

THE AHARONOV BOHM EFFECT IN ECE2.

by

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ABSTRACT

The Aharonov Bohm effect is defined in ECE2 as a region where electric and magnetic fields are absent but in which the vacuum four potential is non-zero. The Aharonov Bohm vacuum is distinguished from the vacuum defined by the absence of charge current density and it is shown that the Aharonov Bohm vacuum contains a vector potential which can cause electron spin resonance and nuclear magnetic resonance in the absence of a magnetic field.

Keywords: ECE2 theory, the Aharonov Bohm effect, ESR caused by the vacuum potential.

UFT 336



## 1. INTRODUCTION

In recent papers of this series {1-12} ECE2 special relativity has been developed for various types of spectroscopy, notably electron spin resonance (ESR) and nuclear magnetic resonance (NMR). ECE2 special relativity is defined in a space with finite torsion and curvature. During the course of development of ECE2 theory (UFT313-UFT320 and UFT322 - UFT335 on [www.aias.us](http://www.aias.us) to date) several new insights have emerged, notably in field theory, cosmology and spectroscopy. In Section 2 of this paper the Aharonov Bohm (AB) vacuum is defined in ECE2 as regions in which electric and magnetic fields are absent but in which the AB vacuum four potential is non-zero. It is shown that the vector potential of the AB vacuum causes electron spin resonance (ESR) in the absence of a magnetic field. The well known Chambers experiment can be adopted for ESR due to the AB vacuum, proving that the vacuum contains a vector potential.

This paper is a summary of the detailed calculations in the five notes accompanying UFT336 on [www.aias.us](http://www.aias.us). In Note 366(1), the relativistic theory of ESR in an electron beam is developed using the quantization method of the Schroedinger equation, i.e. quantization of the classical linear momentum. This is a stepping stone on the way to rigorous relativistic quantization, one in which the relativistic four momentum is quantized. The Dirac equation is quantized in this way. In Note 336(2) the ECE wave equation in the Dirac limit is used to calculate the complete wave function of the electron, a wave function that depends both on the coordinate  $r$  and the time  $t$ . This note uses rigorously relativistic quantization to produce the ESR resonance frequency in an electron beam. The Lorentz factor is defined by the de Broglie / Einstein equations. The Note concludes that this type of ESR can be used as a test of the fundamentals of relativistic quantum mechanics and of the de Broglie / Einstein equations. Note 336(3) makes a preliminary study of the anomalous  $g$  factor of the electron

in terms of the AB vacuum vector potential. Note 336(4) uses the field equations of ECE2 theory (see for example UFT318) to define the AB vacuum in terms of the potentials of ECE2 theory, and distinguishes the AB vacuum from the traditionally defined vacuum in which electric and magnetic fields are non zero, but in which the charge current density is zero. Finally Note 336(5) calculates the ESR resonance frequency due to the AB vacuum vector potential. Section 2 is based on Notes 336(4) and 336(5). Section 3 is a graphical and computational analysis by co author Horst Eckardt.

## 2. CONDITION FOR THE AB EFFECT AND ESR BY THE VACUUM.

The Aharonov Bohm effect is well known {1-12} to be described by the presence of potentials and the absence of electromagnetic fields. Consider the magnetic flux density  $\underline{B}$  in ECE2 theory, it is defined by the  $\underline{W}$  and  $\underline{A}$  potentials of ECE2 (e.g. UFT318) as follows:

$$\underline{B} = \underline{\nabla} \times \underline{W} = \underline{\nabla} \times \underline{A} + 2\underline{\omega} \times \underline{A} \quad - (1)$$

where:

$$\underline{W} = W^{(0)} \underline{\omega}, \quad \underline{A} = A^{(0)} \underline{q}. \quad - (2)$$

Here  $\underline{\omega}$  is the spin connection vector and  $\underline{q}$  is the tetrad vector. Therefore the AB vacuum is defined by the Cartan geometry:

$$\underline{\nabla} \times \underline{\omega} = \underline{0} \quad - (3)$$

and

$$\underline{\nabla} \times \underline{q} = 2\underline{q} \times \underline{\omega}. \quad - (4)$$

Using the identity:

$$\underline{\nabla} \cdot \underline{\nabla} \times \underline{q} = \underline{0} \quad - (5)$$

the AB vacuum geometry can be defined by one equation:

$$\underline{\omega} \cdot \underline{\nabla} \times \underline{q} = 0 \quad - (6)$$

i.e.

$$\underline{W} \cdot \underline{\nabla} \times \underline{A} = 0 \quad - (7)$$

With reference to Note 336(4), the magnetic flux density B and the electric field strength E are defined in ECE2 theory by the spin and orbital curvature vectors (UFT318) as follows:

$$\underline{B} = \underline{W}^{(s)} \underline{R}(\text{spin}) \quad - (8)$$

and

$$\underline{E} = c \underline{W}^{(o)} \underline{R}(\text{orb}). \quad - (9)$$

So the AB effects occur in regions where there is no torsion and no curvature, but in which the tetrad and spin connection are zero. In minimal notation, the AB vacuum geometry {1 - 12} is as follows:

$$T = d\Lambda q + \omega \Lambda q = 0 \quad - (10)$$

$$R = d\Lambda \omega + \omega \Lambda \omega = 0 \quad - (11)$$

so

$$d\Lambda q = -\omega \Lambda q, \quad - (12)$$

$$d\Lambda \omega = -\omega \Lambda \omega, \quad - (13)$$

with

$$T = R = 0. \quad - (14)$$

In Note 336(4), the AB vacuum is carefully distinguished from the traditional vacuum defined by the absence of charge current density, but a vacuum in which electric and magnetic fields and potentials are non zero. In the AB vacuum the electric and magnetic fields, and the charge / current density four vector are all zero, but the potentials are non zero. The Chambers experiment shows that the AB vacuum is a physical vacuum, because the Young diffraction of electron matter waves is affected by potentials in the absence of fields. The

traditional type of vacuum is defined in ECE2 theory by:

$$\left. \begin{aligned} \underline{\nabla} \cdot \underline{B} &= 0 \\ \underline{\nabla} \cdot \underline{E} &= 0 \\ \frac{\partial \underline{B}}{\partial t} + \underline{\nabla} \times \underline{E} &= \underline{0}, \quad \underline{\nabla} \times \underline{B} - \frac{1}{c} \frac{\partial \underline{E}}{\partial t} = \underline{0} \end{aligned} \right\} - (15)$$

and by:

$$\left. \begin{aligned} \underline{\kappa} \cdot \underline{B} &= \underline{\kappa} \cdot \underline{E} = 0 \\ \kappa_0 c \underline{B} + \underline{\kappa} \times \underline{E} &= \underline{0}, \quad \kappa_0 \underline{E} / c + \underline{\kappa} \times \underline{B} = \underline{0} \end{aligned} \right\} - (16)$$

where

$$\kappa_0 = 2 \left( \underline{v}_0 / r^{(0)} - \underline{\omega}_0 \right), \quad - (17)$$

$$\underline{\kappa} = 2 \left( \underline{v} / r^{(0)} - \underline{\omega} \right), \quad - (18)$$

in the notation of the Engineering Model (UFT303) and UFT318. Note 336(4) shows that the

solution:

$$\kappa_0 = 0, \quad \underline{\kappa} = \underline{0} \quad - (19)$$

means that  $\underline{B}$  and  $\underline{E}$  vanish. The simplest solution of Eqs. (15) to (19) is:

$$\underline{E} = \underline{B} = \underline{0}, \quad \kappa_0 = 0, \quad \underline{\kappa} = \underline{0} \quad - (20)$$

in which case the traditional vacuum reduces to the AB vacuum.

If the traditional vacuum theory is accepted, and plane wave solutions used for Eqs. (15), the result is:

$$\underline{\kappa} = \left( \frac{\kappa^2 + \kappa_0^2}{\kappa_x + \kappa_y} \right) (\underline{i} + \underline{j}) \quad - (21)$$

as shown in Note 336(4). Under condition (21) ECE2 allows vacuum electric and magnetic fields to exist in the absence of charge current density. The Aharonov Bohm vacuum on the other hand is defined by Eq. (20).

It is of interest to develop a theory of the interaction of the AB vacuum with one

electron, because this theory leads to the possibility of ESR and NMR in regions where there is no magnetic flux density B. This would be a precise demonstration of the existence of the AB vacuum vector potential, denoted A in the following theory. As shown in detail in Notes 336(5), the relativistic theory of the interaction of one electron with the Aharonov Bohm vacuum is given by the equation:

$$(E - e\phi)^2 = c^2 (\underline{p} - e\underline{A}) \cdot (\underline{p} - e\underline{A}) + m^2 c^4 \quad (22)$$

which is the Einstein energy equation of the electron modified by the minimal prescription:

$$\underline{p}^{\mu} \rightarrow \underline{p}^{\mu} - eA^{\mu} \quad (23)$$

where the AB vacuum four potential is:

$$A^{\mu} = \left( \frac{\phi}{c}, \underline{A} \right) \quad (24)$$

The total relativistic energy and momentum are defined by the well known de Broglie /

Einstein equations:

$$E = \gamma mc^2 = \hbar \omega \quad (25)$$

$$\underline{p} = \gamma \underline{p}_0 = \hbar \underline{k} \quad (26)$$

where  $\omega$  is the angular frequency of the electron matter wave and  $\underline{k}$  its wave vector.

Here  $\hbar$  is the reduced Planck constant. The Lorentz factor is therefore defined by:

$$\gamma = \frac{\hbar \omega}{mc^2} = \left( 1 - \frac{p_0^2}{m^2 c^2} \right)^{-1/2} \quad (27)$$

where the relativistic momentum is defined by:

$$\underline{p} = \gamma \underline{p}_0 = \gamma m \underline{v}_0 \quad (28)$$

It follows that:

$$E - mc^2 = \frac{1}{(1+\gamma)m} \underline{\sigma} \cdot (\underline{p} - e\underline{A}) \left( \frac{1 - e\phi}{(1+\gamma)mc^2} \right)^{-1} \underline{\sigma} \cdot (\underline{p} - e\underline{A}) + e\phi \quad - (29)$$

which reduces to a Dirac type theory when:

$$\gamma \rightarrow 1 \quad - (30)$$

i.e. when the electron's angular frequency is defined by its rest angular frequency:

$$\hbar \omega_0 = mc^2 \quad - (31)$$

The Dirac theory therefore contains a self contradiction, because the electron is not moving.

As shown in immediately preceding papers the Dirac theory produces the unphysical result:

$$H_0 = H - mc^2 = ? 0. \quad - (32)$$

In order to develop Eq. (29) in an analytically tractable way assume that:

$$e\phi \ll (1+\gamma)mc^2 \quad - (33)$$

an approximation that leads to:

$$E - mc^2 \sim \frac{1}{(1+\gamma)m} \underline{\sigma} \cdot (\underline{p} - e\underline{A}) \underline{\sigma} \cdot (\underline{p} - e\underline{A}) + \frac{1}{(1+\gamma)m} \underline{\sigma} \cdot (\underline{p} - e\underline{A}) \frac{e\phi}{(1+\gamma)mc^2} \underline{\sigma} \cdot (\underline{p} - e\underline{A}) + e\phi \quad - (34)$$

The right hand side of this equation contains the ESR term in its first term, and spin orbit

effects in its second term. Relativistic quantization is defined by

$$\hat{p}^\mu \psi = i\hbar \partial^\mu \psi \quad - (35)$$

i.e.

$$E\psi = i\hbar \frac{\partial \psi}{\partial t} \quad - (36)$$

and

$$\underline{p}\psi = -i\hbar \underline{\nabla}\psi \quad - (37)$$

This quantization procedure cannot be proven ab initio. It is purely empirical. So there are many ways of quantizing Eq. (34).

The required ESR term is given by the quantization:

$$\begin{aligned} (E - mc^2)\psi &= \frac{1}{m(1+\gamma)} \left( \underline{\sigma} \cdot (-i\hbar \underline{\nabla} - e\underline{A}) \cdot \left( (\underline{\sigma} \cdot \underline{p} - e\underline{A})\psi \right) \right) \\ &= \frac{ie\hbar}{m(1+\gamma)} \underline{\sigma} \cdot \underline{\nabla} \underline{\sigma} \cdot \underline{A} \psi + \dots \quad - (38) \end{aligned}$$

Using Pauli algebra:

$$\underline{\sigma} \cdot \underline{\nabla} \underline{\sigma} \cdot \underline{A} = \underline{\nabla} \cdot \underline{A} + i \underline{\sigma} \cdot \underline{\nabla} \times \underline{A} \quad - (39)$$

so the relevant real and physical part of Eq. (38) is:

$$(E - mc^2)\psi = -\frac{e\hbar}{m(1+\gamma)} \underline{\sigma} \cdot \underline{\nabla} \times \underline{A} \psi + \dots \quad - (40)$$

The spin angular momentum of the electron is:

$$\underline{S} = \frac{\hbar}{2} \underline{\sigma} \quad - (41)$$

so

$$(E - mc^2)\psi = -\frac{2e}{m(1+\gamma)} \underline{S} \cdot \underline{\nabla} \times \underline{A} \psi + \dots \quad - (42)$$

In quantum theory it is well known that:



$$S_z \psi = \hbar m_s \psi \quad (43)$$

where

$$m_s = -S, \dots, S \quad (44)$$

Here S is the spin angular quantum number of the electron, a fermion, so:

$$S = 1/2 \quad (45)$$

and

$$m_s = \frac{1}{2}, -\frac{1}{2} \quad (46)$$

Therefore:

$$(\hat{E} - mc^2) \psi = -\frac{2e\hbar}{m(1+\gamma)} m_s (\nabla \times \underline{A})_z \quad (47)$$

and electron spin resonance is defined by:

$$\hbar \omega_{res} = -\frac{2e\hbar}{m(1+\gamma)} \left(-\frac{1}{2} - \frac{1}{2}\right) (\nabla \times \underline{A})_z \quad (48)$$

so the resonance frequency is:

$$\omega_{res} = \frac{2e}{m(1+\gamma)} (\nabla \times \underline{A})_z \quad (49)$$

Therefore the effect of the vacuum of AB type is to cause ESR in the absence of a magnetic field, QED.

### 3. COMPUTATIONAL AND GRAPHICAL ANALYSIS

Section by Dr. Horst Eckardt.

# The Aharonov Bohm effect in ECE2

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## 3 Computational and graphical analysis

We demonstrate some properties of the vector potential in order to explain that it is difficult to assess the type of vector potential from its appearance. First we consider a dipole vector field in two dimensions:

$$\mathbf{A}(X, Y) = q_1 \left( \frac{X - X_0}{((X - X_0)^2 + Y^2)^{3/2}} \right) + q_2 \left( \frac{X + X_0}{((X + X_0)^2 + Y^2)^{3/2}} \right). \quad (50)$$

The direction vectors of is field have been graphed Fig. 1, together with the equipotential lines following from the Coulomb type potential

$$V(X, Y) = \frac{q_1}{((X - X_0)^2 + Y^2)^{1/2}} + \frac{q_2}{((X + X_0)^2 + Y^2)^{1/2}}. \quad (51)$$

The two values of charges were chosen different from each other:  $q_1 = 1$ ,  $q_2 = -1.6$ . As a result, the potential and dipole field at the right hand side is much more contracted than at the left, and a rotational structure of directional vectors can be seen at the right hand side. Nevertheless, the curl of a dipole field vanishes as can be calculated from Eq.(50):

$$\nabla \times \mathbf{A}(X, Y) = \mathbf{0}. \quad (52)$$

This seems not to be as expected at a first glance.

As a second example we consider the vector potential of an infinitely extended solenoid in cylindrical coordinates  $(r, \theta, Z)$ . From electrodynamics is known that the corresponding vector potential has only a  $\theta$  component. It rises linearly within the solenoid (until radius  $a$ ) and drops hyperbolically outside:

$$A_\theta = \begin{cases} fr & \text{for } r \leq a \\ f \frac{a^2}{r} & \text{for } r > a \end{cases} \quad (53)$$

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where  $f$  is a factor. The  $Z$  component of the curl of this vector potential is

$$(\nabla \times \mathbf{A})_Z = \begin{cases} 2f & \text{for } r \leq a \\ 0 & \text{for } r > a \end{cases}, \quad (54)$$

the other components are zero. This describes the fact that an idealized infinite solenoid contains a homogeneous magnetic field inside, and the magnetic field vanishes outside as is the case for the Aharonov Bohm effect. The vector potential is graphed in Fig. 2 with some circles describing isolines of the magnitude of  $\mathbf{A}$ . Although the structure of this potential is very regular compared to Fig. 1, the curl does not vanish in the inner part. The curl follows from the coordinate dependence of the  $\mathbf{A}$  components in a quite intricate way.

Fig. 2 may serve as a demonstration of the potential for the Aharonov Bohm effect where the solenoid normally has a torus-like, closed form. Interpreting  $\mathbf{A}$  as a vacuum potential, it may even be permitted that the curl does not vanish. According to newest experimental results there may fluctuating electric and magnetic fields be present in the vacuum on a very short time scale. This justifies the approach for electron spin resonance (Eq.(49)) in this paper.

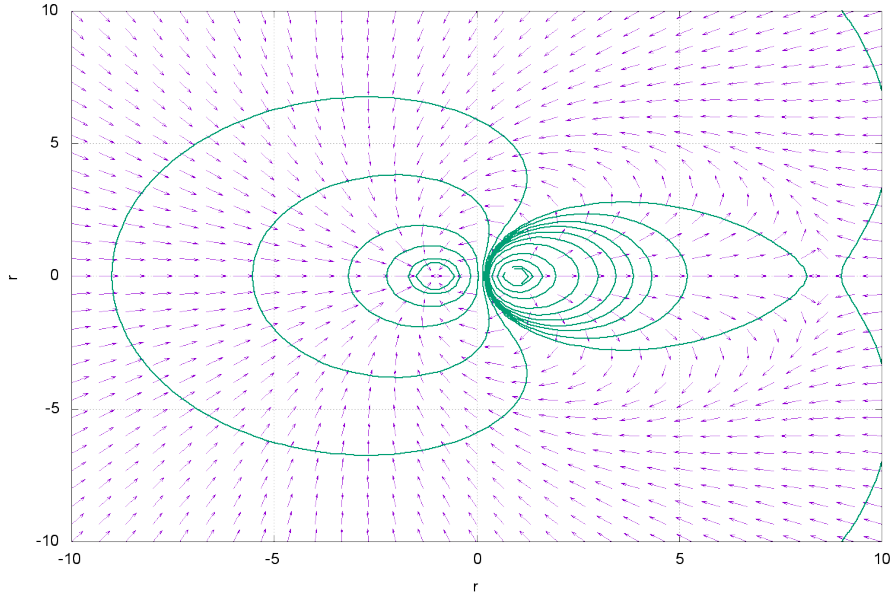


Figure 1: Equipotential lines and direction vectors of a dipole field.

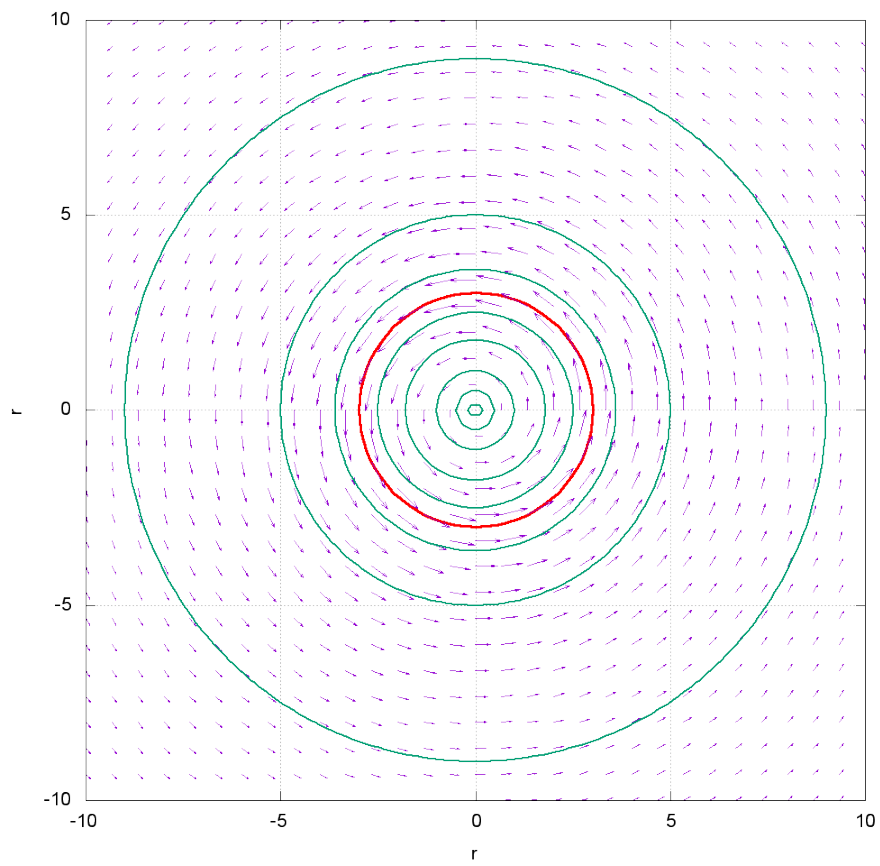


Figure 2: Vector potential and lines of equal magnitude for an infinite solenoid (cross section). The red circle indicates the radius of the solenoid.

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