PRECESSING ELLIPSE FROM THE ECE2 ORBITAL LORENTZ FORCE EQUATION.

by

M. W. Evans and H. Eckardt,

Civil List, AIAS and UPITEC

(www.archive.org, www.webarchive.org.uk, www.aias.us , www.upitec.org, www.atomicprecision.com , www.et3m.net)

ABSTRACT

By considering the minimal prescription of UFT347 it is shown that a new type of precessing ellipse emerges from the relevant hamiltonian, so this is the simplest way of describing any precession. The Leibniz force equation is augmented by terms which include the observed precession frequency and is a Lorentz force equation. The relation between the hamiltonian and lagrangian is based on the canonical momentum. For a uniform gravitomagnetic field the force equation can be derived from a simple lagrangian, and the former can be expressed as a Binet equation.

Keywords, ECE2 relativity, precessing orbit, orbital Lorentz force and Binet equations.

UFT 348

1. INTRODUCTION

In the immediately preceding paper (UFT347) of this series {1-12} it was shown that the minimal prescription produces an orbital Lorentz force equation which gives rise to orbital precession through the intermediacy of the gravitomagnetic field. The precession frequency is half the magnitude of the gravitomagnetic field. In Section 2 of this paper it is shown that the hamiltonian corresponding to the Lorentz force equation gives a precessing ellipse of a type hitherto unknown. This precessing ellipse is a rigorous and accurate description of the observed orbit because the observed precession frequency is used in the equations. The calculated precessing ellipse is similar in structure to that of x theory {1 - 12} but x depends on the plane polar angle theta. The method uses the canonical momentum of UFT347. For a uniform gravitomagnetic field the conserved angular momentum can be calculated straightforwardly, and the Lorentz force equation reduces to a precessional Binet equation. The orbit calculated from the hamiltonian can be used in the Binet equation to give the force. Section 3 is a description with graphics of the methods used to produce the precessing orbit and also gives numerical self consistency checks of the procedures used in this paper.

This paper is a short synopsis of detailed calculations found in the notes accompanying UFT348 on www.aias.us. Note 348(1) is a detailed description of the derivation of the precessing elliptical orbit from the hamiltonian relevant to the orbital Lorentz force equation. Note 348(2) gives some details of the development of the Lorentz force equation. Note 348(3) is a development of the precessional lagrangian, this note is developed numerically in Section 3. Notes 348(4) and 348(5) develop the force equation for a uniform gravitomagnetic field and deduce the precessional Binet equation.

2. DYNAMICAL DEVELOPMENTS

Consider the minimal prescription of UFT347:

$$\underline{p} \rightarrow \underline{p} + m\underline{v}_g \quad -(1)$$

where

$$\underline{W}_g = \underline{v}_g \quad -(2)$$

is the gravitational vector potential of ECE2 relativity. This gravitational potential has the units of linear velocity. Here m is the mass of an object in orbit around an object of mass M. For a uniform gravitomagnetic field {1 - 12}:

$$v_g^2 = \Omega^2 r^2 \quad -(3)$$

where Ω is the observed precession frequency. The latter is half the magnitude of the gravitomagnetic field defined by:

$$\underline{\Omega}_g = \underline{\nabla} \times \underline{v}_g . \quad -(4)$$

This equation is directly analogous with the definition of the magnetic flux density as the curl of the electromagnetic vector potential $\underline{W}$ of ECE2 relativity.

The hamiltonian defined by the minimal prescription ($\underline{1}$) is:

$$H = \frac{1}{2m}\left(\underline{p} + m\underline{v}_g\right) \cdot \left(\underline{p} + m\underline{v}_g\right) + U(r) \quad -(5)$$

where U (r) is potential energy of attraction between m and M:

$$U(r) = -\frac{mMG}{r} \quad -(6)$$

where G is Newton's constant and r the magnitude of the distance between m and M. As in Notes for UFT347 the hamiltonian may be developed as:

$$H = \frac{1}{2}m\left(v^2 + v_g^2\right) + \Omega L + U(r) \quad -(7)$$

where L is the constant magnitude of the angular momentum:

$$\underline{L} = \underline{r} \times \underline{p} \quad -(8)$$

and where Ω is the observed precession frequency, considered as a Larmor frequency {1 - 12}.

Eqs. (3) and (7) give:

$$H = \frac{1}{2} m \left(v^2 + \Omega^2 r^2 \right) + \Omega L + U(r) \quad -(9)$$

where:

$$v^2 = \left(\frac{dr}{dt} \right)^2 + r^2 \left(\frac{d\theta}{dt} \right)^2 . \quad -(10)$$

Therefore the hamiltonian is:

$$H = \frac{1}{2} m \left(\left(\frac{dr}{dt} \right)^2 + r^2 \left(\left(\frac{d\theta}{dt} \right)^2 + \Omega^2 \right) \right) + \Omega L + U(r) \quad -(11)$$

or

$$H_1 = H - \Omega L = \frac{1}{2} m \left(\left(\frac{dr}{dt} \right)^2 + r^2 \left(\frac{d\theta_1}{dt} \right)^2 \right) + U(r) \quad -(12)$$

where

$$\left(\frac{d\theta_1}{dt} \right)^2 = \left(\frac{d\theta}{dt} \right)^2 + \Omega^2 . \quad -(13)$$

Using well known methods {1 - 10} a hamiltonian of the type (12) leads to the conic section orbit:

$$r = \frac{\alpha}{1 + \epsilon \cos \theta_1} \quad -(14)$$

where α is the half right latitude and where ϵ is the eccentricity.

Denote:

$$\omega_1 = \frac{d\theta_1}{dt} , \quad \omega = \frac{d\theta}{dt} \quad -(15)$$

to find that:

$$\omega_1^2 = \omega^2 + \Omega^2 \quad -(16)$$

If:

$$\Omega << \omega \quad -(17)$$

then to an excellent approximation:

$$\omega_1 \sim \omega\left(1 + \frac{1}{2}\left(\frac{\Omega}{\omega}\right)^2\right), \quad -(18)$$

i.e.

$$\frac{d\theta_1}{dt} = \left(1 + \frac{1}{2}\left(\frac{\Omega}{\omega}\right)^2\right)\frac{d\theta}{dt}. \quad -(19)$$

So

$$d\theta_1 = \left(1 + \frac{1}{2}\left(\frac{\Omega}{\omega}\right)^2\right) d\theta \quad -(20)$$

and:

$$\theta_1 = \int \left(1 + \frac{1}{2}\left(\frac{\Omega}{\omega}\right)^2\right) d\theta. \quad -(21)$$

In Eq. (21), ω is the angular velocity corresponding to the hamiltonian

$$H_0 = \frac{p_0^2}{2m} + U(r). \quad -(22)$$

This angular velocity is:

$$\omega = \frac{L_0}{mr^2} \quad -(23)$$

where the angular momentum $\underline{L_0}$ is a constant of motion. In Eq. (23):

$$r = \frac{\alpha_0}{1 + \epsilon_0 \cos\theta} \quad -(24)$$

therefore:

$$\frac{1}{\omega^2} = \frac{m^2 r^4}{L_0^2} = \frac{m^2 \alpha_0^4}{L_0^2 (1+\epsilon_0 \cos\theta)^4} \quad -(25)$$

and:

$$\theta_1 = \int \left(1 + \frac{m^2 \alpha_0^4 \Omega^2}{2(1+\epsilon_0 \cos\theta)^4 L_0^2} \right) d\theta . \quad -(26)$$

This integral is evaluated numerically in Section 3. The orbit is:

$$r = \frac{\alpha}{1+\epsilon \cos\theta_1} \quad -(27)$$

and in Section 3 it is demonstrated numerically and graphically that this is a precessing orbit, Q. E. D.

Therefore the minimal prescription (1) is enough to produce a precessing orbit.

In the x theory of previous UFT papers {1 - 12} it was assumed that:

$$\theta_1 = x\theta \quad -(28)$$

where x is a constant. In this more accurate analytical theory it is seen that x depends on θ . The most accurate theory of precession in ECE2 relativity is UFT328, which is rigorously relativistic and uses the canonical momentum to solve the relativistic hamiltonian and lagrangian simultaneously. However UFT328 uses a numerical scatter plot method to produce precession and does not have a known analytical solution. The method in this paper uses ECE2 relativity to give an analytical solution in the classical limit of the hamiltonian and lagrangian. This paper has the advantage of giving an analytical result in terms of the experimental precession frequency, Ω , so the method of this paper is valid for ANY precession observed in astronomy.

In the above equations the constant angular momenta are:

$$L = mr^2 \frac{d\theta_1}{dt}, \quad -(29)$$

$$L_0 = mr^2 \frac{d\theta}{dt}. \quad -(30)$$

The half right latitudes are:

$$\alpha = \frac{L^2}{m^2 MG}, \quad \alpha_0 = \frac{L_0^2}{m^2 MG}, \quad -(31)$$

and the eccentricities are:

$$\epsilon^2 = 1 + \frac{2H_1 L^2}{m^3 M^2 G^2}, \quad -(32)$$

and

$$\epsilon_0^2 = 1 + \frac{2HL_0^2}{m^3 M^2 G^2}. \quad -(33)$$

In general:

$$\theta_1 = \theta + \frac{m^2 \alpha_0^4 \Omega^2}{2L_0^2} \int \frac{d\theta}{(1+\epsilon_0 \cos\theta)^4} \quad -(34)$$

and the integral can be evaluated analytically using computer algebra as in Section 3. If it is assumed that:

$$\theta_1 = x\theta \quad -(35)$$

then:

$$x = 1 + \frac{1}{2\theta} \left(\frac{m\alpha_0^2 \Omega}{L_0} \right)^2 \int \frac{d\theta}{(1+\epsilon_0 \cos\theta)^4} \quad -(36)$$

and the orbit can be put in the format of an orbit of x theory:

$$r = \frac{\alpha}{1+\epsilon\cos(x\theta)} \quad -(37)$$

However, x is not constant in general.

The orbit (27) is plotted in Section 3 and is shown to precess, Q. E. D. It is equivalent to the orbital Lorentz force equation of UFT347:

$$\underline{F} = m\ddot{\underline{r}} = -\frac{mMG}{r^2}\underline{e}_r + m\frac{\partial \underline{v}_g}{\partial t} - m\dot{\underline{r}} \times \underline{\Omega}_g \quad -(38)$$

in which the canonical momentum is defined by:

$$m\dot{\underline{r}} = \underline{p} + m\underline{v}_g . \quad -(39)$$

In the absence of the gravitational vector potential $\underline{v}_g$, Eq. (38) reduces to the Leibnitz equation:

$$\underline{F} = m(\ddot{r} - r\dot{\theta}^2)\underline{e}_r = -\frac{mMG}{r^2}\underline{e}_r \quad -(40)$$

in which:

$$\ddot{\underline{r}} = \frac{d\dot{\underline{r}}}{dt} = \frac{d\underline{v}}{dt} . \quad -(41)$$

Therefore the Leibnitz equation is:

$$\ddot{r} - r\dot{\theta}^2 = F(r) \quad -(42)$$

and can be transformed to the Binet equation:

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r} = -\frac{mr^2}{L^2}F(r) . \quad -(43)$$

The Leibnitz equation gives the non precessing conic section:

$$r = \frac{\alpha}{1+\epsilon\cos\theta} \quad -(44)$$

Eqs. (38) and (39) give:

$$\underline{F} = m\underline{\ddot{r}} = m\left(\frac{d\underline{v}}{dt} + \frac{d\underline{v}_g}{dt}\right) = -\frac{mMG}{r^2}\underline{e}_r + m\frac{d\underline{v}_g}{\partial t} - m\underline{\dot{r}} \times \underline{\Omega}_g. \quad -(45)$$

To develop this equation consider:

$$\frac{dv_x}{dt} = \frac{\partial v_x}{\partial t} + \frac{\partial X}{\partial t}\frac{\partial v_x}{\partial X} + \dots \quad -(46)$$

then to first order:

$$\frac{d\underline{v}}{dt} = \frac{\partial \underline{v}}{\partial t} + \left(\underline{\dot{R}} \cdot \underline{\nabla}\right)\underline{v} \quad -(47).$$

where:

$$\underline{\dot{R}} = \frac{\partial X}{\partial t}\underline{i} + \frac{\partial Y}{\partial t}\underline{j} + \frac{\partial Z}{\partial t}\underline{k} \quad -(48)$$

It follows that:

$$\frac{d\underline{v}}{dt} - \frac{\partial \underline{v}}{\partial t} = \left(\underline{\dot{R}} \cdot \underline{\nabla}\right)\underline{v} \quad -(49)$$

so Eq. (45) becomes:

$$\underline{F} = m\frac{d\underline{v}}{dt} = -\frac{mMG}{r^2}\underline{e}_r - \left(\underline{\dot{R}} \cdot \underline{\nabla}\right)\underline{v}_g - m\underline{\dot{r}} \times \underline{\Omega}_g$$
$$= m\left(\ddot{R} - r\dot{\theta}^2\right)\underline{e}_r \quad -(50)$$

Here:

$$\underline{R} = X\underline{i} + Y\underline{j} + Z\underline{k} = R\underline{e}_r \quad -(51)$$

$$\underline{v} = \frac{d\underline{R}}{dt} = \dot{R}\,\underline{e}_r + R\dot{\theta}\,\underline{e}_\theta = \dot{\underline{R}} \quad -(52)$$

and:
$$\ddot{\underline{R}} = \left(\ddot{R} - R\dot{\theta}^2\right)\underline{e}_r \quad -(53)$$

for a planar orbit.

Therefore the orbital Lorentz force equation becomes:

$$\underline{F} = m\left(\ddot{R} - R\dot{\theta}^2\right)\underline{e}_r = -\frac{mMG}{r^2}\underline{e}_r - (\dot{\underline{R}}\cdot\underline{\nabla})\underline{v}_g - m\dot{\underline{r}}\times\underline{\Omega}_g \quad -(54)$$

in which the canonical momentum is:

$$m\dot{\underline{r}} = m(\underline{v} + \underline{v}_g) = m(\dot{\underline{R}} + \underline{v}_g). \quad -(55)$$

Some considerations of the lagrangian corresponding to this general equation are given in Note 348(3) and Section 3.

However, the hamiltonian (7) and the orbit (14) are based on a uniform gravitomagnetic field defined by:

$$\underline{\Omega}_g = \underline{\nabla}\times\underline{v}_g \quad -(56)$$

and:
$$\underline{v}_g = \frac{1}{2}\underline{\Omega}_g\times\underline{r}. \quad -(57)$$

As in Note 347(2):

$$v_g^2 = \frac{1}{4}\underline{\Omega}_g\times\underline{r}\cdot\underline{\Omega}_g\times\underline{r} = \frac{1}{4}\left(\Omega_g^2 r^2 - (\underline{\Omega}_g\cdot\underline{r})(\underline{\Omega}_g\cdot\underline{r})\right) \quad -(58)$$

and if the gravitomagnetic field is perpendicular to the plane of the orbit:

$$v_g^2 = \frac{1}{4}\Omega_g^2 r^2 = \Omega^2 r^2 \quad -(59)$$

which is Eq. (3). In general the lagrangian is:

$$\mathcal{L} = \frac{1}{2m}\left(\underline{p} + m\underline{v}_g\right)\cdot\left(\underline{p} + m\underline{v}_g\right) - \left(\underline{p} + m\underline{v}_g\right)\cdot\underline{v}_g - U(r) \quad -(60)$$

which can be written as:

$$\mathcal{L} = \frac{1}{2}m\left(v^2 - v_g^2\right) - U(r) \quad -(61)$$

From Eqs. (59) and (61) the lagrangian becomes:

$$\mathcal{L} = \frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\theta}^2 - \Omega^2 r^2\right) - U(r). \quad -(62)$$

The rotational Euler Lagrange equation is:

$$\frac{\partial \mathcal{L}}{\partial \theta} = 0 = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\theta}} \quad -(63)$$

from which the conserved angular momentum is defined:

$$L = mr^2\dot{\theta}. \quad -(64)$$

The other Euler Lagrange equation of the system is:

$$\frac{\partial \mathcal{L}}{\partial r} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{r}} \quad -(65)$$

which gives the force equation:

$$F(r) = -\frac{\partial U}{\partial r} = m\left(\ddot{r} - r\left(\dot{\theta}^2 - \Omega^2\right)\right) \quad -(66)$$

in which:

$$U = -\frac{mMG}{r}, \quad F = -\frac{mMG}{r^2}. \quad -(67)$$

This is the Lorentz force equation of the precessing orbit (14), and as in Note 348(5) can be transformed to the precessional Binet equation:

$$F(r) = -\frac{L^2}{mr^2}\left(\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r}\right) - m\Omega^2 r \quad (68)$$

3. NUMERICAL AND GRAPHICAL ANALYSIS

(Section by Dr. Horst Eckardt)

Precessing ellipse from the ECE2 orbital force equation

M. W. Evans*, H. Eckardt†
Civil List, A.I.A.S. and UPITEC

(www.webarchive.org.uk, www.aias.us,
www.atomicprecision.com, www.upitec.org)

3 Graphical and further computational analysis

The precession factor x, defined by Eq. (36), is not constant and dependent on the angle θ. The integral can be solved analytically, yielding a lot of terms, whose leading term is a periodic atan function:

$$\begin{aligned} x &= 1 + \frac{A}{\theta} \int \frac{d\theta}{(1+\epsilon_0 \cos\theta)^4} \\ &= 1 + a\frac{A}{\theta} \operatorname{atan}\left(\frac{(\epsilon_0 - 1)\, \sin\theta}{\sqrt{1-\epsilon_0^2}\, (\cos\theta + 1)} \right) + \ldots \end{aligned} \tag{69}$$

with constants A and a. This function $x(\theta)$ is graphed in Fig. 1. It scales to be slightly larger than unity. The periodicity of 2π is visible but not exact because (36) is an approximation. The factor of $1/2\theta$ in (36) obviously outperforms the integral. The orbit (37) is shown in Fig. 2 for $x = 1$ and $x(\theta)$ given by Eq. (69). This results in a precessing ellipse as can clearly be seen.

Some variants of the Lagrangian for the Lorentz force equation are investigated in the following. First we consider a direct method where Lagrangian and Hamiltonian are given by

$$\mathscr{L} = T - U \tag{70}$$

$$H = T + U \tag{71}$$

with kinetic energy T and potential energy U. Replacing $\mathbf{p}$ by the canonical momentum

$$\mathbf{p} = m\mathbf{v} \rightarrow \mathbf{p} + m\mathbf{v}_g \tag{72}$$

then leads to

$$\mathscr{L}_1 = \frac{1}{2m}(\mathbf{p} + m\mathbf{v}_g)^2 - U(r), \tag{73}$$

$$H_1 = \frac{1}{2m}(\mathbf{p} + m\mathbf{v}_g)^2 + U(r). \tag{74}$$

*email: emyrone@aol.com
†email: mail@horst-eckardt.de

In a more general case, however, we can use the relation between Hamiltonian and Lagrangian known from Hamilton's equations. Then:

$$H = \sum_j p_j \dot{q}_j - \mathscr{L} \tag{75}$$

where q_j are the generalized coordinates and p_j the generalized momenta. In our case we only have one generalized momentum (72) – in vector form – which is obtained by the minimal prescription. Since q is the space position vector $\mathbf{r}$, its time derivative is the velocity $\mathbf{v}$. Evaluating (75) has to give the Hamiltonian (74). To achieve this result, the Lagrangian has to be defined as

$$\mathscr{L}_2 = \frac{1}{2m}(\mathbf{p} + m\mathbf{v}_g)^2 - (\mathbf{p} + m\mathbf{v}_g)\mathbf{v}_g - U(r) \tag{76}$$

so that we obtain again:

$$H_2 = H_1 = \frac{1}{2m}(\mathbf{p} + m\mathbf{v}_g)^2 + U(r). \tag{77}$$

For an example we will use plane polar coordinates (r, θ) in which the linear velocity is given by

$$\mathbf{v} = \begin{bmatrix} \dot{r} \\ r\dot{\theta} \end{bmatrix}. \tag{78}$$

For the extra velocity derived from the gravitomagnetic field we use two variants. First we use

$$\mathbf{v}_{g1} = \begin{bmatrix} \dot{r}_g \\ r_g\dot{\theta} \end{bmatrix} \tag{79}$$

where the angular coordinate is the same as for the orbit, i.e. the particle m and velocity $\mathbf{v}_g$ rotate in the same frame. In the second case we use a completely independent $\mathbf{v}_g$ with both coordinates independent from the orbital motion:

$$\mathbf{v}_{g2} = \begin{bmatrix} \dot{r}_g \\ r_g\dot{\theta}_g \end{bmatrix}. \tag{80}$$

The Lagrangians of all four combinations $\mathscr{L}_{1,2}, \mathbf{v}_{g1,2}$ are listed in Table 1. For $\mathscr{L}_1$, mixed terms in $r \cdot r_g$ appear. This leads to corresponding combinations in the constants of motion (angular momentum) shown in Table 2. These have been calculated by the Lagrangian method, Eq. (63). A similar result appears in the third line of Table 2. However in the fourth line the angular momentum of a particle without precession appears. This astonishing result means that the angular motion is not impacted by the precessional velocity $\mathbf{v}_{g2}$. The reason is that in the Lagrangian the term $\dot{\theta}^2$ appears without a coupling factor to r_g so this result is plausible. The Hamiltonians (Table 3) are equal for $\mathscr{L}_1$ and $\mathscr{L}_2$ as expected. They only differ in the appearance of $\dot{\theta}_g$ by the different definitions of $\mathbf{v}_{g1}$ and $\mathbf{v}_{g2}$.

$\mathscr{L}$	$\mathbf{v}_g$	$\mathscr{L}$
$\mathscr{L}_1$	$\mathbf{v}_{g1}$	$\frac{m}{2}\left(\left(r\,\dot\theta + r_g\,\dot\theta\right)^2 + (\dot r_g + \dot r)^2\right) - U(r)$
	$\mathbf{v}_{g2}$	$\frac{m}{2}\left(\left(r\,\dot\theta + r_g\,\dot\theta_g\right)^2 + (\dot r_g + \dot r)^2\right) - U(r)$
$\mathscr{L}_2$	$\mathbf{v}_{g1}$	$\frac{m}{2}\left(\dot r^2 + \left(r^2 - r_g^2\right)\dot\theta^2 - \dot r_g^2\right) - U(r)$
	$\mathbf{v}_{g2}$	$\frac{m}{2}\left(\dot r^2 + r^2\dot\theta^2 - \dot r_g^2 - r_g^2\dot\theta_g^2\right) - U(r)$

Table 1: Lagrangians for different $\mathbf{v}_g$ models.

$\mathscr{L}$	$\mathbf{v}_g$	L
$\mathscr{L}_1$	$\mathbf{v}_{g1}$	$m(r+r_g)^2\,\dot\theta$
	$\mathbf{v}_{g2}$	$m\,r\,(r\,\dot\theta + r_g\,\dot\theta_g)$
$\mathscr{L}_2$	$\mathbf{v}_{g1}$	$m(r+r_g)(r-r_g)\,\dot\theta$
	$\mathbf{v}_{g2}$	$m\,r^2\,\dot\theta$

Table 2: Constants of motion L for different Lagrangians and $\mathbf{v}_g$ models.

$\mathscr{L}$	$\mathbf{v}_g$	H
$\mathscr{L}_1$	$\mathbf{v}_{g1}$	$\frac{m}{2}\left(\dot r^2 + r^2\dot\theta^2 + \dot r_g^2 + r_g^2\dot\theta^2\right) + U(r)$
	$\mathbf{v}_{g2}$	$\frac{m}{2}\left(\dot r^2 + r^2\dot\theta^2 + \dot r_g^2 + r_g^2\dot\theta_g^2\right) + U(r)$
$\mathscr{L}_2$	$\mathbf{v}_{g1}$	$\frac{m}{2}\left(\dot r^2 + r^2\dot\theta^2 + \dot r_g^2 + r_g^2\dot\theta^2\right) + U(r)$
	$\mathbf{v}_{g2}$	$\frac{m}{2}\left(\dot r^2 + r^2\dot\theta^2 + \dot r_g^2 + r_g^2\dot\theta_g^2\right) + U(r)$

Table 3: Hamiltonians H for different Lagrangians and $\mathbf{v}_g$ models.

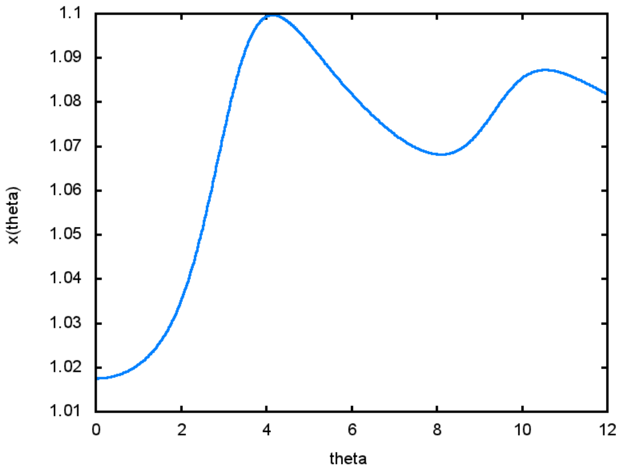

Figure 1: Angular dependence of x factor.

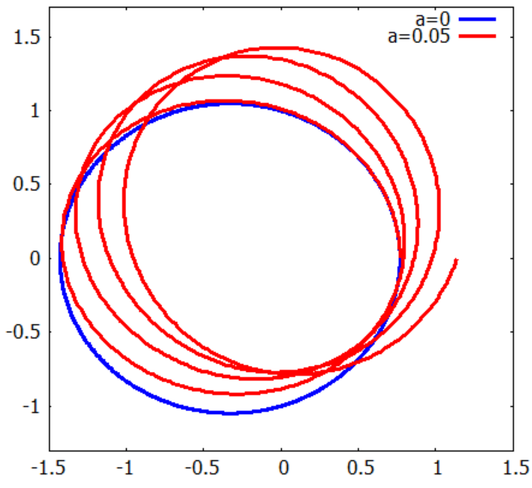

Figure 2: Elliptic orbitals for $a = 0$ and $a = 0.05$ (with precession).

ACKNOWLEDGMENTS

The British Government is thanked for a Civil List Pension, and the staff of AIAS and others for many interesting discussions. Dave Burleigh, CEO of Annexa Inc., is thanked for histing www.aias.us, site maintenance, voluntary posting and feedback software maintenance. Alex Hill is thanked for translation and broadcasting, and Robert Cheshire for broadcasting.

REFERENCES

{1} M .W. Evans, H. Eckardt, D. W. Lindstrom and S. J. Crothers, "The Principles of ECE" (open source and New Generation Publishing, London, in press).

{2} M. W. Evans, S. J. Crothers, H. Eckardt and K. Pendergast, "Criticisms of the Einstein Field Equation" (CEFE, UFT301 and Cambridge International (CISP) 2010).

{3} L. Felker, "The Evans Equations of Unified Field Theory" (UFT302 and Abramis 2007, translated into Spanish by Alex Hill on www.aias.us).

{4} H. Eckardt, "The ECE Engineering Model" (UFT303).

{5} M. W. Evans, "Collected Scientometrics" (UFT307 and New Generation 2015).

{6} M. W. Evans, H. Eckardt and D. W. Lindstrom, "Generally Covariant Unified Field Theory" (Abramis 2005 - 2011 in seven volumes, and open source on www.aias.us)

{7} M. W. Evans, Ed., J. Found. Phys. Chem., (open source on www.aias.us, CISP 2011).

{8} M. W. Evans, Ed., "Definitive Refutations of the Einsteinian General Relativity", special issue six of ref. (7), open source on www.aias.us.

{9} M. W. Evans and L. B. Crowell, "Classical and Quantum Electrodynamics and the B(3) Field" (World Scientific 2001 and Omnia Opera of www.aias.us)

{10} M. W. Evans and S. Kielich, Eds., "Modern Nonlinear Optics" (Wiley Interscience, New York, 1992, 1993, 1997, 2001) in two editions and six volumes.

{11} M. W. Evans and J.-P. Vigier, “The Enigmatic Photon” (Kluwer 1994 to 2002 and Omnia Opera section of www.aias.us, in five volumes each, hardback and softback.)

{12} M. W. Evans and A. A. Hasanein, “The Photomagneton in Quantum Field Theory” (World Scientific 1994).