GEODETIC AND LENSE THIRRING PRECESSIONS FROM THE ECE2 GRAVITATIONAL FIELD EQUATIONS.

by

M. W. Evans and H. Eckardt.

Civil List, AIAS and UPITEC

(www.archive.org, www.webarchive.org.uk, www.aias.us, www.upitec.org, ww.atomicprecision.com, www.et3m.net)

ABSTRACT

The geodetic and Lense Thirring precessions are calculated in the dipole approximation from the ECE2 gravitational field equations, using the concepts of gravitomagnetostatics in a Lorentz covariant theory in a mathematical space with finite torsion and curvature. The Lense Thirring precession is in exact agreement with the experimental data from Gravity Probe B using an averaging procedure. The geodetic precession is in good agreement and can be refined to exact agreement with additional assumptions.

Keywords: ECE2, geodetic and Lense Thirring precessions, Gravity Probe B.

UFT 345

1. INTRODUCTION

In recent papers of this series {1-12}, the field equations of ECE2 unified field theory have been applied to precession phenomena in astronomy, and to other phenomena such as electromagnetic deflection by gravitation, in papers such as UFT324 and UFT328. Recently the Evans Eckardt theorem has been inferred and used to describe the Thomas precession and Lense Thirring precession from the foundational definition of relativistic velocity in a Lorentz covariant theory such as ECE2. The latter unifies special and general relativity by developing a Lorentz covariant structure in a mathematical space with finite torsion and curvature. This Lorentz covariant structure (UFT313 - 320, 322 - 344 on www.aias.us) gives the field equations of gravitation in precisely the same format as the field equations of electromagnetism. This is achieved with finite torsion and curvature. It is well known that the claims of the obsolete Einsteinian era of gravitational physics are incorrect because of the neglect of torsion. As shown in UFT99 and associated proofs on www.aias..us, if torsion is zero so is curvature, and there is no geometry, reductio ad absurdum. This means that any geometrical theory of gravitation must be based on non zero torsion and curvature. The claims to precision of the Einsteinian era cannot be correct. This has been accepted at leading universities in van der Merwe's "post Einsteinian paradigm shift" which has generated an estimated 500 to 1000 million readings off www.aias.us and www.upitec.org (See for example UFT307) since 2002. These sites are archived on www.archive.org and www.webarchive.org.uk.

This paper is a synopsis of calculations in the notes posted with UFT345 on www.aias.us. Notes 345(1) to 345(4) are preliminary groundwork, Note 345(5) is the calculation of the Lense Thirring effect in a simplified geometry which is developed in Note 345(6) and in Section 3, where an average value of the effect is calculated. Note 345(6) gives the groundwork for the calculation of the geodetic effect, this calculation is refined in Note

345(8).

In Section 2 the ECE2 field equations are applied to the geodetic and Lense Thirring precessions {1 - 12} mistakenly attributed to the incorrect Einstein theory. The gravitational equivalent of magnetostatics is used to describe both effects from the same starting equation, that of the gravitomagnetic field in the dipole approximation. Gravity Probe B measured both effects experimentally using precision gyroscopes, which are gravitomagnetic dipole moments. The earth's gravitomagnetic field creates a torque with the magnetic dipole moment on board Gravity Probe B. In the Lense Thirring effect the gravitomagnetic field is that of the spinning earth in a static frame fixed at the centre of the earth. In the geodetic effect it is the gravitomagnetic field of the earth in a spinning frame, the spinning of the earth as seen from Gravity Probe B.

In Section 3 the results are analysed numerically and graphically and checked with computer algebra.

2. PRECESSION THEORIES.

The calculation of the Lense Thirring precession in ECE was initiated in UFT117 on www.aias.us. The major advance since then is the emergence of the ECE2 gravitational field equations. Consider, as in UFT117, the gravitomagnetic field of the earth in the dipole approximation:

$$\underline{\Omega} = \frac{2}{5}\frac{MGR^2}{c^2 r^3}\left(\underline{\omega} - 3\underline{n}\left(\underline{\omega}\cdot\underline{n}\right)\right) \quad -(1)$$

were the earth is considered to be a spinning sphere. Here M is the mass of the earth, G is Newton's constant, R is the radius of the earth, r is the distance from the centre of the earth to Gravity Probe B, $\underline{\omega}$ is the angular velocity vector of the earth, and $\underline{n}$ is the unit vector defined by:

$$\underline{n} = \frac{\underline{r}}{r} \quad -(2)$$

The Gravity Probe B satellite was in polar orbit, orbiting in a plane perpendicular to the equator in a geocentric orbit. The angular velocity vector $\underline{\omega}$ of the spinning earth is:

$$\underline{\omega} = \omega \underline{k} \quad -(3)$$

because the earth spins around the $\underline{k}$ axis perpendicular to the equator. The distance between the centre of the earth and Gravity Probe B is defined in the plane perpendicular to $\underline{k}$:

$$\underline{r} = Y \underline{j} + Z \underline{k} \quad -(4)$$

so:

$$\underline{\omega} - 3\underline{n}(\underline{\omega} \cdot \underline{n}) = \omega \left(\left(1 - 3\frac{Z^2}{r^2} \right) \underline{k} - \frac{3YZ}{r^2} \underline{j} \right) \quad -(5)$$

The experimental inclination of Garvity Probe B was almost exactly 90°.

Therefore in the dipole approximation (see UFT117):

$$\underline{\Omega} = \frac{2}{5} \frac{MGR^2 \omega}{c^2 r^3} \left(\left(1 - 3\frac{Z^2}{r^2} \right) \underline{k} - \frac{3YZ}{r^2} \underline{j} \right). \quad -(6)$$

The Gravity Probe B spacecraft carried precision gyroscopes which are currents of mass and which are therefore gravitomagnetic dipole moments ($\underline{m}$). The torque between the earth and the spacecraft is:

$$\underline{Tq} = \underline{m} \times \underline{\Omega}. \quad -(7)$$

As in UFT344 this torque produces the Larmor precession frequency:

$$\Omega_{LT} = \frac{1}{2} |\underline{\Omega}| \quad -(8)$$

which is the Lense Thirring precession. This paper uses the following data:

$$M = 5.98 \times 10^{24} \text{ kg}$$
$$R = 6.37 \times 10^{6} \text{ m}$$
$$r = 7.02 \times 10^{6} \text{ m}$$
$$c = 2.998 \times 10^{8} \text{ m s}^{-1}$$
$$G = 6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$$
$$\omega = 7.292 \times 10^{-5} \text{ rad s}^{-1}.$$

At the equator:

$$\underline{\omega} \cdot \underline{n} = 0 \quad -(9)$$

and the magnitude of the gravitomagnetic field of the earth from Eq. (6), in radians per second is:

$$\Omega = 1.52 \times 10^{-14} \text{ rad s}^{-1} \quad -(10)$$

compared with the experimental value (UFT117) of

$$\Omega(\text{exp}) = 1.26 \times 10^{-14} \text{ rad s}^{-1}. \quad -(11)$$

More generally:

$$\underline{\omega} \cdot \underline{n} = \frac{Z}{r} \omega \underline{k} \quad -(12)$$

and

$$3\underline{n}(\underline{\omega} \cdot \underline{n}) = \frac{3Z\omega}{r^2}(Y\underline{j} + Z\underline{k}). \quad -(13)$$

It therefore follows that:

$$\underline{\omega} - 3\underline{n}\left(\underline{\omega}\cdot\underline{n}\right) = \omega\underline{k} - \frac{3\omega Z}{r}\left(\frac{Y\underline{j}}{r} + \frac{Z\underline{k}}{r}\right). \quad -(14)$$

Defining:

$$\sin\theta = \frac{Z}{r}, \quad \cos\theta = \frac{Y}{r} \quad -(15)$$

then:

$$\underline{\omega} - 3\underline{n}\left(\underline{\omega}\cdot\underline{n}\right) = \omega\left(\left(1 - 3\sin^2\theta\right)\underline{k} - 3\sin\theta\cos\theta\,\underline{j}\right). \quad -(16)$$

Therefore the Lense Thirring precession is, from these equations:

$$\Omega_{LT} = \frac{MGR^2\omega}{5c^2r^3}\left|\left(1 - 3\sin^2\theta\right)\underline{k} - 3\sin\theta\cos\theta\,\underline{j}\right|. \quad -(17)$$

In Section three an average value of the precession is worked out and the latitude defined for precise agreement with the experimental data by Stanford / NASA. It is assumed that the experimental result is an average, because the Lense Thirring precession in general depends on the latitude. It is not clear how the Lense Thirring precession is isolated experimentally from the geodetic precession. This paper accepts the experimental claims uncritically.

The analogue of Eq. (1) in magnetostatics {1 - 12} is

$$\underline{B} = \frac{\mu_0}{4\pi r^3}\left(\underline{m} - 3\underline{n}\left(\underline{m}\cdot\underline{n}\right)\right) \quad -(18)$$

where $\underline{B}$ is the magnetic flux density, and where μ_0 is the magnetic permeability. Here $\underline{m}$ is the magnetic dipole moment:

$$\underline{m} = -\frac{e}{2m}\underline{L} \quad -(19)$$

where -e is the charge on the electron, m is the mass of the electron, and $\underline{L}$ the orbital angular momentum. The gravitomagnetic permeability of the ECE2 field equations is:

$$\mu_{0G} = \frac{4\pi G}{c^2} \quad -(20)$$

where c is the vacuum speed of light, a universal constant. So:

$$\underline{\Omega} = \frac{G}{c^2 r^3}\left(\underline{m}_g - 3\underline{n}\left(\underline{m}_g \cdot \underline{n}\right)\right) \quad -(21)$$

where the gravitomagnetic dipole moment $\underline{m}_g$ is defined in analogy to Eq. (19) by replacing -e by m, so:

$$\underline{m}_g = \frac{1}{2}\underline{L}. \quad -(22)$$

The angular momentum of the spinning earth considered as a sphere (UFT117) is:

$$\underline{L} = \frac{2}{5} M R^2 \underline{\omega} \quad -(23)$$

so

$$\Omega = \frac{MGR^2\omega}{5c^2r^3} = 49.4 \text{ milliarcseconds per year} \quad -(24)$$

Q. E. D.

The geodetic precession is calculated from the same starting equation as the Lense-Thirring precession:

$$\underline{\Omega} = \frac{MG}{2c^2 r}\left|\underline{\omega} - 3\underline{n}\left(\underline{\omega} \cdot \underline{n}\right)\right|. \quad -(25)$$

The vector $\underline{r}$ is defined by Eq. (4) because Gravity Probe B was in a polar orbit once every ninety minutes, giving an angular velocity of

$$\omega = \frac{2\pi}{90 \times 60} = 1.164 \times 10^{-3} \text{ rad s}^{-1} \quad -(26)$$

As seen from a frame of reference fixed on Gravity Probe B, the earth rotates at a given angular velocity, generating the angular momentum:

$$\underline{L} = \underline{M} r \underline{\omega} \quad - (27)$$

for an assumed circular orbit, a good approximation to the orbit of Gravity Probe B. If it is assumed that:

$$\underline{\omega} = \omega_X \underline{i} \quad - (28)$$

perpendicular to the polar orbit, then the theoretical geodetic precession is:

$$\Omega = \frac{MG\omega}{2c^2 r} \quad - (29)$$

For the earth:

$$\frac{MG}{2c^2} = 2.2175 \times 10^{-3} \, m. \quad - (30)$$

If it is assumed that r is the distance from the centre of the earth to Gravity Probe B then:

$$r = 7.02 \times 10^6 \, m. \quad - (31)$$

This gives a theoretical result of:

$$\Omega = 3.677 \times 10^{-13} \, rad \, s^{-1} \quad - (32)$$

The experimental claim by NASA / Stanford is:

$$\Omega(exp) = 1.016 \times 10^{-12} \, rad \, s^{-1} \quad - (33)$$

It is assumed that this experimental value is an average.

The theory is in good agreement with the experimental claim. It ahs been assumed that the angular momentum needed for Eq. (25) is generated by a static earth in a rotating frame. The latter is the passive rotation equivalent to the active rotation of Gravity Probe B around the centre of the earth in a polar orbit once every ninety minutes. Exact agreement with the experimental claim can be obtained by assuming an effective gravitomagnetic Landé factor, or by assuming that the rotating frame is described more generally by:

$$\underline{\omega} = \omega_X \underline{i} + \omega_Y \underline{j} \quad -(34)$$

and

$$\underline{r} = Y\underline{j} + Z\underline{k}. \quad -(35)$$

In Section 3, computer algebra and graphics are used to evaluate the magnitude:

$$x = \left| \underline{\omega} - 3\underline{n}(\underline{\omega} \cdot \underline{n}) \right| \quad -(36)$$

from Eqs. (34) and (35). Therefore exact agreement with the Lense Thirring and geodetic precessions can be obtained from the gravitational field equations of ECE2, which is a Lorentz covariant theory developed in a mathematical space with identically non zero torsion and curvature as required for any valid geometry, and any valid theory of relativity based on geometry.

3. AVERAGING THEORY, COMPUTER ALGEBRA AND GRAPHICS.

Geodetic and Lense Thirring precession from the ECE2 gravitational field equations

M. W. Evans*, H. Eckardt†
Civil List, A.I.A.S. and UPITEC

(www.webarchive.org.uk, www.aias.us, www.atomicprecision.com, www.upitec.org)

3 Averaging theory, computer algebra and graphics

Lense Thirring effect

For the Lense Thirring effect the gravitomagnetic field is calculated from Eq. (1) which can be written

$$\mathbf{\Omega} = \frac{2}{5}\frac{MGR^2}{c^2r^3}\omega\mathbf{x} \tag{37}$$

with the angular vectorial factor

$$\mathbf{x} = \boldsymbol{\omega}_n - 3\mathbf{n}(\boldsymbol{\omega}_n \cdot \mathbf{n}) \tag{38}$$

containing the unit vector of angular momentum $\boldsymbol{\omega}_n$. If $\boldsymbol{\omega}_n$ is perpendicular to the radius unit vector $\mathbf{n}$, it is

$$|\mathbf{x}| = 1 \tag{39}$$

while for $\boldsymbol{\omega}_n$ being parallel to $\mathbf{n}$, we have

$$|\mathbf{x}| = 2 \tag{40}$$

so the modulus of $\mathbf{x}$ varies between 1 and 2. It is assumed that the experimental value of $\mathbf{\Omega}$ is an angular averaged value. We can determine this average value as follows. For the special geometry of $\boldsymbol{\omega}$ in direction of the Z axis and $\mathbf{n}$ in the Y-Z plane, we have according to Eqs. (12-16):

$$\mathbf{x} = \begin{bmatrix} 0 \\ -3\sin\theta\cos\theta \\ 1-3\sin^2\theta \end{bmatrix} \tag{41}$$

*email: emyrone@aol.com
†email: mail@horst-eckardt.de

and

$$x^2 = 9\sin^2\theta\cos^2\theta + \left(1 - 3\sin^2\theta)^2\right)^2 \qquad (42)$$
$$= 4 - 3\cos^2\theta.$$

The angular dependencies of x^2 and x are graphed in Fig. 1, showing how x varies between 1 and 2. In addition, the Y and Z component of the angular vector $\mathbf{x}$ are shown. They are both crossing zero but at different angles θ, therefore the modulus of $\mathbf{x}$ is always greater than unity.

Taking the average of x^2:

$$\langle x^2 \rangle = \frac{1}{\pi/2}\int_0^{\pi/2} (4 - 3\cos^2\theta)\, d\theta \qquad (43)$$

gives the result 5/2. Assuming

$$\langle x^2 \rangle = \langle x \rangle^2 \qquad (44)$$

then we obtain

$$\langle x \rangle = \sqrt{\langle x^2 \rangle} = \sqrt{\frac{5}{2}} = 1.5811. \qquad (45)$$

Multiplying the theoretical result ($1.52 \cdot 10^{-14}$ rad/s) by this value gives

$$\Omega = 2.40 \cdot 10^{-14}\,\text{rad/s} \qquad (46)$$

and a ratio

$$\frac{\Omega_{\text{theory}}}{\Omega_{\text{exp}}} = 1.91, \qquad (47)$$

this could correspond to an effective gravitomagnetic g factor of the Larmor frequency:

$$g = 2 \cdot 1.91 = 3.82. \qquad (48)$$

Another – perhaps more realistic – explanation of the deviation would be that the momentum of inertia for the earth was calculated assuming a homogeneous sphere, but the earth core has a much higher mass density than the earth mantle so the angular momentum is smaller than that of a homogeneous sphere of equal mass. The radii of the core and the average outer radius are

$$R_{\text{core}} = 3.485 \cdot 10^6\text{m}, \qquad (49)$$
$$R_{\text{earth}} = 6.371009 \cdot 10^6\text{m}. \qquad (50)$$

About 35% of the earth mass is concentrated in the core, therefore the masses of the core and the outer spherical shell (the earth mantle) are

$$M_{\text{core}} = 0.35 \cdot 5.97219 \cdot 10^{24}\text{kg} = 2.090 \cdot 10^{24}\text{kg}, \qquad (51)$$
$$M_{\text{mantle}} = 0.65 \cdot 5.97219 \cdot 10^{24}\text{kg} = 3.882 \cdot 10^{24}\text{kg}. \qquad (52)$$

The moments of inertia of earth core and mantle (a sphere and a spherical shell) are

$$I_{\rm core} = \frac{2}{5} M_{\rm core} R_{\rm core}^2 = 1.015 \cdot 10^{37} {\rm kg\,m}^2, \tag{53}$$

$$I_{\rm mantle} = \frac{2}{5} M_{\rm mantle} \frac{R_{\rm earth}^5 - R_{\rm core}^5}{R_{\rm earth}^3 - R_{\rm core}^3} = 7.167 \cdot 10^{37} {\rm kg\,m}^2. \tag{54}$$

The sum is smaller than the moment of inertia taken simply by the earth mass and earth radius:

$$I_{\rm earth} = \frac{2}{5} M_{\rm earth} R_{\rm earth}^2 = 9.696 \cdot 10^{37} {\rm kg\,m}^2 \tag{55}$$

so that the ratio of both models is

$$\frac{I_{\rm core} + I_{\rm mantle}}{I_{\rm earth}} = 0.8439\,. \tag{56}$$

So we have to multiply the results obtained for the gravitomagnetic field by this value. From the second line in Table 1, where the results are listed, we see that the minimal theoretical value (Lense-Thirring effect at the equator) coincides very well with the experimental value within 1.6%.

Geodetic precession

The same calculation for the angular average as above can be done for the geodetic effect, described by Eq. (25). The angular factor is the same as for the Lense Thirring effect so the angular average is identical to Eq. (45). The theoretical value ($3.675 \cdot 10^{-13}$ rad/s, Eq. (32)) here is lower than the experimental value ($1.016 \cdot 10^{-12}$ rad/s, Eq. (33)), therefore applying the average x factor leads to

$$\Omega = 5.811 \cdot 10^{-13}\,{\rm rad/s} \tag{57}$$

which is nearer to, but still below the experimental value. The averaging method has been repeated with a more general position of the angular momentum axis:

$$\boldsymbol{\omega} = \omega_Y \mathbf{i} + \omega_Y \mathbf{j} \tag{58}$$

i.e. the position has been tilted from the X axis. The highest value of angular average x is obtained for a 45 degree's tilting (i.e. $\omega_X = \omega_Y$):

$$\langle x \rangle = 1.8587 \tag{59}$$

corresponding to

$$\Omega = 6.831 \cdot 10^{-13}\,{\rm rad/s}. \tag{60}$$

Even for the maximum value $x = 2$ the result remains below the experimental value. Translating $x = 1.5811$ into a geodetic gravitomagnetic g factor gives

$$g = \frac{\Omega_{\rm theory}}{\Omega_{\rm exp}} = 1.49 \tag{61}$$

and could be an explanation of the deviation. We conclude that all theoretical results are close to the experimental findings. The numbers are comprehensively listed in Table 1. A final assessment can only be done after all details of the experiments have been understood.

	Theory min.	Theory max.	Theory av.	Exp.
Lense-Thirring effect, std. I	$1.52 \cdot 10^{-14}$	$3.04 \cdot 10^{-14}$	$2.40 \cdot 10^{-14}$	$1.26 \cdot 10^{-14}$
Lense-Thirring effect, improved I	$1.28 \cdot 10^{-14}$	$2.56 \cdot 10^{-14}$	$2.02 \cdot 10^{-14}$	
geodetic effect	$3.677 \cdot 10^{-13}$	$7.354 \cdot 10^{-13}$	$5.811 \cdot 10^{-13}$	$1.016 \cdot 10^{-12}$
geodetic effect, modified $\boldsymbol{\omega}$			$6.831 \cdot 10^{-13}$	

Table 1: Theoretical and experimental values of gravitomagnetic field in units of rad/s. I is moment of inertia, see text.

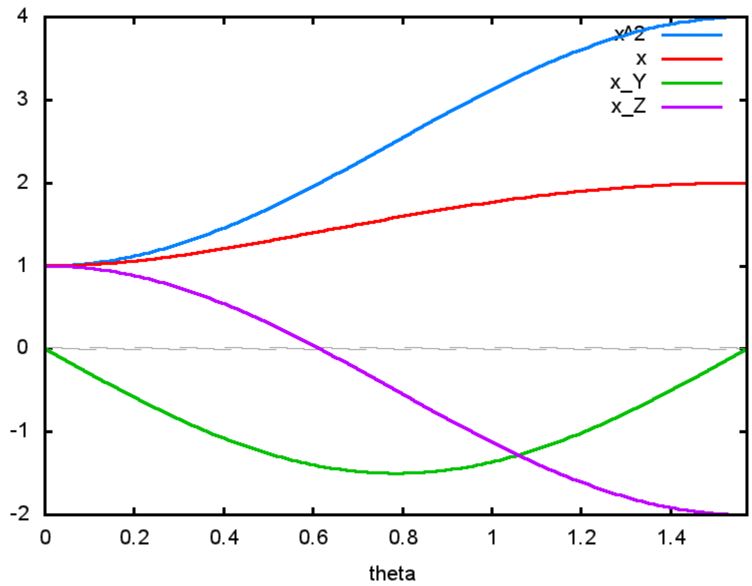

Figure 1: Angular factors x and x^2, and Y and Z components of vector $\mathbf{x}$.

ACKNOWLEDGMENTS

The British Government is thanked for a Civil List Pension, and the staff of AIAS and others for many interesting discussions. Dave Burleigh is thanked for posting, feedback software and maintenance, and for site maintenance and hosting. Alex Hill is thanked for translation and broadcasting, and Robert Cheshire for broadcasting.

REFERENCES

{1} M. W. Evans, H. Eckardt, D. W. Lindstrom and S. J. Crothers, "The Principles of ECE" (UFT281 - UFT288, New Generation, London, 2016 in prep.)

{2} M. W. Evans, "The Book of Scientometrics" (UFT307 and Filtered Statistics Section on www.aias.us and New Generation 2015).

{3} H. Eckardt, "The Engineering Model" (UFT303 on www.aias.us).

{4} M. W. Evans, H. Eckardt and D. W. Lindstrom, "Generally Covariant Unified Field Theory" (relevant UFT papers on www.aias.us and Abramis 2005 to 2011 in seven volumes)

{5} M. W. Evans, S. J. Crothers, H. Eckardt and K. Pendergast, "Criticisms of the Einstein Field Equation" (UFT301 and Cambridge International, CISP, 2010)

{6} M. W. Evans (Ed.), J. Found. Phys. Chem. (relevant UFT papers and CISP 2011).

{7} M. W. Evans (Ed.), "Definitive Criticisms of the Einsteinian General Relativity" (relevant UFT papers and material on www.aias.us and CISP 2012).

{8} L. Felker, "The Evans Equations of Unified Field Theory (Abramis 2007, UFT302, Spanish translation by Alex Hill on www.aias.us).

{9} M. W. Evans and L. B. Crowell, "Classical and Quantum Electrodynamics and the B(3) Field" (World Scientific 2001 and Omnia Opera of www.aias.us).

{10} M .W. Evans and S. Kielich (Eds.), "Modern Nonlinear Optics" (Wiley Interscience, 1992, 1993, 1997 and 2001) in two editions and six volumes.

{11} M. W. Evans and J.-P. Vigier, "The Enigmatic Photon" (Kluwer 1994 to 2002 and Omnia Opera of www.aias.us) in five volumes hardback and five volumes softback.

{12} M. W. Evans and A. A. Hasanein, "The Photomagneton in Quantum Field Theory" (World Scientific 1994).