A GENERAL THEORY OF PRECESSIONS IN TERMS OF CARTAN GEOMETRY

by

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ABSTRACT

The gravitational field equations of ECE2 theory are used to derive a theory of any astronomical precession in terms of the magnitude of vorticity. The result is expressed in terms of the tetrad and spin connection of Cartan geometry. The theory can be applied to any type of precession, notably the Lense Thirring, geodetic and perihelion precessions, to give exact agreement in each case in terms of the vorticity of the mathematical space of the field equations, which are Lorentz covariant in a space with finite torsion and curvature.

Keywords: ECE2 theory, gravitational field equations, precession, vorticity.

UFT 346

1. INTRODUCTION

In recent papers of this series {1 - 12} the theory of astronomical precessions has been developed in terms of the gravitational field equations of ECE2 theory. Any precession observed in astronomy is described in terms of gravitomagnetostatics and is half the magnitude of the gravitomagnetic field. The Lense Thirring precession has been described precisely using the spin angular momentum of a gravitating object, i.e. the angular momentum of the object about its own axis. For example the Lense Thirring precession of Gravity Probe B in polar orbit around the earth. The geodetic precession has been described using an orbital angular momentum. For example the geodetic precession of Gravity Probe B has been described in terms of the orbital angular momentum of the earth as seen from a frame of reference fixed on Gravity Probe B.

In this paper these concepts are extended to perihelion precession, which is considered to be a type of geodetic precession. For example the perihelion precession of the earth around the sun is developed in terms of the orbital angular momentum of the sun as seen from the earth. In every day experience the sun appears to revolve about the earth, and this motion generates an orbital angular momentum. Good agreement is obtained when the observed astronomical parameters are used, and precise agreement can always be obtained for any precession using an effective angular momentum. The Lense Thirring precession is calculated of the earth about the sun, and it is found to be orders of magnitude smaller than the perihelion precession.

It is shown that any precession observed in astronomy can be developed in terms of the magnitude of the vorticity of spacetime, the vorticity being defined in terms of Cartan geometry using a combination of the terad and spin connection vector. This combination appears in the ECE2 gravitational field equations.

This paper is a short synopsis of detailed calculations which are posted in the background notes accompanying UFT346 on www.aias.us. In Notes 346(1) and 346(2) precession is calculated due to a localized distribution of a current of mass. The theory parallels exactly the well known theory of magnetostatics in the dipole approximation. The gravitomagnetic dipole moment is half the angular momentum. In consequence any precession can be described in terms of an angular momentum in an approximation such as the dipole approximation. Using the well known concept of principal axes of moment of inertia, the angular momentum can always be expressed as the product of a scalar moment of inertia and an angular velocity vector defined by the axis of spin. Note 346(2) gives sample calculations of moment of inertia. In Note 346(3) the Lense Thirring precession of the earth about the sun is calculated to be:

$$\Omega = 7.201 \times 10^{-7}\,'' \text{ a year} \qquad -(1)$$

which is five orders of magnitude smaller than the perihelion precession of the earth about the sun:

$$\Omega = 0.05 \pm 0.012\,'' \text{ a year.} \qquad -(2)$$

In Note 364(4) the perihelion precession of the earth is calculated from the ECE2 gravitational field equations using the orbital angular momentum of the sun as seen from the earth, and using a circular orbit in the first approximation. Finally Note 364(5) develops a precise and general theory of all precessions in terms of the vorticity of ECE2 spacetime. The vorticity is defined in terms of Cartan geometry.

Section 2 of this paper is based on Notes 346(4) and 346(5). Section 3 is a numerical analysis.

2. PRECESSION IN TERMS OF ANGULAR MOMENTUM AND VORTICITY.

As shown in Notes 346(1) to 346(3), and in preceding papers of this series

{1 - 12}, any precession can be described in the dipole approximation in terms of angular momentum $\underline{L}$:

$$\Omega = \frac{G}{2c^2}\left|\underline{\nabla} \times \left(\frac{\underline{L}\times\underline{r}}{r^3}\right)\right| \quad -(3)$$

$$= \frac{G}{2c^2 r^3}\left|\frac{3\underline{r}}{r}\left(\frac{\underline{r}}{r}\cdot\underline{L}\right) - \underline{L}\right| .$$

When this equation is applied to the perihelion precession of the earth, $\underline{r}$ is the distance from the sun to the earth. Here G is Newton's constant and c the speed of light in vacuo. The sun rotates about an axis $\underline{k}$. The plane of the earth's orbit is inclined to the plane perpendicular to $\underline{k}$ at an angle θ of 7.25°. So as in Note 346(3):

$$\underline{r} = \underline{i}X\cos\theta - \underline{k}X\sin\theta + Y\underline{j} . \quad -(4)$$

The observed precession rate of the earth's perihelion is:

$$\Omega = (0.05 \pm 0.012)'' \text{ a year} = 7.681\times 10^{-15} \text{ rad s}^{-1} . \quad -(5)$$

From the earth, the sun appears to be orbiting with an angular momentum:

$$\underline{L} = M r^2 \underline{\omega} \quad -(6)$$

where M is the mass of the sun, and where $\underline{\omega}$ is the angular frequency of the orbit. The earth rotates around the sun once every year, i.e. once every 3.156×10^7 sec. So:

$$\omega = \frac{2\pi}{3.157\times 10^7} \text{ rad s}^{-1} \quad -(7)$$

Using:

$$r = 1.4958\times 10^{11} \text{ m}, \quad -(8)$$
$$\frac{MG}{c^2} = 1.475\times 10^3 \text{ m},$$

the perihelion precession is found to be:

$$\Omega = 0.981 \times 10^{-15} \ rad\,s^{-1} \quad -(9)$$

The experimental result is:

$$\Omega(exp) = 7.681 \times 10^{-15} \ rad\,s^{-1} \quad -(10)$$

Exact agreement can be obtained by using an effective angular momentum $\underline{L}$. The above theory has used a circular orbit in the first approximation.

The fundamental assumption is that the orbit of the earth about the sun produces a torque:

$$\underline{Tq} = \underline{m}_g \times \underline{\Omega} \quad -(11)$$

where $\underline{m}_g$ is the gravitomagnetic dipole moment:

$$\underline{m}_g = \frac{1}{2}\underline{L} \quad -(12)$$

and where $\underline{\Omega}$ is the gravitomagnetic field. Here $\underline{L}$ is the the orbital angular momentum (6). The gravitomagnetic field is the curl of the gravitomagnetic vector potential:

$$\underline{A_g} = \frac{G}{2c^2 r^3}\,\underline{L} \times \underline{r} \quad -(13)$$

so:

$$\underline{\Omega}_g = \underline{\nabla} \times \underline{A}_g . \quad -(14)$$

In direct analogy the Lense Thirring precession of the earth with respect to the sun is due to the latter's spin angular momentum about its own axis. The sun spins once every 27 days about its own axis. So the relevant angular momentum in this case is the spin angular momentum of the sun:

$$\underline{L}_s = \frac{2}{5} M R^2 \underline{\omega} = I \underline{\omega}. \quad -(15)$$

Similarly the Lense Thirring precession of Gravity Probe B is due to the spin of the earth every 24 hours. This spin produces a mass current and a gravitomagnetic dipole moment:

$$\underline{m}_g(\text{spin}) = \frac{1}{2} \underline{L}_s \quad -(16)$$

due to spin angular momentum.

The perihelion precession is developed as a geodetic precession caused by an orbital angular momentum, the orbital angular momentum of the sun, which is observed in a frame of reference fixed on the earth. In direct analogy an electron orbiting a nucleus has spin and orbital angular momenta.

These concepts are developed in ECE2 theory, which is a theory of relativity that is Lorentz covariant in a space with finite torsion and curvature (UFT315 to UFT319 on www.aias.us). In ECE2, the magnetic flux density can be defined by curvature through the $\underline{W}$ potential:

$$\underline{B} = \underline{\nabla} \times \underline{W} = \underline{\nabla} \times \underline{A} + 2 \underline{\omega}_s \times \underline{A}. \quad -(17)$$

The $\underline{A}$ potential is defined by torsion, and $\underline{\omega}_s$ is the spin connection vector. In precise analogy, the gravitomagnetic field is defined as follows:

$$\underline{\Omega}_g = \underline{\nabla} \times \underline{W}_g = \underline{\nabla} \times \underline{A}_g + 2 \underline{\omega}_s \times \underline{A}_g \quad -(18)$$

where:

$$A^\mu_g = (\Phi_g, c\underline{A}_g) \quad -(19)$$

and where Φ_g is the scalar potential of gravitation.

Note that $\underline{W}_g$ has the units of linear velocity $\underline{v}_g$, so:

$$\underline{\Omega}_g = \underline{\nabla} \times \underline{v}_g \quad -(20)$$

which defines the gravitomagnetic field as a vorticity. This is the vorticity of the spacetime of ECE2 relativity. It follows that any precession can be described precisely as follows:

$$\Omega = \frac{1}{2} \left| \underline{\Omega}_g \right| = \frac{1}{2} \left| \underline{\nabla} \times \underline{v}_g \right| \quad -(21)$$

So all precessions in the universe are due to the vorticity of the ECE2 spacetime, Q. E. D. If it is assumed that:

$$\underline{\nabla} \cdot \underline{v}_g = 0 \quad -(22)$$

then the spacetime is that of an inviscid ECE2 spacetime, in precise analogy with the theory fo an inviscid fluid in hydrodynamics.

The equations of gravitomagnetostatics are:

$$\underline{\nabla} \times \underline{\Omega}_g = \underline{\kappa} \times \underline{\Omega}_g - \frac{4\pi G}{c^2} \underline{J}_m \quad -(23)$$

where:

$$\underline{\kappa} = \frac{1}{r^{(0)}} \underline{q} - \underline{\omega}_S . \quad -(24)$$

Here, $\underline{J}_m$ is the current density of mass, $\underline{q}$ is the tetrad vector and $\underline{\omega}_S$ is the spin connection and where $r^{(0)}$ is a scalar with the units of length. It follows from Eq. (23) that:

$$\underline{\nabla} \cdot \left(\underline{\nabla} \times \underline{\Omega}_g \right) = \underline{\nabla} \cdot \left(\underline{\kappa} \times \underline{\Omega}_g \right) = 0. \quad -(25)$$

Now use:

$$\underline{\nabla}\cdot(\underline{\kappa}\times\underline{\Omega}_g)=\underline{\Omega}_g\cdot(\underline{\nabla}\times\underline{\kappa})-\underline{\kappa}\cdot(\underline{\nabla}\times\underline{\Omega}_g)$$

$$= 0 \quad -(26)$$

so:

$$\underline{\Omega}_g\cdot(\underline{\nabla}\times\underline{\kappa})=\underline{\kappa}\cdot\underline{\nabla}\times\underline{\Omega}_g. \quad -(27)$$

One possible solution of Eq. (27) is:

$$\underline{\Omega}_g = v_g\,\underline{\kappa} \quad -(28)$$

where:

$$v_g = |\underline{v}_g|. \quad -(29)$$

Therefore the ECE2 equation of any precession is:

$$\underline{\Omega}_g = \underline{\nabla}\times\underline{v}_g = v_g\,\underline{\kappa} \quad -(30)$$

where:

$$\Omega = \frac{1}{2}|\underline{\nabla}\times\underline{v}_g| = \frac{1}{2}v_g\left(\frac{q}{r^{(0)}}-\omega_s\right) \quad -(31)$$

These are generally valid equations without any approximation, and are based on Cartan geometry.

In the dipole approximation:

$$\underline{\Omega}_g = \frac{G}{2c^2}\underline{\nabla}\times\left(\frac{\underline{L}\times\underline{r}}{r^3}\right) \quad -(32)$$

and comparing Eqs. (30) and (32):

$$\underline{v}_g = \frac{G}{2c^2r^3}\underline{L}\times\underline{r} \quad -(33)$$

This is also an expression for the $\underline{W}_g$ potential. Finally:

$$\underline{\kappa} = \frac{G}{2c^2 v_g}\, \underline{\nabla} \times \left(\frac{\underline{L} \times \underline{r}}{r^3} \right) \tag{34}$$
$$= r^3\, \underline{\nabla} \times \left(\frac{\underline{L} \times \underline{r}}{r^3} \right) \Big/ \left| \underline{L} \times \underline{r} \right|$$

which shows that any precession is due to Cartan geometry, Q. E.D.

3. NUMERICAL ANALYSIS

Section by Dr. Horst Eckardt.

A general theory of precessions in terms of Cartan geometry

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3 Numerical analysis

The gravitomagnetic field in dipole approximation has been analysed numerically and graphically in three dimensions. In ECE2 theory the gravitomagnetic field is defined by

$$\mathbf{\Omega}_g = \nabla \times \mathbf{W}_g \tag{35}$$

where $\mathbf{W}_g$ is the gravitomagnetic vector potential. This is – in analogy to electromagnetism – given by

$$\mathbf{W}_g = \frac{G}{c^2 r^3} \mathbf{m}_g \times \mathbf{r} \tag{36}$$

where $\mathbf{m}_g$ is a gravitational dipole moment. For a given $\mathbf{m}_g$ the vector potential and gravitomagnetic field can be computed in three dimensions by computer algebra. (The equations are quite complicated and not shown.) It would be desirable to draw equipotential lines because these are perpendicular to the field lines and demonstrate the curving of the field clearly. As the name says, these are results of a scalar potential which is not available in the gravitomagnetic case. However, it is known from electrodynamics that dipole fields can be defined by a magnetic scalar potential Φ in analogy to the electric case:

$$\mathbf{\Omega}_g = -\nabla \Phi. \tag{37}$$

Therefore we defined a dipole by two distinct charges $\pm q$ and a potential

$$\Phi(\mathbf{r}) = C \left(\frac{q}{|\mathbf{r} - \mathbf{r}_1|} - \frac{q}{|\mathbf{r} - \mathbf{r}_2|} \right) \tag{38}$$

with the "charges" at positions $\mathbf{r}_1$ and $\mathbf{r}_2$, here placed on the Z axis. The corresponding field $\mathbf{\Omega}_g$ is rotationally symmetric around this axis and is shown in Fig. 1, together with the equipotential lines of Φ.

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It may be interesting how the field looks like if two dipoles are positioned upon one another. This is a linear arrangement without a quadrupole moment which would occur in other cases. The results are graphed in Fig. 2. There is a recess in the XY plane which is shown in more detail in Fig. 3. Field lines do not go straight from pole to pole in this region, contrary to the single dipole. It has been reported by Johnson in the seventies that such a behaviour has been found for certain magnets.

The other Figures have been calculated from Eqs. (35,36) directly without using a scalar potential. Positioning the dipole in the Z direction:

$$\mathbf{m}_g = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{39}$$

leads to a rotationally symmetric vector potential in the XY plane which is graphed in Fig. 4. The lines of constant values are circles. There are no Z components of $\mathbf{W}_g$. This can be seen in the 3D vector plot of Fig. 5. The vector potential is a type of spherical vortex, being strongest in the centre. According to Eq. (20), the vector potential corresponds to a velocity vector whose vorticity is the gravitomagnetic field. Therefore the arrows in Fig. 5 can be interpreted as velocities directly, showing a hydrodynamical vortex.

The question is what the torque

$$\mathbf{T}_q = \mathbf{m}_g \times \mathbf{\Omega}_g \tag{40}$$

looks like. This has been graphed in Fig. 6. It has a shape similar to the vector potential but with an essential difference: the torque is zero in the equatorial plane $Z = 0$ because it changes sign from below to above and vice versa. This can be seen when comparing the arrows in Fig.6 with those of Fig. 5. If a planet moves on a non-equatorial orbit around the centre, there is a torque which takes both directions for one revolution cycle. If the orbit is a circle, the effects will cancel out but there are changes of velocity of both signs during one revolution.

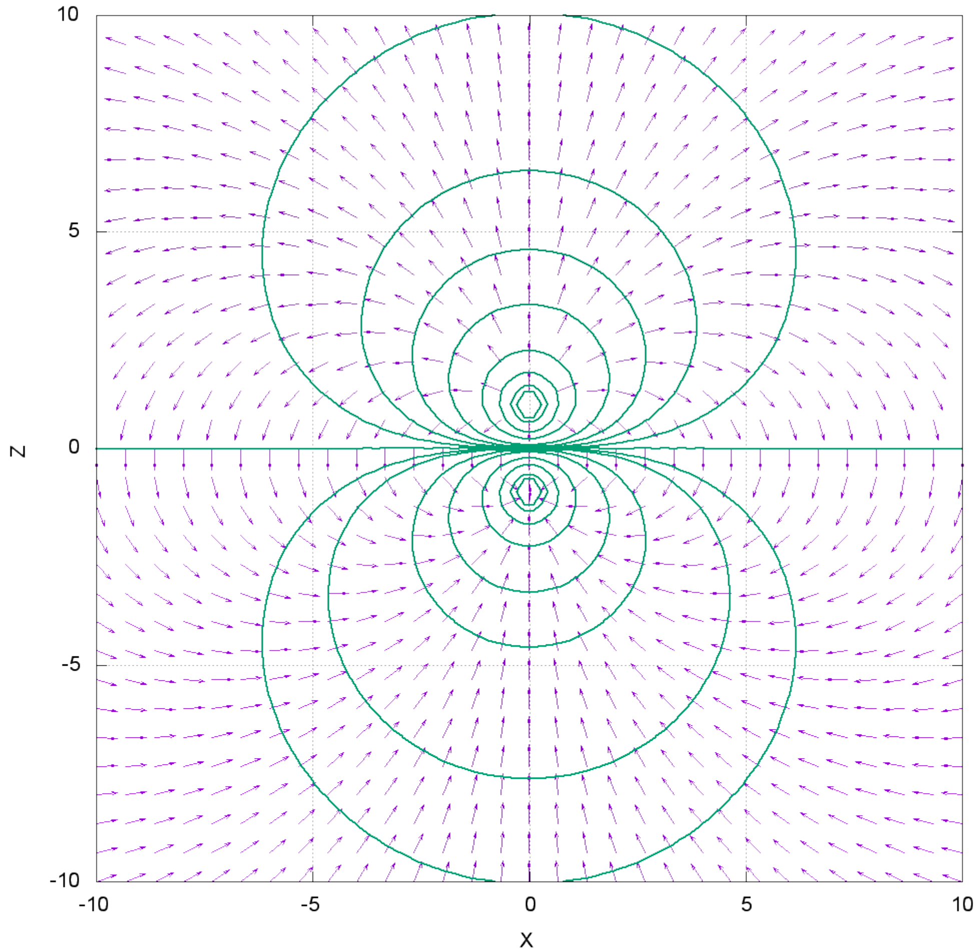

Figure 1: Dipole vector field $\mathbf{\Omega}_g$ (directional vectors only) in XZ plane and equipotential lines.

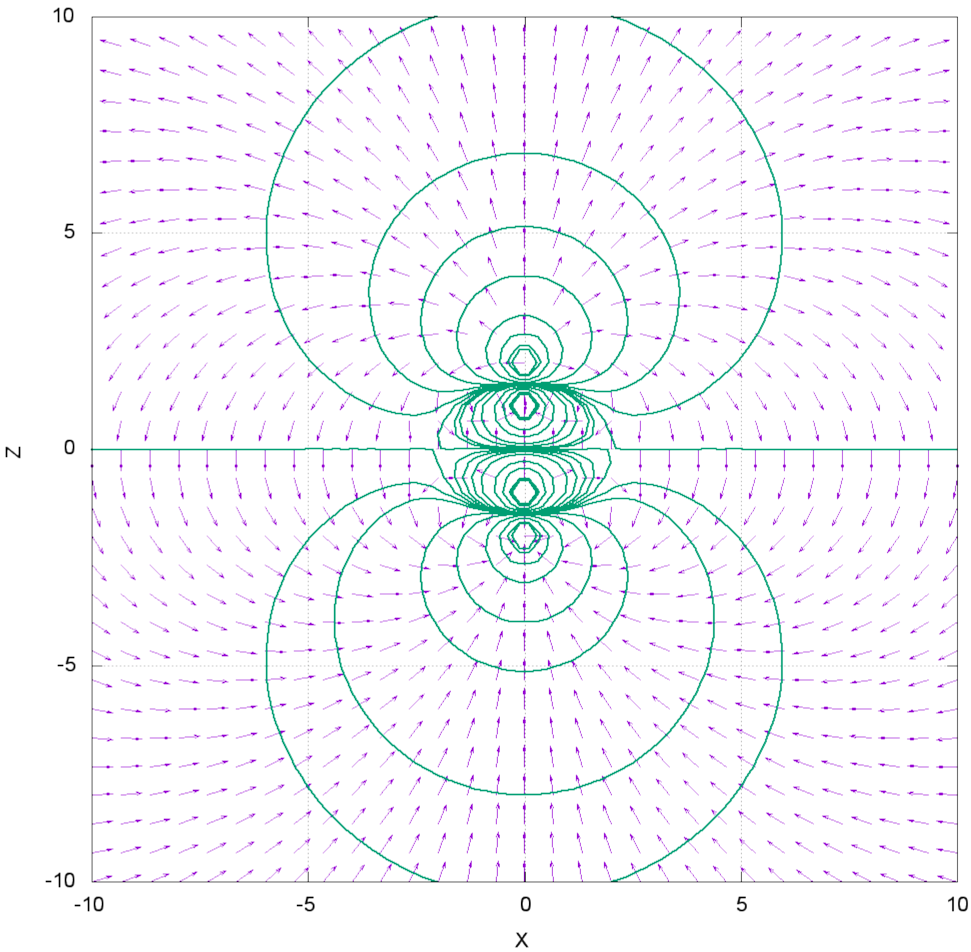

Figure 2: Double-dipole vector field in XZ plane and equipotential lines.

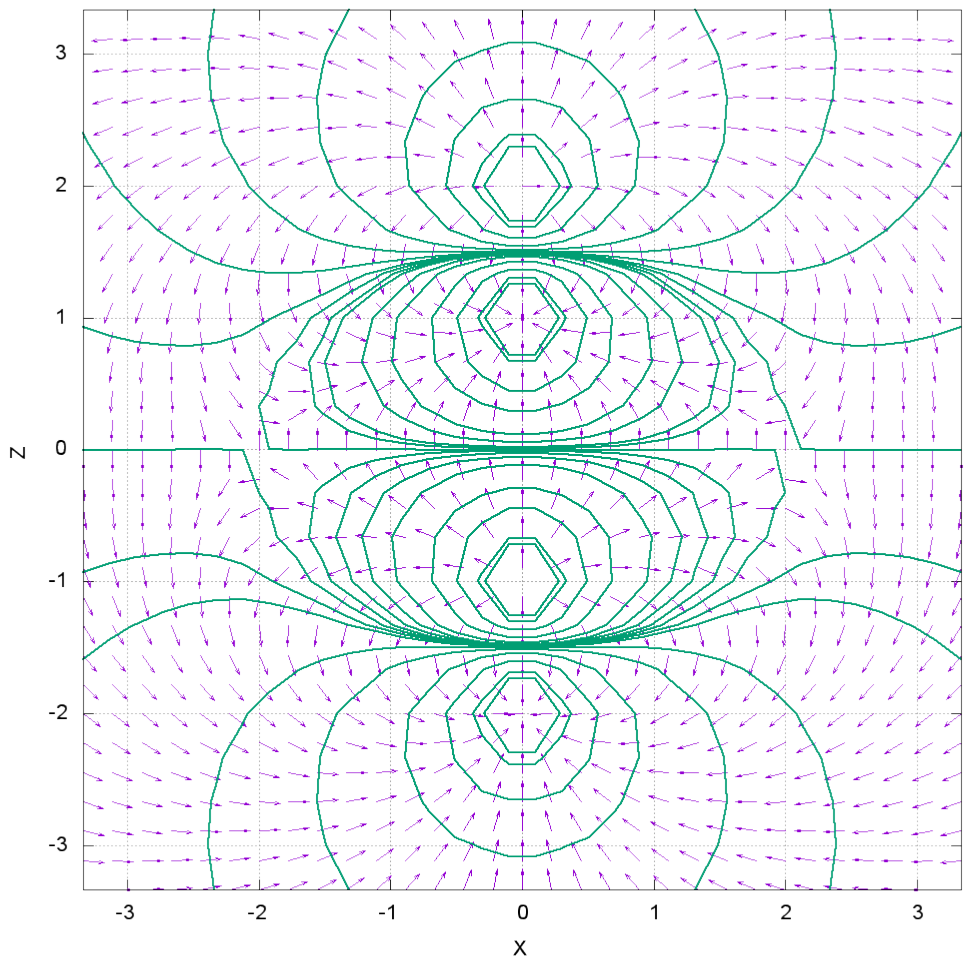

Figure 3: Double-dipole vector field and equipotential lines, enlarged view.

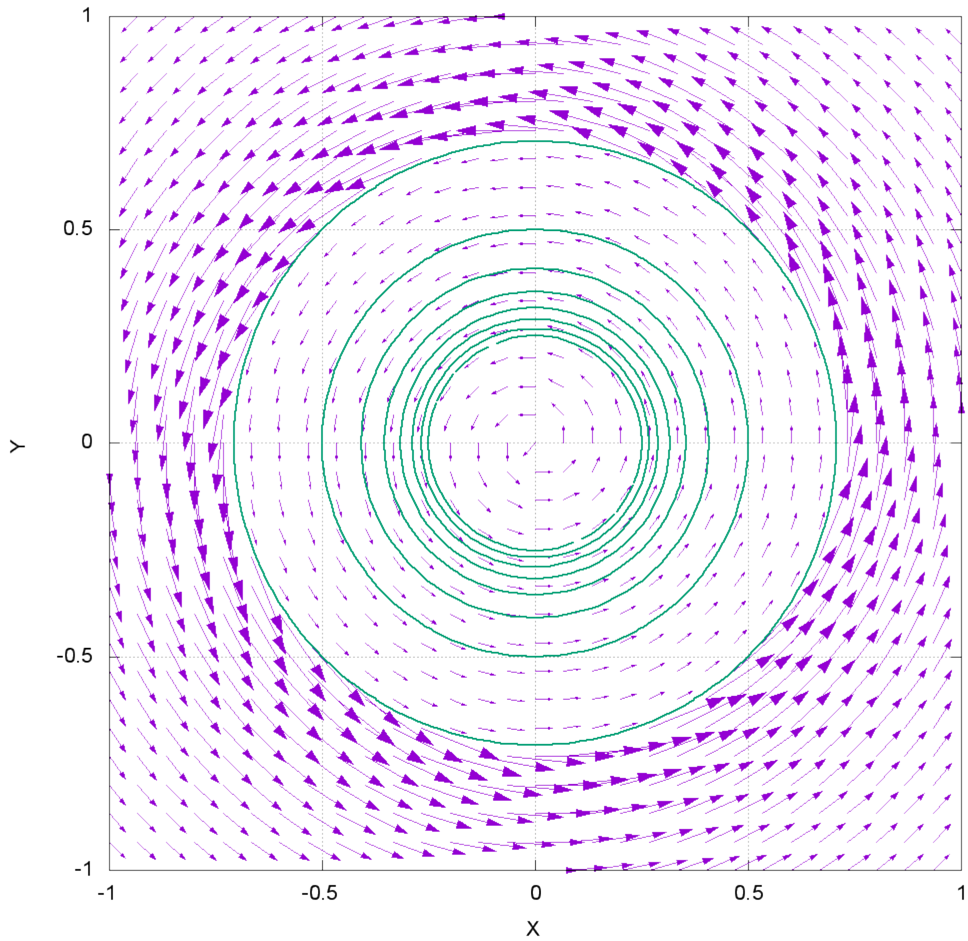

Figure 4: Dipole vector potential $\mathbf{W}_g$ in XY plane, only directional vectors shown within lines of constant absolute value.

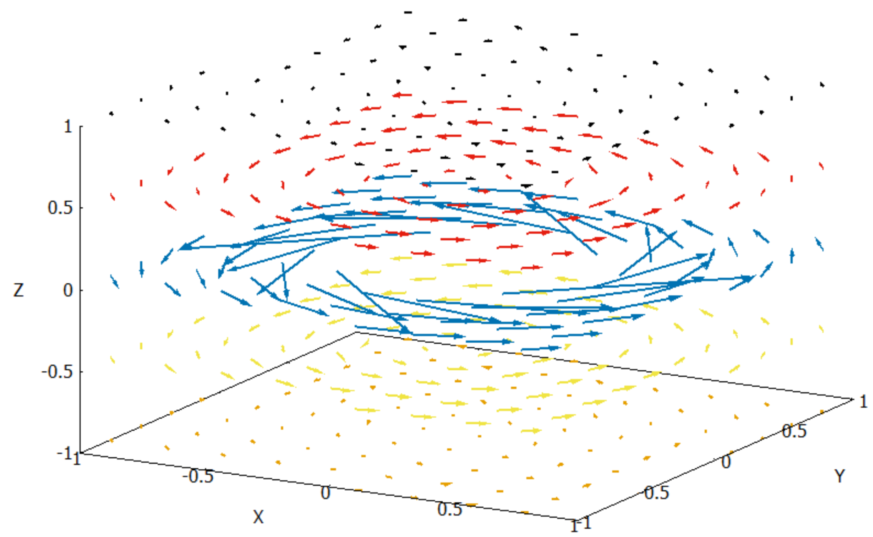

Figure 5: 3D view of dipole vector potential $\mathbf{W}_g$.

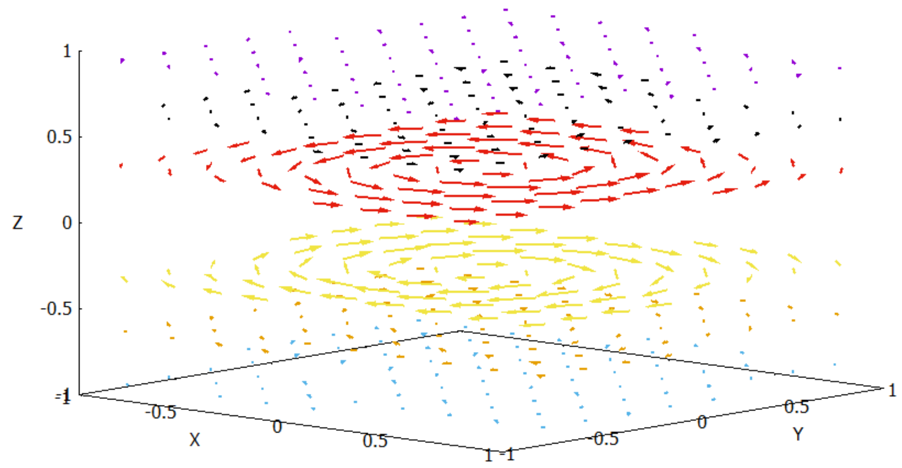

Figure 6: 3D view of dipole torque field $\mathbf{T}_q$.

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REFERENCES

{1} M. W. Evans, H. Eckardt, D. W. Lindstrom and S. J. Crothers, "Principles of ECE Theory" (UFT281 - UFT288 and zipped file on www.aias.us, New Generation in prep.)

{2} M. W. Evans, "The Book of Scientometrics" (UFT307 and New Generation 2015).

{3} H. Eckardt, "The ECE Engineering Model" (UFT303).

{4} M. W. Evans, S. J. Crothers, H. Eckardt and K. Pendergast "Criticisms of the Einstein Field Equation" (UFT301 and Cambridge International (CISP) 2010).

{5} L. Felker, "The Evans Equations of Unified Field Theory" (UFT302 and Abramis 2007, Spanish translation by Alex Hill on www.aias.us).

{6} M . W. Evans, H. Eckardt and D. W. Lindstrom, "generally Covariant Unified Field Theory (relevant UFT papers and Abramis 2005 to 2011 in seven volumes).

{7} M. W. Evans, Ed., J. Found. Phys. Chem., relevant UFT papers on www.aias.us and CISP 2011).

{8} M. W. Evans, Ed., "Definitive Refutations of the Einsteinian General Relativity" (relevant www.aias.us papers and material, and CISP 2012).

{9} M. W. Evans and L. B. Crowell, "Classical and Quantum Electrodynamics and the B(3) Field" (World Scientific 2001, and Omnia Opera of www.aias.us).

{10} M .W. Evans and S. Kielich, Eds., “Modern Nonlinear Optics” (Wiley Interscience, New York, 1992, reprinted 1993, softback 1997, second edition 2001) in two editions and six volumes.

{11} M W. Evans and J.-P. Vigier, “The Enigmatic Photon” (Kluwer 1994 - 2002 in five volumes hardback and five volumes softback, Omnia Opera section of www.aias.us).

{12} M .W. Evans and A. S. Hasanein, “The Photomagneton in Quantum Field theory” (World Scientific 1994).