quad -(16)$$

The relativistic Leibnitz equation of orbits is:

$$F(r) = -\frac{\partial U}{\partial r} = \frac{d}{dt}\left(\gamma m \dot{r} \right) - \gamma m r \dot{\theta}^2 \quad -(17)$$

which must be solved with:

$$L = \gamma m r^2 \dot{\theta} = \text{constant} . \quad -(18)$$

From the infinitesimal line element of special relativity, Eq. (4), it is found that:

$$\left(\frac{p}{L} \right)^2 = \frac{1}{r^4} \left(\left(\frac{dr}{d\theta} \right)^2 + r^2 \right) \quad -(19)$$

where p / L is the relativistic ratio. It is shown graphically in Section 3 that the relativistic

ratio is not the same as the classical ratio p_0 / L_0 , and the relativistic ratio must be computed from Eqs. (7) and (8) which originate in the lagrangian (1).

The hamiltonian (2) of special relativity can be written as:

$$H = \left(p^2 c^2 + m^2 c^4 \right)^{1/2} + U \quad -(20)$$

using

$$E^2 = \gamma^2 m^2 c^4 = p^2 c^2 + m^2 c^4. \quad -(21)$$

In this expression p is the relativistic momentum, so:

$$E = \gamma m c^2 , \; \underline{p} = \gamma m \underline{v_0} . \quad -(22)$$

From Eq. ( ):

$$H_0 = H - mc^2 = \frac{p^2 c^2}{E + mc^2} + U \quad -(23)$$

so:

$$H_0 \rightarrow \frac{1}{2} m v_0^2 + U \quad -(24)$$

In the limit:

$$v_0 << c . \quad -(25)$$

Eq. (20) reduces to the classical hamiltonian:

$$H_0 = H - mc^2 = \frac{p_0^2}{2m} + U. \quad -(26)$$

Therefore the transition from classical dynamics to special relativity may be described as follows:

$$p_0^2 \to 2\left(\frac{E^2}{mc^2(E+mc^2)}\right)p_0^2 \quad -(27)$$

The classical kinetic energy transforms as follows:

$$T = \frac{p_0^2}{2m} \to \left(\frac{E^2}{mc^2(E+mc^2)}\right)\frac{p_0^2}{m} \quad -(28)$$

and the classical hamiltonian transforms as:

$$H_0 \to \left(\frac{E^2}{mc^2(E+mc^2)}\right)\frac{p_0^2}{m} + U \quad -(29)$$

In general, the Hamilton and Euler Lagrange equations may be applied to the problem and in general E depends on the Lorentz factor defined from the metric (4) as:

$$\gamma = \frac{dt}{d\tau} = \left(1 - \frac{v_0^2}{c^2}\right)^{-1/2} \quad -(30)$$

In the Dirac approximation it is assumed that:

$$E \sim mc^2, \quad U \ll mc^2 \quad -(31)$$

and so E is approximated by the rest energy:

$$E \sim E_0 = mc^2. \quad -(32)$$

This is a rough approximation which is accepted because it works. The accurate results are given in Section 3 from the numerical method. These can be applied not only to precessing orbits but also to the Dirac and Sommerfeld atoms, and that will be the subject of UFT329.

As can be seen from Section 3, the Lorentz factor gamma is defined directly from the infinitesimal line element in terms of the classical velocity:

$$v_0^2 = \left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\theta}{dt}\right)^2. \quad -(33)$$

However the graphics show clearly that the relativistic ratio p / L is not the same as the classical ratio p_0 / L_0. These considerations are developed in the notes. The true orbit is the one obtained from Eqs. (7) and (8). The true orbit can be compared with models such as the x theory:

$$r = \frac{\alpha}{1+\epsilon\cos(x\theta)} \quad -(34)$$

and a model of the general precessing orbit:

$$r = \frac{\alpha}{1+\epsilon\cos(\theta_1(\theta))} \quad -(35)$$

and various analytical approximations to the numerical orbit can be obtained as described in detail in the notes for UFT328 on www.aias.us.

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Precessing elliptical orbits in ECE2 special relativity, theory and computation

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3 Numerical analysis

The Lagrangian of special relativity is

$$\mathscr{L} = -\frac{m\,c^2}{\gamma} - U \tag{36}$$

with potential energy

$$U = -\frac{m\,M\,G}{r}. \tag{37}$$

The resulting Lagrange equations are Eqs.(7-8) of this paper which were derived in UFT paper 325.

The orbital derivative is given by (setting $\dot{\theta} = \omega$):

$$\frac{dr}{d\theta} = \frac{dr}{d\tau}\frac{d\tau}{d\theta} = \frac{dr}{dt}\frac{dt}{d\theta} = \frac{\dot{r}}{\omega}. \tag{38}$$

We obtain for the relativistic ratio of p/L:

$$\frac{p}{L} = \frac{\gamma m v_0}{\gamma m r^2 \omega} = \frac{v_0}{\omega r^2} \tag{39}$$

with constituting equations

$$v_0 = \sqrt{\dot{r}^2 + r^2\omega^2}, \tag{40}$$

$$v = \gamma v_0, \tag{41}$$

$$\gamma = \frac{1}{\sqrt{1 - v_0^2/c^2}}. \tag{42}$$

The numerical results are compared with the corresponding results of the non-relativistic, Newtonian Lagrangian

$$\mathscr{L}_N = \frac{1}{2} m v_0^2 - U. \tag{43}$$

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The relativistic and non-relativistic calculations started at $\theta = 0$ with the same radius and initial angular velocity. Therefore the angular momenta were not the same at the starting point. It is however not possible to use the non-relativistic L_0 in the relativistic equation because this is not a constant of motion there. From Fig. 1 (orbits) it can be seen that the relativistic orbit is significantly larger for identical initial conditions. This is a hint that it makes no sense to use an equation for the non-relativistic orbit in a relativistic context. The orbital derivative $\frac{dr}{d\theta}$ is graphed in Fig. 2. Since the derivative takes both signs, there are two overlapping elliptic curves in the polar plot (negative values are represented by an angular shift of π).

The graph of $\dot{r}$ (Fig. 3) is a circle in the non-relativistic case which is run through twice because of the symmetry with sign change for a full ellipse. In the relativistic case the precession leads to a splitting of the circle which can well be observed in the figure. The angular velocity (Fig. 4) remains positive and shows the relativistic precessing behaviour as do nearly all other curves.

Fig. 5 shows $\gamma(\theta)$, this varies only between 1.00 and 1.03 for this particular orbit although the orbital precession (graphed in Fig. 1) is significant. The ratio v/c (Fig. 6) is dominated by the angular velocity component of v and therefore resembles ω (Fig. 4). The ratio p/L (Fig. 7) looks also very similar due to its dependence on v. There is always a bend in the curves at the aphelion. The differences between Newtonian and relativistic results for linear momentum, angular momentum and force have already be shown in UFT paper 325.

It is of some interest to inspect the angular dependence of the orbits described by Eqs.(34) and (35). The first problem is to find a meaningful method for comparing the Newtonian and relativistic case since the maximum radius (as well as the effective ϵ and time dependence) are different. Therefore we used the orbital derivatives

$$\frac{dr_1}{d\theta} = \frac{\dot{r_1}}{\omega_1} \tag{44}$$

and

$$\frac{dr_2}{d\theta} = \frac{\dot{r_2}}{\omega_2} \tag{45}$$

for the Newtonian (r_1) and relativistic case (r_2). Both curves are crossing zero at perihelion and aphelion and have been normalized so that they look identical except their dependence on angle θ, see Fig. 8. The horizontal difference between both for a given ordinate value is a measure of the progression of angular precession, see Fig. 8. The difference

$$\Delta\theta = \theta_2 - \theta_1 \quad \text{for} \quad \frac{dr_1}{d\theta_1} = \frac{dr_2}{d\theta_2} \tag{46}$$

has also been plotted in Fig. 8. It can be seen that there is no linearly growing $\Delta\theta$ as assumed in x theory (Eq.(34)). Within the first orbit round (0-2π) the difference becomes even negative just before approaching 2π. This is the most realistic calculation of precession we have done in all papers so far.

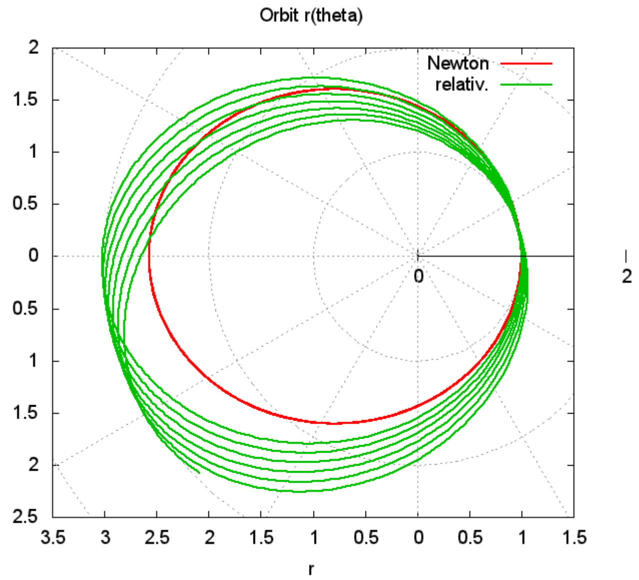

Figure 1: Orbit $r(\theta)$.

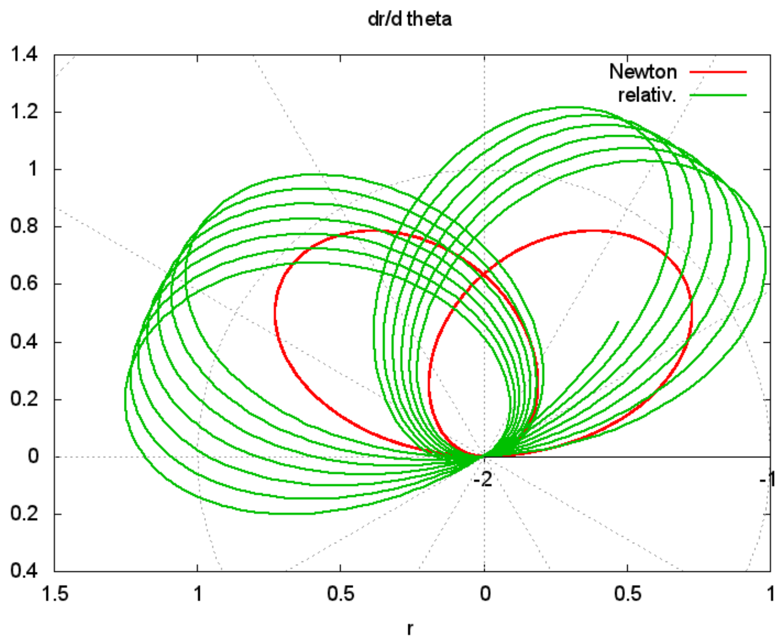

Figure 2: Orbit derivative $dr/d\theta$.

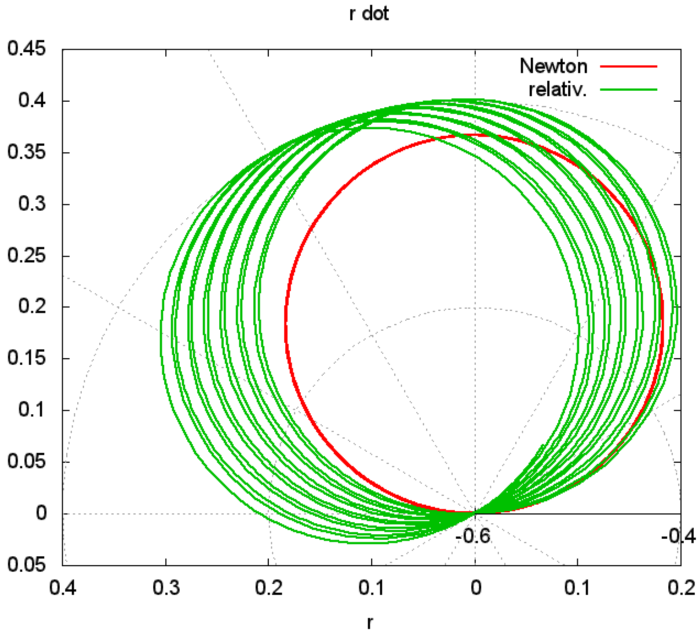

Figure 3: Radial derivative $\dot{r}$.

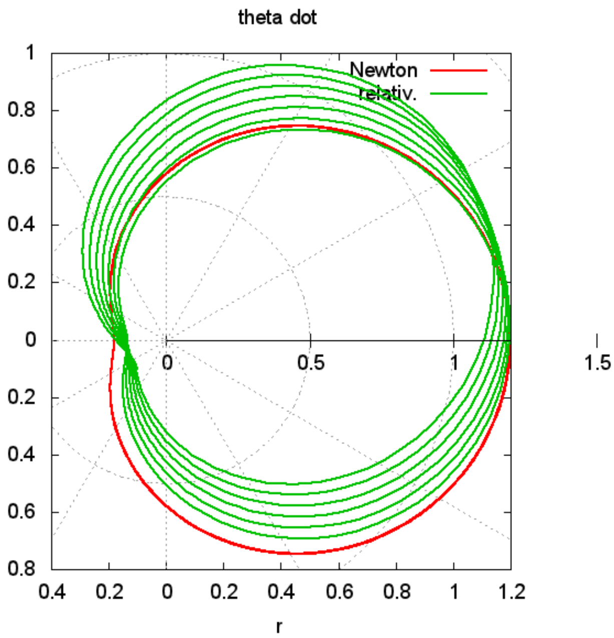

Figure 4: Angular velocity $\dot{\theta} = \omega$.

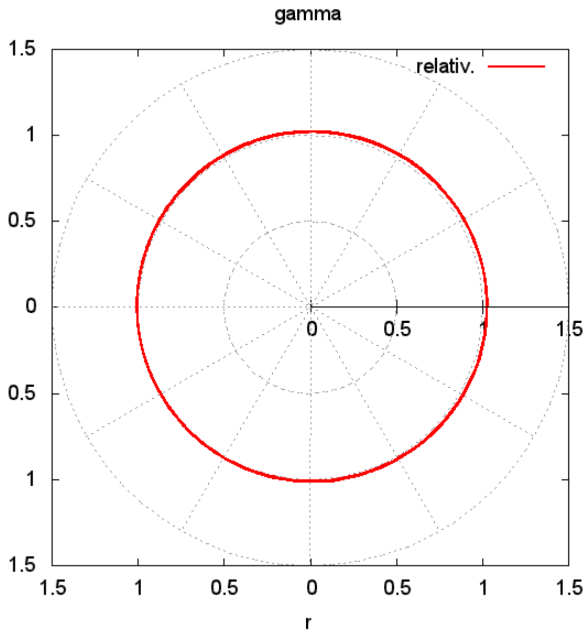

Figure 5: Relativistic γ factor.

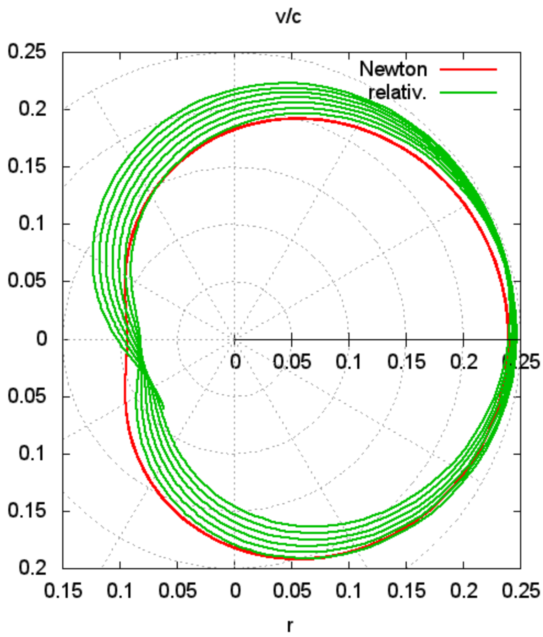

Figure 6: Ratio v/c.

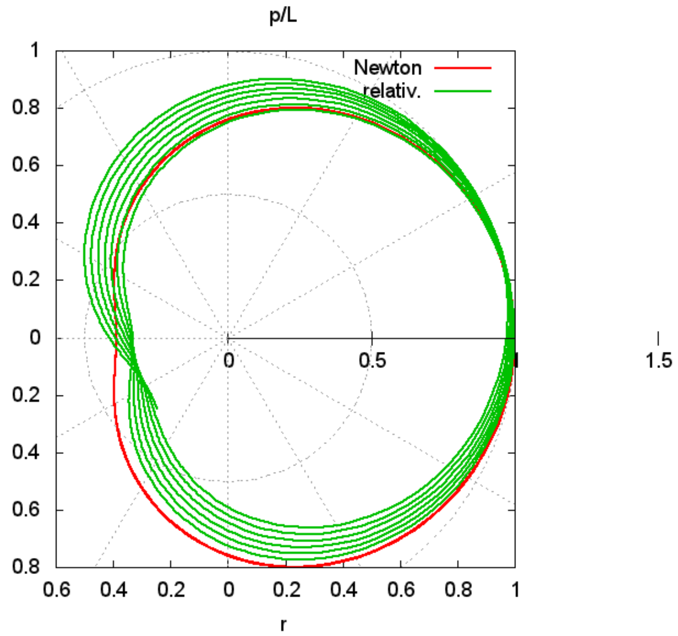

Figure 7: Ratio p/L.

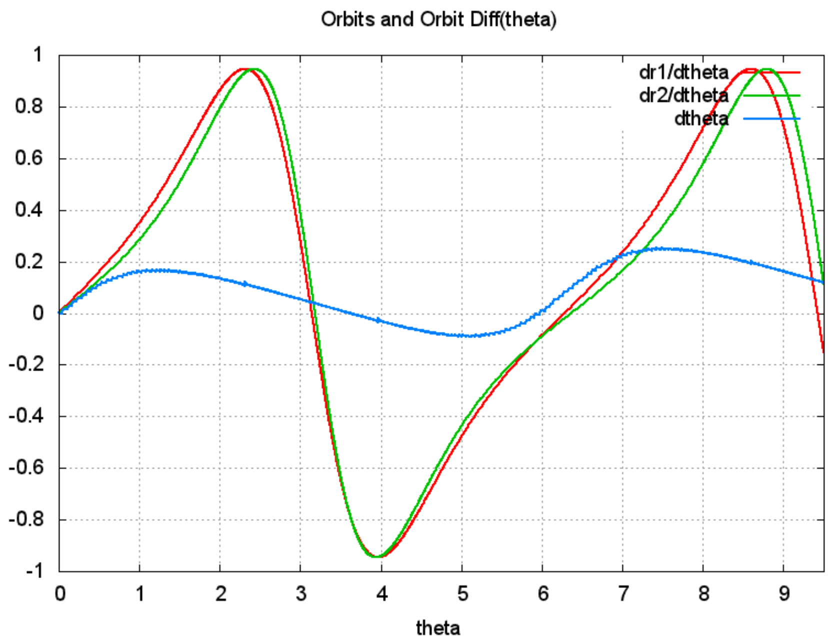

Figure 8: Normalized $dr/d\theta$ for Newtonian and relativistic calculation, and difference $\Delta\theta(\theta)$.

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