

Chapter 9

Derivation Of The Evans Lemma And Wave Equation From The First Cartan Structure Relation

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Abstract

A new theorem of differential geometry is proven: the first Cartan structure equation is the commutator of the tetrad postulate. Conversely the appropriate interchange of base manifold indices in the tetrad postulate gives the first Cartan structure equation. The latter can be written as an equality of two tetrad postulates with reversed indices. Therefore the Evans Lemma and wave equation can be obtained directly from the first Cartan structure equation, which is thereby shown to be the source of all wave equations in generally covariant physics, both relativity and quantum mechanics.

Key words: Cartan structure equation; tetrad postulate; Evans Lemma; Evans wave equation.

9.1 Introduction

Recently a generally covariant unified field theory has been developed which unifies general relativity and quantum mechanics in terms of standard differential geometry [1]– [12]. In this theory the wave equations of physics are derived from the tetrad postulate and the field equations of electrodynamics and gravitation from the Cartan structure equations and the Bianchi identities of differential geometry [13]. In this paper a simple theorem of differential geometry is proven in Section 9.2 which shows that the first Cartan structure equation is an equality of two tetrad postulates. If the appropriate base manifold indices are interchanged in the tetrad postulate, the result is the first Cartan structure equation. The tetrad postulate is the source of the Evans Lemma of differential geometry [1]– [12], an identity which states that scalar curvature is the eigenvalue of the tetrad eigenfunction. The Eigen operator in the Evans Lemma is the d’Alembertian and the tetrad is the fundamental field of the Palatini variation of general relativity [1, 15]. The Lemma is a theorem of differential geometry which serves as the subsidiary proposition that gives the Evans wave equation using the Einstein field equation in index contracted form. The Evans wave equation gives all the well known wave equations of physics in appropriate limits [1]– [12], notably the Dirac equation in the special relativistic limit.

Therefore it is shown in this paper that the source of all the wave equations of quantum mechanics is the first Cartan structure equation itself. The latter is also the source of the tetrad postulate and vice versa. This means that physics is geometry, as inferred by Einstein and many others.

In Section 9.3 an example of this new inference at work is given in the context of the Aharonov Bohm effects, which are described straightforwardly in the Evans unified field theory [1]– [12].

9.2 Proof Of The Theorem

In condensed notation the first Cartan structure equation is

$$D \wedge q^a = T^a \tag{9.1}$$

which is shorthand for

$$d \wedge q^a + \omega^a_b \wedge q^b = T^a. \tag{9.2}$$

Reinstating the unwritten [13] indices of the base manifold

$$d \wedge q^a_\lambda + \omega^a_{\mu b} \wedge q^b_\lambda = T^a_{\mu\lambda} \tag{9.3}$$

and writing out Eq.(9.3) in full we obtain

$$\partial_\mu q^a_\lambda - \partial_\lambda q^a_\mu + \omega^a_{\mu b} q^b_\lambda - \omega^a_{\lambda b} q^b_\mu = T^a_{\mu\lambda}. \tag{9.4}$$

In these equations $D \wedge$ is the covariant exterior derivative, q^a is the tetrad one-form and T^a is the torsion two-form of differential geometry. Therefore the

first Cartan structure equation states that the torsion form is the covariant exterior derivative of the tetrad form. Eq.(9.4) is Eq.(9.1) written out in full using the spin connection $\omega^a_{\mu b}$. In order to define the torsion form correctly the spin connection must be identically non-zero. It is seen from Eq.(9.4) that the torsion form $T^a_{\mu\nu}$ is defined in terms of commutators or wedge products.

Now express the torsion form in terms of the torsion tensor $T^\nu_{\mu\lambda}$ of Riemann geometry [13]:

$$T^\nu_{\mu\lambda} = \Gamma^\nu_{\mu\lambda} - \Gamma^\nu_{\lambda\mu}. \quad (9.5)$$

The relation between the torsion form $T^a_{\mu\lambda}$ of differential geometry and the torsion tensor $T^\nu_{\mu\lambda}$ of Riemann geometry is [13]

$$T^a_{\mu\lambda} = T^\nu_{\mu\lambda} q^a_\nu \quad (9.6)$$

i.e.

$$T^a_{\mu\lambda} = \Gamma^\nu_{\mu\lambda} q^a_\nu - \Gamma^\nu_{\lambda\mu} q^a_\nu. \quad (9.7)$$

From Eqs.(9.4) and (9.7):

$$\begin{aligned} \partial_\mu q^a_\lambda - \partial_\lambda q^a_\mu + \omega^a_{\mu b} q^b_\lambda - \omega^a_{\lambda b} q^b_\mu \\ = \Gamma^\nu_{\mu\lambda} q^a_\nu - \Gamma^\nu_{\lambda\mu} q^a_\nu. \end{aligned} \quad (9.8)$$

Eq.(9.8) is the difference of two tetrad postulates [1]– [15]:

$$\partial_\mu q^a_\lambda + \omega^a_{\mu b} q^b_\lambda = \Gamma^\nu_{\mu\lambda} q^a_\nu \quad (9.9)$$

and

$$\partial_\lambda q^a_\mu + \omega^a_{\lambda b} q^b_\mu = \Gamma^\nu_{\lambda\mu} q^a_\nu. \quad (9.10)$$

Eqs.(9.9) and (9.10) are respectively:

$$D_\mu q^a_\lambda = 0 \quad (9.11)$$

and

$$D_\lambda q^a_\mu = 0. \quad (9.12)$$

Therefore the first Cartan structure equation is a commutator of two tetrad postulates

$$D_\mu q^a_\lambda - D_\lambda q^a_\mu = 0 \quad (9.13)$$

i.e.

$$D_\mu q^a_\lambda = D_\lambda q^a_\mu = 0 \quad (9.14)$$

Q.E.D.

The Evans Lemma is obtained [1]– [12] from the identity:

$$D^\mu D_\mu q^a_\lambda = D^\lambda D_\lambda q^a_\mu := 0 \quad (9.15)$$

and so the Lemma is obtained directly from the first Cartan structure equation. The Lemma is the subsidiary geometrical proposition:

$$\square q^a_\lambda = R q^a_\lambda \quad (9.16)$$

where R is a the scalar curvature defined in the Einstein field equation [13]. The index contracted form of the latter equation is [13]:

$$R = -kT \quad (9.17)$$

where T is the index contracted canonical energy-momentum tensor and where k is the Einstein constant. Note that Eq.(9.17) is valid for all radiated and matter fields [16] not just gravitation. Using Eq.(9.17) in Eq.(9.16) gives the Evans wave equation of generally covariant unified field theory [1]– [12]:

$$(\square + kT) q^a{}_{\mu} = 0. \quad (9.18)$$

The source of the Evans wave equation has therefore been shown in this paper to be the first Cartan structure equation itself.

9.3 Application To The Class Of Aharonov Bohm Effects

The fundamental ansatz that transforms from geometry to physics in the unified field theory is [1]– [12]:

$$A^a{}_{\mu} = A^{(0)} q^a{}_{\mu} \quad (9.19)$$

where $A^a{}_{\mu}$ is the vector potential magnitude. Similarly the electromagnetic field tensor follows from Eq.(9.19):

$$F^a{}_{\mu\nu} = A^{(0)} T^a{}_{\mu\nu}. \quad (9.20)$$

Thus, the electromagnetic potential $A^a{}_{\mu}$ is the tetrad form within a premultiplier $A^{(0)}$, and the electromagnetic field is the torsion form within the same premultiplier $A^{(0)}$. Using the the ansatz (9.19) the first Cartan structure equation gives the relation between field and potential in two ways. Firstly

$$F^a{}_{\mu\nu} = (d \wedge A^a)_{\mu\nu} + \omega^a{}_{\mu b} \wedge A^b{}_{\nu} \quad (9.21)$$

and secondly, using Eq.(9.6):

$$F^a{}_{\mu\nu} = T^{\rho}{}_{\mu\nu} A^a{}_{\rho}. \quad (9.22)$$

The class of Aharonov Bohm effects have been explained straightforwardly [1]– [12] using Eq.(9.21) as being due to the term $\omega^a{}_{\mu b} \wedge A^b{}_{\nu}$. This term is also responsible for the Evans spin field [1]– [12] and is the origin of polarization and magnetization [1]– [12]. In simple analogy, the iron whisker of the Chambers experiment, for example, acts as a stirring rod, and sets up a whirlpool of spacetime in its vicinity, i.e. in regions where the magnetic field does not exist, the term $\omega^a{}_{\mu b} \wedge A^b{}_{\nu}$ results in the observed electron diffraction fringe shift. This explanation means that there exists a hitherto unobserved electromagnetic

Aharonov Bohm effect due to the Evans spin field [1]– [12]. This is a close relative of the inverse Faraday effect and would be of probable interest for RADAR and stealth technology.

The additional inference into the Aharonov Bohm effects given by the new geometrical theorem of this paper is summarized in Eq.(9.22), which shows that the electromagnetic field is the inner product of the torsion tensor and the electromagnetic potential. The torsion tensor vanishes in the Maxwell Heaviside field theory, because the latter is constructed in a flat or Minkowski spacetime, but Eq.(9.22) is generally covariant as required by relativity theory. The combined result of Eqs.(9.21) and (9.22) is therefore:

$$F^a_{\mu\nu} = T^{\rho}_{\mu\nu} A^a_{\rho} = (d \wedge A^a)_{\mu\nu} + \omega^a_{\mu b} \wedge A^b_{\nu} \quad (9.23)$$

and shows that the Aharonov Bohm effects are due to a term $\omega^a_{\mu b} \wedge A^b_{\nu}$ which does not exist in Maxwell Heaviside theory and does not exist in the standard model. Nevertheless this term is the result of a rigorously objective theory of electromagnetism based on general relativity, not special relativity. This is the generally covariant unified field theory, which is therefore preferred experimentally and philosophically over the Maxwell Heaviside field theory.

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