RIGOROUS QUANTIZATION OF THE HAMILTONIAN OF ECE2 SPECIAL RELATIVITY.

by

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ABSTRACT

It is shown that the hamiltonian of ECE2 special relativity can be quantized using at least four different classification schemes, each leading to different spectral results. The method used by Dirac uses an empirical choice of approximation which results in a zero classical hamiltonian and so is very restrictive if not unphysical. The schemes are illustrated with the rigorous quantization of the class one hamiltonian. If the spectral detail predicted by rigorous quantization is not observed, there is a major crisis in physics, because the philosophy of the Dirac equation would have been refuted, and along with it most of quantum field theory. In ECE theory the Dirac equation can be derived from Cartan geometry.

Keywords: ECE special relativity, quantization of the ECE2 hamiltonian.

4FT 333

1. INTRODUCTION

In recent papers of this series {1 - 12} ECE2 theory has been developed in UFT314-320 and UFT322 - UFT332 from the Jacobi Cartan Evans (JCE) identity of UFT313. It has been shown that ECE2 leads to a type of special relativity in a space with non zero torsion and curvature. Therefore the well known equations of special relativity can be used in a new way, notably the lagrangian and hamiltonian. In immediately preceding papers it has been shown that orbital precession in a plane can be produced by simultaneous solution of the lagrangian and hamiltonian of ECE2 special relativity. The ECE2 hamiltonian is mathematically the same as the one used by Dirac to produce relativistic quantum mechanics using the SU(2) basis. For over ninety years it was thought that the procedure used by Dirac is rigorous and foundational, but it has been shown in UFT330 ff. that it depends on a subjective or empirical choice of approximation. When the hamiltonian is quantized rigorously, different spectral detail emerge from each choice of quantization. There are at least four classes of hamiltonian in the SU(2) basis. Therefore the Dirac equation is not foundational. This finding catalyzes a major crisis in physics because it shows that a new insight is needed in order to forge a rigorous relativistic quantum mechanics.

This paper is a short synopsis of detailed calculations given in the notes accompanying UFT333 on www.aias.us. Note 333(1) describes the classification scheme and proves that the classical hamiltonian H_0 vanishes in the approximation used by Dirac. This approximation is therefore unphysical because it restricts the classical hamiltonian to zero, i.e. the hamiltonian can take only one value. This famous approximation happens to work but it is not known why it works. It is unphysical because the classical hamiltonian is obviously not restricted to zero in general. Note 333(2) works out the class four hamiltonian in terms of the Lande factor. Note 333(3) develops the class four hamiltonian without approximation.

Note 333(4) is a development of the classical energy levels of the H atom, which can be expressed in terms of the rest energy of special relativity multiplied by $(\alpha / n)^2$ where α is the fine structure constant and n the principal quantum number of the H atom. Notes 333(5) to 333(7) describe the rigorous development of the class one hamiltonian and form the basis of Section 2 of this paper. Section 3 is a computational and graphical analysis.

2. RIGOROUS QUANTIZATION OF THE CLASS ONE HAMILTONIAN

The classification scheme is based on the following four types of SU(2) hamiltonian in ECE2 special relativity:

$$H_0 = \frac{1}{m}\, \boldsymbol{\sigma} \cdot \boldsymbol{p}_0 \frac{\gamma^2}{1+\gamma} \boldsymbol{\sigma} \cdot \boldsymbol{p}_0 + U \quad -(1)$$

$$H_0 = \frac{\gamma}{m}\, \boldsymbol{\sigma} \cdot \boldsymbol{p}_0 \frac{\gamma}{1+\gamma} \boldsymbol{\sigma} \cdot \boldsymbol{p}_0 + U \quad -(2)$$

$$H_0 = \frac{\gamma^2}{m}\, \boldsymbol{\sigma} \cdot \boldsymbol{p}_0 \frac{1}{1+\gamma} \boldsymbol{\sigma} \cdot \boldsymbol{p}_0 + U \quad -(3)$$

$$H_0 = \frac{1}{m} \frac{\gamma^2}{1+\gamma}\, \boldsymbol{\sigma} \cdot \boldsymbol{p}_0\, \boldsymbol{\sigma} \cdot \boldsymbol{p}_0 + U \quad -(4)$$

Here m is the particle mass, $\boldsymbol{p}_0$ the classical linear momentum, U the potential energy and γ is the Lorentz factor:

$$\gamma = \left(1 - \frac{p_0^2}{m^2 c^2}\right)^{-1/2} \quad -(5)$$

in which c is the vacuum speed of light. The classes one to four hamiltonians are defined by Eqs. (1) to (4) respectively. The classical hamiltonian is defined by:

$$H_0 := H - mc^2 \quad -(6)$$

where H is the relativistic hamiltonian:

$$H = E + U. \quad -(7)$$

where E is the relativistic total energy:

$$E = \gamma mc^2 = \left(c^2 p^2 + m^2 c^4\right)^{1/2}. \quad -(8)$$

Here

$$\underline{p} = \gamma \underline{p_0} \quad -(9)$$

is the relativistic momentum. It follows that:

$$H_0 = H - mc^2 = \frac{c^2 p^2}{E + mc^2} + U \quad -(10)$$

and the Dirac approximation is {1 - 12}:

$$H = mc^2 \quad -(11)$$

which means that the classical hamiltonian vanishes:

$$H_0 = 0. \quad -(12)$$

This is clearly unphysical, but the approximation leads to the famous description of spectral fine structure:

$$H_0 = \frac{c^2 p^2}{2mc^2 - U} + U \quad -(13)$$

through spin orbit interaction.

The rigorous development of the class one hamiltonian is described in Notes 333(5) to 333(7), of which this section is a synopsis. First note that:

$$p_0^2 = 2m(H_0 - U) \quad -(14)$$

so it follows that:

$$\frac{\gamma^2}{1+\gamma} = \left(\left(1 - \frac{2(H_0 - U)}{mc^2}\right) + \left(1 - \frac{2(H_0 - U)}{mc^2}\right)^{1/2}\right)^{-1} \quad -(15)$$

In the H atom, using the non relativistic hydrogenic orbitals in a first approximation:

$$\langle U \rangle = -2\langle H_0 \rangle = -mc^2\left(\frac{\alpha}{n}\right)^2 \quad -(16)$$

as in Note 333(4). Quantization takes place by using:

$$-i\hbar \underline{\nabla} \phi = \underline{p_0} \phi \quad -(17)$$

for the first $\underline{p_0}$ in Eq. (1), and by using the function for the second $\underline{p_0}$. This is in fact an arbitrary procedure, but is the one which has been used for ninety years. This procedure gives:

$$H_0 \phi = -\frac{i\hbar}{m}\underline{\sigma}\cdot\underline{\nabla}\left(\left(1 - \frac{2(H_0 - U)}{mc^2} + \left(1 - \frac{2(H_0 - U)}{mc^2}\right)^{1/2}\right)^{-1}\underline{\sigma}\cdot\underline{p_0}\,\phi\right) + U\phi \quad -(18)$$

Using computer algebra it is found that:

$$\underline{\nabla}\left(\frac{\gamma^2}{1+\gamma}\right) = -\left(\frac{2 + \left(1 - \frac{p_0^2}{m^2c^2}\right)^{-1/2}}{\left(1 - \frac{p_0^2}{m^2c^2} + \left(1 - \frac{p_0^2}{m^2c^2}\right)^{1/2}\right)^2}\right)\frac{e^2}{4\pi\epsilon_0 m^2 c^2}\frac{\underline{r}}{r^3} \quad -(19)$$

for the Coulomb potential between the electron and proton of the H atom:

$$U = -\frac{e^2}{4\pi\epsilon_0 r} \quad -(20)$$

Defining:

$$A := \frac{2 + \left(1 - \frac{p_0^2}{m^2c^2}\right)^{-1/2}}{\left(\left(1 - \frac{p_0^2}{m^2c^2}\right)^{1/2} + 1 - \frac{p_0^2}{m^2c^2}\right)^2} \quad -(21)$$

it is found that:

$$H_0\phi = -\frac{ie^2\hbar A}{4\pi\epsilon_0 m^2c^2r^3}\,\underline{\sigma}\cdot\underline{r}\,\underline{\sigma}\cdot\underline{p}_0\,\phi + U\phi + \ldots \quad -(22)$$

By Pauli algebra:

$$\underline{\sigma}\cdot\underline{r}\,\underline{\sigma}\cdot\underline{p}_0 = \underline{r}\cdot\underline{p}_0 + i\,\underline{r}\times\underline{p}_0 \quad -(23)$$

in which the classical orbital angular momentum is:

$$\underline{L} = \underline{r}\times\underline{p}_0. \quad -(24)$$

So the rigorous class one spin orbit hamiltonian is:

$$E = \langle H_{SO}\rangle = \frac{e^2\hbar A}{4\pi\epsilon_0 m^2c^2}\left\langle\frac{\underline{\sigma}\cdot\underline{L}}{r^3}\right\rangle. \quad -(25)$$

This reduces to the Dirac result in the limit:

$$\gamma \rightarrow 1 \quad -(26)$$

If $\underline{p}0$ and A are regarded as functions, the Dirac fine structure is shifted as described in Section 3. For relativistic $\underline{p}_0$ the shift becomes very large and should be experimentally observable. Such an experiment is a rigorous test of the Dirac equation, or more accurately, the quantization of the hamiltonian of special relativity and the very foundations of relativistic quantum mechanics.

If the expectation value:

$$\left\langle \frac{p_0^2}{m^2c^2} \right\rangle = \frac{1}{2}\left(\frac{\alpha}{n}\right)^2 = \frac{2.662567 \times 10^{-5}}{n^2} \qquad -(27)$$

is used an entirely different spectrum emerges from Eq. (1). The energy levels of this spectrum are:

$$E = \langle H_{so} \rangle = \frac{e^2 \hbar A}{4\pi \epsilon_0 m^2 c^2} \left\langle \frac{\sigma \cdot L}{r^3} \right\rangle \qquad -(28)$$

$$= \frac{e^2 A}{16\pi \epsilon_0 m^2 c^2} \left(\frac{J(J+1) - L(L+1) - S(S+1)}{a_0^3 n^3 L\left(L+\frac{1}{2}\right)(L+1)} \right).$$

in which the total angular momentum quantum number J is defined by:

$$J = L+S,\ L+S-1,\ \ldots,\ |L-S| \qquad -(29)$$

where L is the orbital angular momentum quantum number and S the spin quantum number.

In Eq. (28) A is defined by expectation values:

$$A = \frac{2 + \left(1 - \left\langle \frac{p_0^2}{m^2c^2} \right\rangle\right)^{-1/2}}{\left(\left(1 - \left\langle \frac{p_0^2}{m^2c^2} \right\rangle\right)^{1/2} + 1 - \left\langle \frac{p_0^2}{m^2c^2} \right\rangle\right)^2}. \qquad -(30)$$

The selection rules for such a spectrum are:

$$\Delta J = 0, \pm 1, \qquad -(31)$$

and

$$\Delta m_J = 0, \pm 1. \quad -(32)$$

Recall that Eq. (28) is the rigorous consequence of:

$$H_0 = \frac{1}{m} \underline{\sigma} \cdot \underline{p}_0 \frac{\gamma^2}{1+\gamma} \underline{\sigma} \cdot \underline{p}_0 + U \quad -(33)$$

which reduces to te classical hamiltonian:

$$H_0 \xrightarrow[\gamma \to 1]{} \frac{p_0^2}{2m} + U \quad -(34)$$

in the limit:

$$\gamma \to 1. \quad -(35)$$

In this development the classical hamiltonian can tae any value as required, this eliminating the unphysical Dirac approximation QED.

The energy levels from Eq. (28) are plotted in Section 3. If these are not observed the ninety year old philosophy of the Dirac equation fails at a basic level.

3. COMPUTATIONAL AND GRAPHICAL ANALYSIS

ACKNOWLEDGMENTS

The British Government is thanked for a Civil List pension, and the staff of AIAS and others for many interesting discussions. Dave Burleigh is thanked for website maintenance and design, feedback software and maintenance, and posting. Alex Hill is thanked for translation and broadcasting, and Robert Cheshire for broadcasting.

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3 Computational and graphical analysis analysis

In Eq.(19) the gradient of $\gamma^2/(1+\gamma)$ is given for the Coulomb potential (20) which has no angular dependence in spherical coordinates. If the potential has a Z dependence exclusively, the gradient takes the general form:

$$\frac{d}{dZ}\left(\frac{\gamma^2}{1+\gamma}\right) = -\left(\frac{\frac{d}{dZ}\,\mathrm{U}(Z)}{m\,c^2\sqrt{1-\frac{2\,(H_0-\mathrm{U}(Z))}{m\,c^2}}} + \frac{2\,\left(\frac{d}{dZ}\,\mathrm{U}(Z)\right)}{m\,c^2}\right) \cdot \left(-\frac{2\,(H_0-\mathrm{U}(Z))}{m\,c^2} + \sqrt{1-\frac{2\,(H_0-\mathrm{U}(Z))}{m\,c^2}} + 1\right)^{-2} \tag{36}$$

with potential $U(Z)$.

The energy splitting of spin-orbit coupling has been demonstrated in Paper 332. In Hydrogen the spin-orbit splitting is small ($\approx 10^{-5}$ eV). In heavy atoms the splitting becomes high and the linear momentum is significantly larger than in Hydrogen. Therefore the A factor defined by Eq.(21) grows significantly, giving additional enlargement of splittings in Eq.(25). To demonstrate the effect, we plottet the A factor in dependence of a normalized p_0, i.e. a variable

$$\bar{p}^2 = \frac{p_0^2}{2m}, \tag{37}$$

see Fig. 1. The factor A goes to infinity for $\bar{p}$ reaching unity which corresponds to velocity $v_0 = c$. For comparison the gamma factor (5) is plotted. It is seen that A rises much faster than the relativistic gamma factor. An effect should be detectable in spectra of heavy elements.

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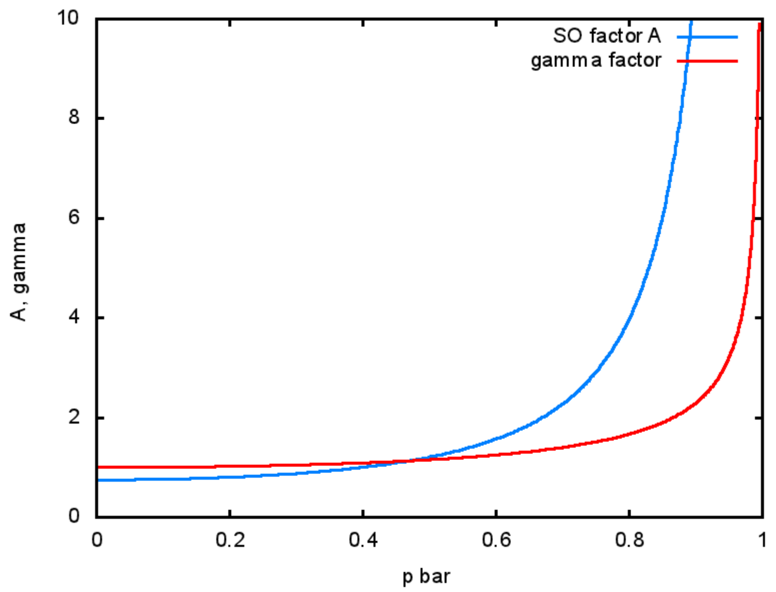

Figure 1: Functions $A(\bar{p})$ and $\gamma(\bar{p})$.

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