KAMBE EQUATIONS GENERALIZED WITH VISCOUS EFFECTS:

THE STRUCTURE OF FLUID ELECTRODYNAMICS.

by

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ABSTRACT

The Kambe equations of fluid dynamics used in immediately preceding papers are generalized to include viscous force and other effects present in the most general format of the Navier Stokes and vorticity equations. The resulting structure is that of ECE2 relativity, fluid dynamics in general is shown to have the same structure as electrodynamics and gravitation in a space with finite torsion and curvature. It is shown that the whole of fluid dynamics can be reduced to one wave equation.

Keywords: ECE2 relativity, fluid dynamics, generalized Kambe field equations, the wave equation of fluid electrodynamics.

UFT 353

1. INTRODUCTION

In immediately preceding papers of this series {1 - 12} the subject of fluid electrodynamics has been developed from the Kambe equations of fluid dynamics (google "Kambe fluid Maxwell Equations"). The Kambe equations have the same structure as ECE2 electrodynamics and gravitation, theories which are developed in a space with finite torsion and curvature. In this paper the Kambe equations are developed to include the viscous force and other terms which appear in the most general Navier Stokes and vorticity equations, well known and highly developed equations of fluid dynamics. It is shown in Section 2 that the general Navier Stokes equation reduces to field equations with the structure of ECE2 relativity. These field equations can be expressed as one wave equation. So the entire subject of fluid dynamics reduces to one wave equation. In Section 3 sample computations and animations of these new field and wave equations are discussed.

This paper is a short synopsis of detailed calculations found in the notes accompanying UFT353 on www.aias.us. . Note 353(1) is a summary description of the most general Navier Stokes and vorticity equations, with an explanation of the various terms. Notes 353(2) and 353(3) derive a useful simplification of the vorticity equation. This is suitable for computation as in UFT349, UFT352 and UFT352. Section 2 is based on Notes 353(4) to 353(6) and derives field equations of fluid dynamics which are ECE2 covariant. It is shown that these field equations can be combined into one second order wave equation which is suitable for animation.

2. FIELD AND WAVE EQUATIONS

Consider the general Navier Stokes equation {1 - 12}:

$$\frac{D\underline{v}}{Dt} = \frac{\partial \underline{v}}{\partial t} + (\underline{v} \cdot \underline{\nabla})\underline{v} = -\underline{\nabla} h - \underline{\nabla} \phi + \underline{f}_{visc} \quad -(1)$$

where $\underline{v}$ is the velocity field of a fluid, h is enthalpy per unit mass, $\underline{f}$ is the viscous force, and ϕ is a potential such as the gravitational potential. The gradient of h is defined by:

$$\underline{\nabla} h = (\underline{\nabla} P)/\rho \quad -(2)$$

where ρ is the mass density of the fluid, and where P is the pressure in non S. I. Units as is customary in fluid dynamics. Kambe used the equation:

$$\frac{D\underline{v}}{Dt} = -\underline{\nabla} h \quad -(3)$$

and so omitted two terms. The correct definition of Kambe's "fluid electric field" is

$$\underline{E}_F = -\underline{\nabla} h - \underline{\nabla} \phi + \underline{f}_{visc} - \frac{\partial \underline{v}}{\partial t}. \quad -(4)$$

If the viscous force is defined most generally ("Vector Analysis Problem Solver") as:

$$\underline{f}_{visc} = \mu \nabla^2 \underline{v} + (\mu + \mu')\underline{\nabla}(\underline{\nabla} \cdot \underline{v}) \quad -(5)$$

it follows that:

$$\underline{\Phi} = h + \phi - (\mu + \mu')\underline{\nabla} \cdot \underline{v} - \phi_1 \quad -(6)$$

where μ and μ' are coefficients to be determined. Here:

$$\underline{E}_F = -\nabla \underline{\Phi} - \partial \underline{v}/\partial t, \quad -(7)$$

$$\underline{\nabla} \phi_1 = \mu \nabla^2 \underline{v}. \quad -(8)$$

With this definition of the scalar potential $\underline{\Phi}$, the Kambe field equations follow:

$$\underline{\nabla} \cdot \underline{B}_F = \underline{\nabla} \cdot \underline{w} = \underline{\nabla} \cdot \underline{\nabla} \times \underline{v} = 0 \quad -(9)$$

$$\underline{\nabla} \times \underline{E}_F + \frac{\partial \underline{B}_F}{\partial t} = \underline{0} \quad -(10)$$

$$\underline{\nabla} \cdot \underline{E}_F = q_F \quad -(11)$$

$$a_0^2 \underline{\nabla} \times \underline{B}_F - \frac{\partial \underline{E}_F}{\partial t} = \underline{J}_F . \quad -(12)$$

It has been shown that the most general form of fluid dynamics can be expressed as ECE2 covariant field equations, the above four equations, Q. E. D. Here a_0 is the constant speed of sound as used by Kambe. More generally, a_0 is not constant.

Note carefully that the above derivation of the four field equations does not use the most general vorticity equation of fluid dynamics, which is:

$$\frac{\partial \underline{w}}{\partial t} + \underline{\nabla} \times (\underline{\nabla} \times \underline{w}) + \frac{1}{\rho^2} \underline{\nabla} \rho \times \underline{\nabla} P + \frac{\mu}{\rho} \nabla^2 \underline{w} \quad -(13)$$

Kambe omitted the second and third terms on the right hand side of Eq. (13), the baroclinic and Reynolds number terms.

In the field equations (9) to (12) the fluid charge is defined by:

$$q_F = \underline{\nabla} \cdot \underline{E}_F = \underline{\nabla} \cdot ((\underline{v} \cdot \underline{\nabla}) \underline{v}) \quad -(14)$$

and the fluid current by:

$$\underline{J}_F = a_0^2 \underline{\nabla} \times (\underline{\nabla} \times \underline{v}) - \frac{\partial}{\partial t} ((\underline{v} \cdot \underline{\nabla}) \underline{v}) . \quad -(15)$$

As in note 353(5) it follows that:

$$\nabla^2 \underline{\Phi} + \frac{\partial}{\partial t} (\underline{\nabla} \cdot \underline{v}) = -q_F \quad -(16)$$

and:

$$\square \underline{v} + \underline{\nabla} \left(\underline{\nabla} \cdot \underline{v} + \frac{1}{a_0^2} \frac{\partial \Phi}{\partial t} \right) = \frac{1}{a_0^2} \underline{J}_F \quad -(17)$$

where the d'Alembertian is defined by:

$$\Box := \frac{1}{a_0^2}\frac{\partial^2}{\partial t^2} - \nabla^2 \quad -(18)$$

These equations can be simplified using the continuity equation:

$$\frac{\partial q_F}{\partial t} + \underline{\nabla} \cdot \underline{J}_F = 0 \quad -(19)$$

As shown in detail in Note 353(6), this is an exact consequence of the definitions (14) and (15). With the definitions:

$$J_F^\mu = \left(a_0 q_F, \underline{J}_F\right), \quad -(20)$$

$$\partial_\mu = \left(\frac{1}{a_0}\frac{\partial}{\partial t}, \underline{\nabla}\right), \quad -(21)$$

and (19), the continuity equation can be written as:

$$\partial_\mu J_F^\mu = 0. \quad -(22)$$

Now define the velocity four vector:

$$v^\mu = \left(\frac{\Phi}{a_0}, \underline{v}\right) \quad -(23)$$

and assume:

$$\partial_\mu v^\mu = \frac{1}{a_0^2}\frac{\partial \Phi}{\partial t} + \underline{\nabla} \cdot \underline{v} = 0. \quad -(24)$$

This is the Lorenz gauge assumption of fluid electrodynamics. With the assumption (24), Eqs. (16) and (17) reduce to:

$$\Box \Phi = q_F \quad -(25)$$

and:

$$\Box \underline{v} = \frac{1}{a_0^2} \underline{J}_F \quad -(26)$$

which can be combined into the single wave equation:

$$\Box v^\mu = \frac{1}{a_0^2} J_F^\mu \quad -(27)$$

Q. E. D.

The relevant S. I. Units are as follows:

$$\Box = m^{-2}, \ \Phi = m^2 s^{-2}, \ \rho_F = s^{-2}, \quad -(28)$$
$$\underline{J}_F = m s^{-3}, \ a_0 = m s^{-1}.$$

From Eqs. (19), (25) and (26):

$$\frac{1}{a_0^2} \frac{\partial}{\partial t} (\Box \Phi) + \underline{\nabla} \cdot \Box \underline{v} = 0. \quad -(29)$$

By commutativity of differential operators:

$$\Box \left(\frac{1}{a_0^2} \frac{\partial \Phi}{\partial t} + \underline{\nabla} \cdot \underline{v} \right) = 0. \quad -(30)$$

The Lorenz gauge (24) is a possible solution of Eq. (30), Q. E. D. So the analysis is self consistent.

It has been shown that the entire subject of fluid dynamics can be reduced to one soluble wave equation (27) which has the structure of the ECE wave equation:

$$(\Box + R) v^\mu = 0 \quad -(31)$$

provided that the scalar curvature R is defined by:

$$R v^\mu = -\frac{1}{a_0^2} J_F^\mu. \quad -(32)$$

Kambe equations generalized with viscous effects. The structure of fluid electrodynamics

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3 Graphics and animation with sample results

In this paper the wave equations of fluid electrodynamics have been developed. In case of an external current density $\mathbf{J}$ the wave equation (26) reads

$$\frac{1}{a_0^2}\frac{\partial^2 \mathbf{v}}{\partial t^2} - \nabla^2 \mathbf{v} = \frac{1}{a_0^2}\mathbf{J}. \tag{33}$$

Assuming a harmonic time dependence, we define

$$\mathbf{v}(\mathbf{r}, t) = \mathbf{v}_S(\mathbf{r}) \exp(i\omega t) \tag{34}$$

and

$$\mathbf{J}(\mathbf{r}, t) = \mathbf{J}_S(\mathbf{r}) \exp(i\omega t) \tag{35}$$

with a time frequency ω and only space-dependent velocities $\mathbf{v}_S$ and current densities $\mathbf{J}_S$. Then Eq. (33) reads

$$-\frac{\omega^2}{a_0^2}\mathbf{v}_S - \nabla^2 \mathbf{v}_S = \frac{1}{a_0^2}\mathbf{J}_S \tag{36}$$

which is an eigenvalue equation. For vanishing current density it can be written in the standard form

$$\nabla^2 \mathbf{v}_S + \lambda \mathbf{v}_S = \mathbf{0} \tag{37}$$

with positive eigenvalues

$$\lambda := \frac{\omega^2}{a_0^2} \tag{38}$$

which correspond to acoustic eigen frequencies for example. This equation can be solved numerically by the finite element method. In our example we re-adopt the 3D flow box of UFT papers 351 and 352 with corresponding boundary

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conditions, irrespective of further utility considerations. The first six eigenvalues (in arbitrary units) are listed in Table 1. There is a degeneracy between first and second eigenvalue and between fifth and sixth eigenvalue. This is due to internal symmetry of the flow box. The values are lying near to each other. The modulus of the first and sixth velocity eigen state has been graphed in Figs. 1-4 for two planes of symmetry ($Z = 0$ and $Y = 0$). The sixth eigen state has a node in the middle plane of symmetry. This symmetry is also present in the vorticity vectors, see Figs. 5-6. The divergence of velocity has been graphed in Figs. 7 and 8. The divergence is not restricted to the boundary regions and is more pronounced for the higher eigen state.

For a correct treatment of the wave equation within fluid electrodynamics, we have to include the current density (15):

$$\mathbf{J}_F = a_0^2 \boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times \mathbf{v}) - \frac{\partial}{\partial t} ((\mathbf{v} \cdot \boldsymbol{\nabla})\mathbf{v}) . \tag{39}$$

The second term is not linear in $\mathbf{v}$ so the time-harmonic approach is only possible for the first term. From (36)this gives the more general eigenvalue equation

$$\nabla^2 \mathbf{v}_S + \boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times \mathbf{v}_S) + \lambda \mathbf{v}_S = \mathbf{0}. \tag{40}$$

As a result, the eigenvalues are very small, compared to Eq. (37), and there are a lot more turbulences. The numerical calculation takes half an hour on a standard PC but converges. The numerical precision, however, is not satisfactory, therefore these result can only show a tendency. The first six eigenvalues are listed in Table 2. There is no degeneration any more. In Figs. 9-14 the vorticity in the plane $Y = 0$ has been graphed, this can be compared with Figs. 5 and 6. Obviously Eq. (40) incurs a lot more of turbulence structures. One can see that eigen state n possesses $n + 1$ vortices. This seems to be a particularity of Eq. (40).

A time-dependent calculation has been tried by assuming that the second-order time derivative can be neglected against the first-order time derivative in the current density:

$$\nabla^2 \mathbf{v}_S = -\boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times \mathbf{v}) + \frac{\partial}{\partial t} ((\mathbf{v} \cdot \boldsymbol{\nabla})\mathbf{v}) . \tag{41}$$

Adding a pressure term $\boldsymbol{\nabla} p$ as described in previous papers gives a non-singular equation but no time solution. Obviously the nonlinearity prevents a solution – at least for this special problem of boundary values considered.

Coming back to the solution of Eq. (40), this seems to be the first time that an ECE2 wave equation of type

$$(\Box + R)\mathbf{v} = \mathbf{0} \tag{42}$$

(see Eq. (31)) has been solved for a curvature R which in turn depends on the variable $\mathbf{v}$. This is certainly a step beyond contemporary standard equations of physics, e.g. the Dirac equation, where always a constant curvature has been assumed. The numerical problems, however, are enormous and a lot of work will be required to develop this field of ECE2 physics.

No.	Eigenvalue
1	12.1031274
2	12.1031274
3	12.1919561
4	13.2402655
5	13.3685992
6	13.3685992

Table 1: Eigenvalues of Eq. (37).

No.	Eigenvalue
1	2.56351677e-3
2	2.68244759e-3
3	4.08141046e-3
4	6.27378404e-3
5	7.79542935e-3
6	8.34876355e-3

Table 2: Eigenvalues of Eq. (40).

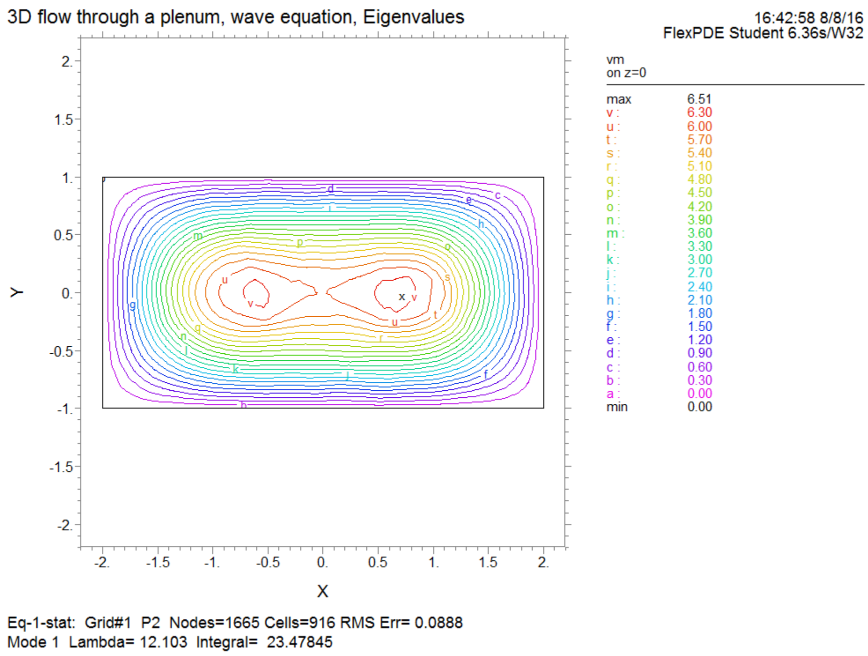

Figure 1: Velocity modulus of Eq. (37) on Z=0, eigen state 1.

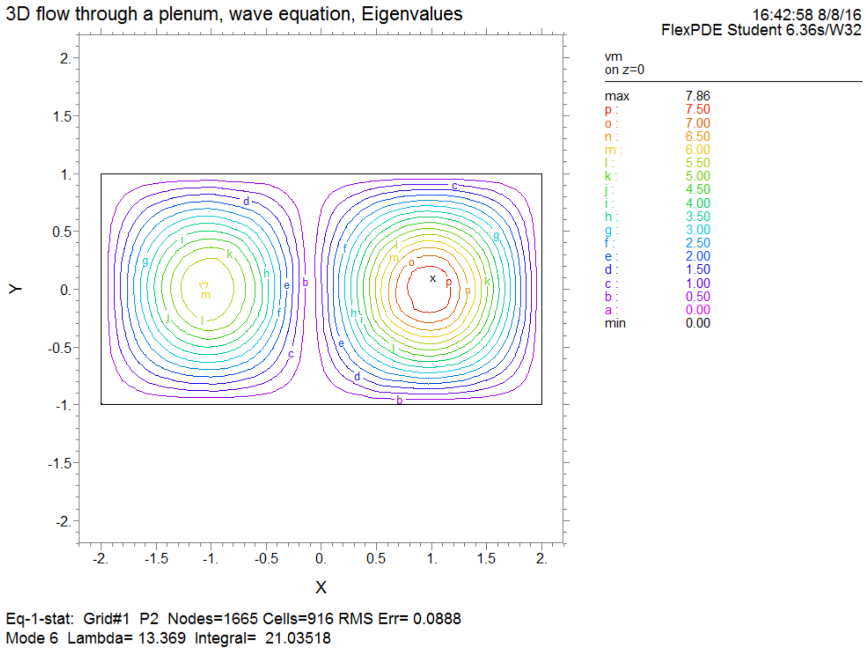

Figure 2: Velocity modulus of Eq. (37) on Z=0, eigen state 6.

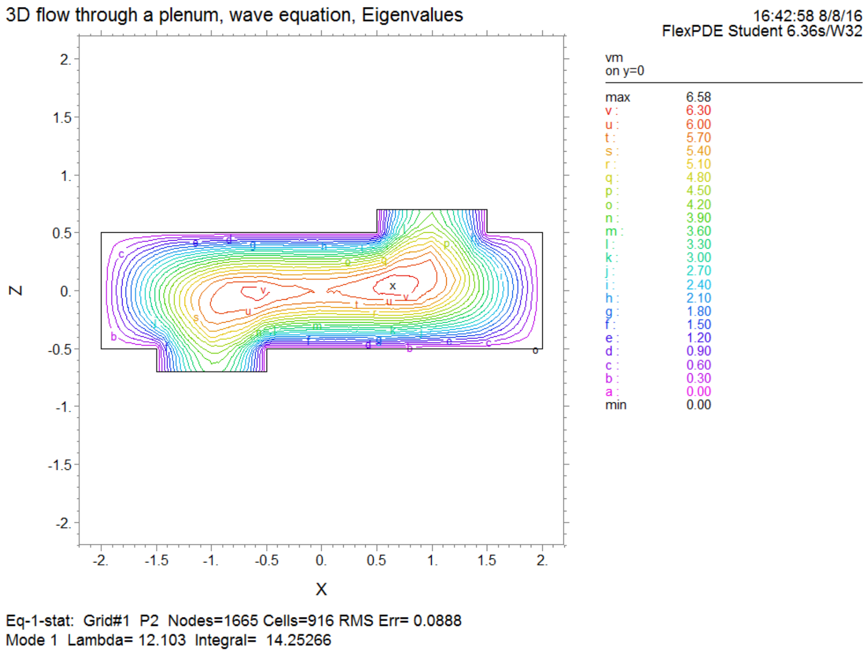

Figure 3: Velocity modulus of Eq. (37) on Y=0, eigen state 1.

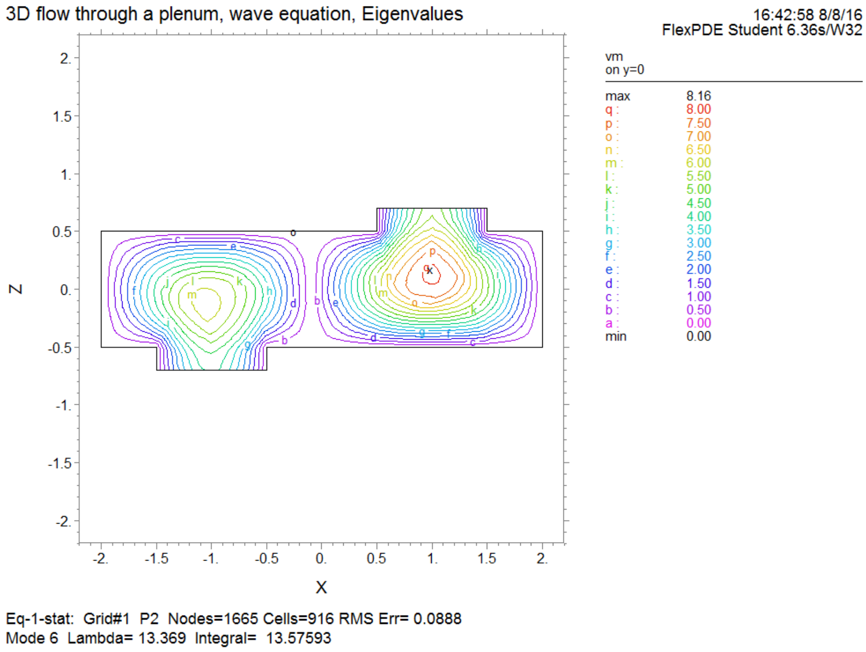

Figure 4: Velocity modulus of Eq. (37) on Y=0, eigen state 6.

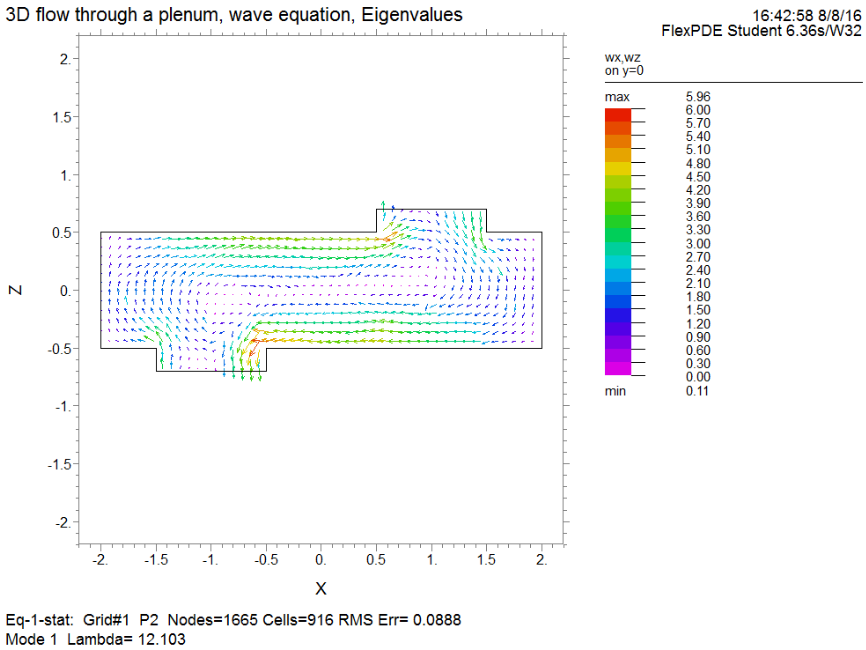

Figure 5: Vorticity of Eq. (37) on Y=0, eigen state 1.

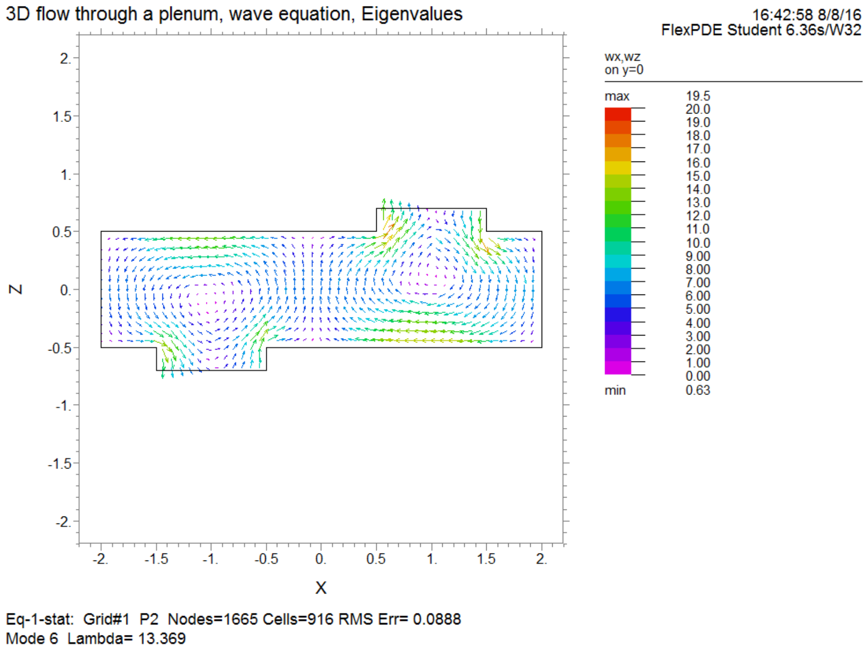

Figure 6: Vorticity of Eq. (37) on Y=0, eigen state 6.

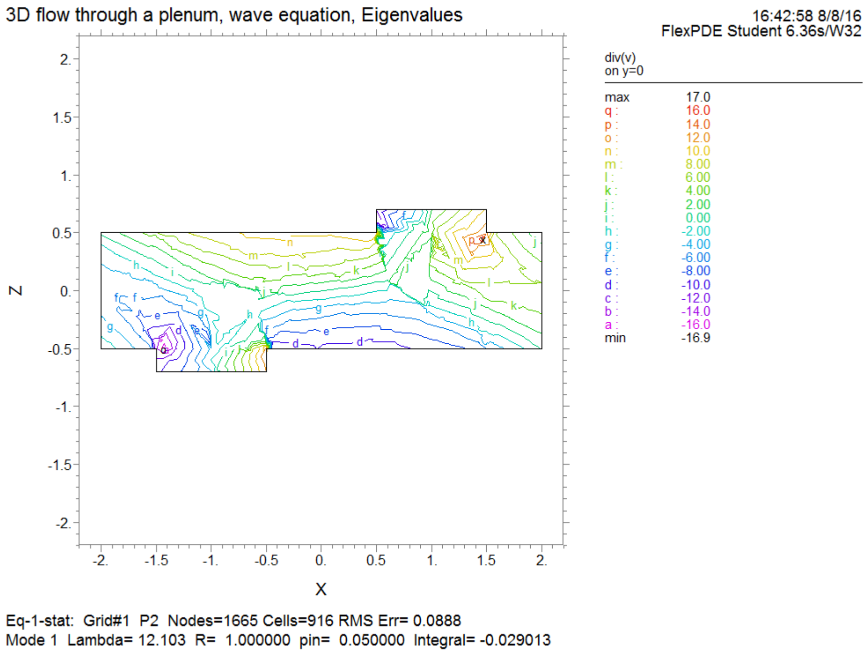

Figure 7: Divergence of velocity of Eq. (37) on Y=0, eigen state 1.

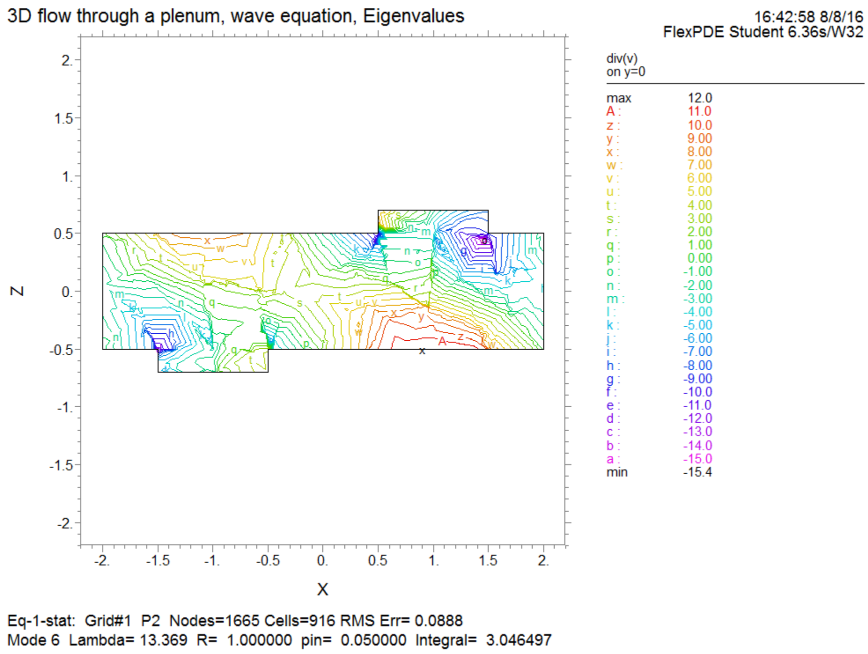

Figure 8: Divergence of velocity of Eq. (37) on Y=0, eigen state 6.

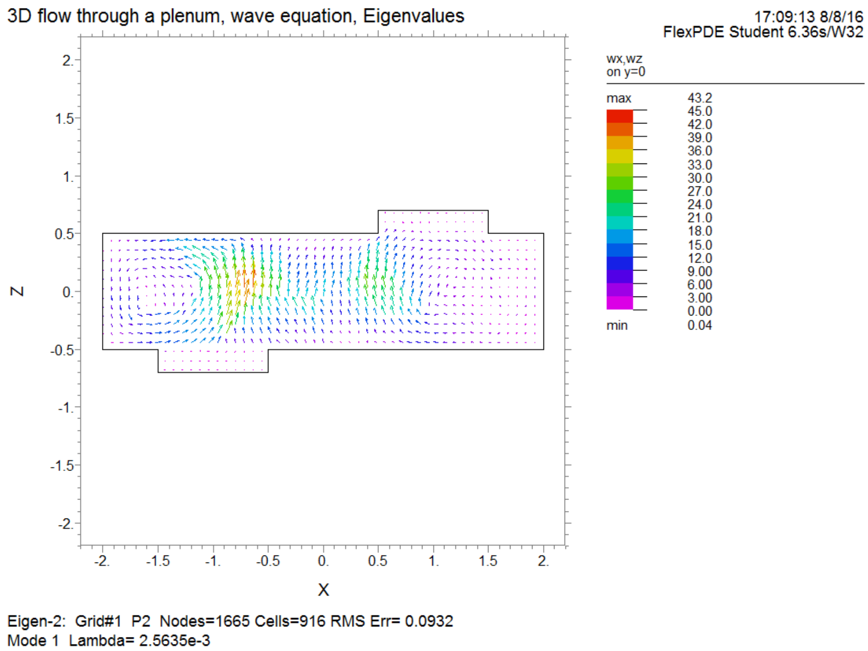

Figure 9: Vorticity of Eq. (40) on Y=0, eigen state 1.

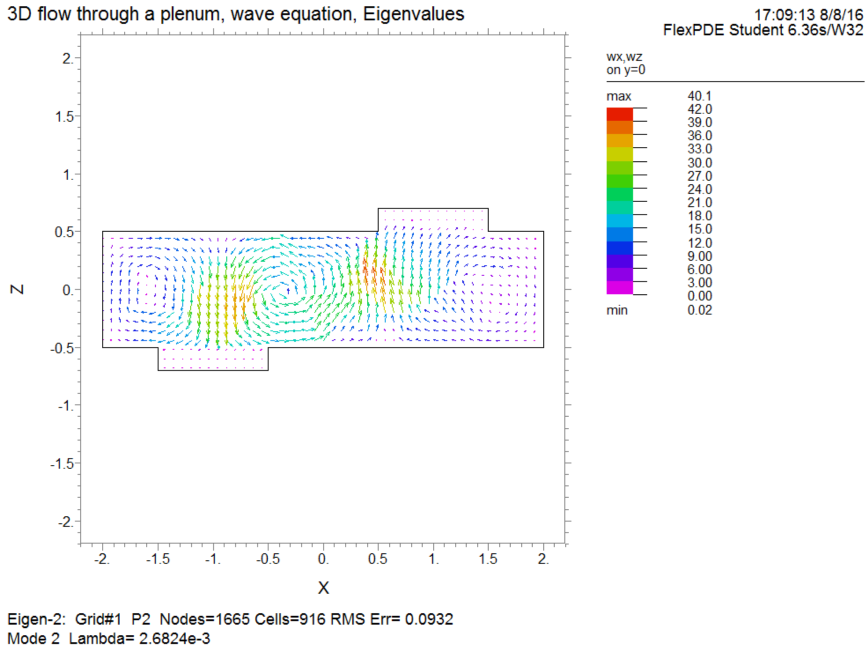

Figure 10: Vorticity of Eq. (40) on Y=0, eigen state 2.

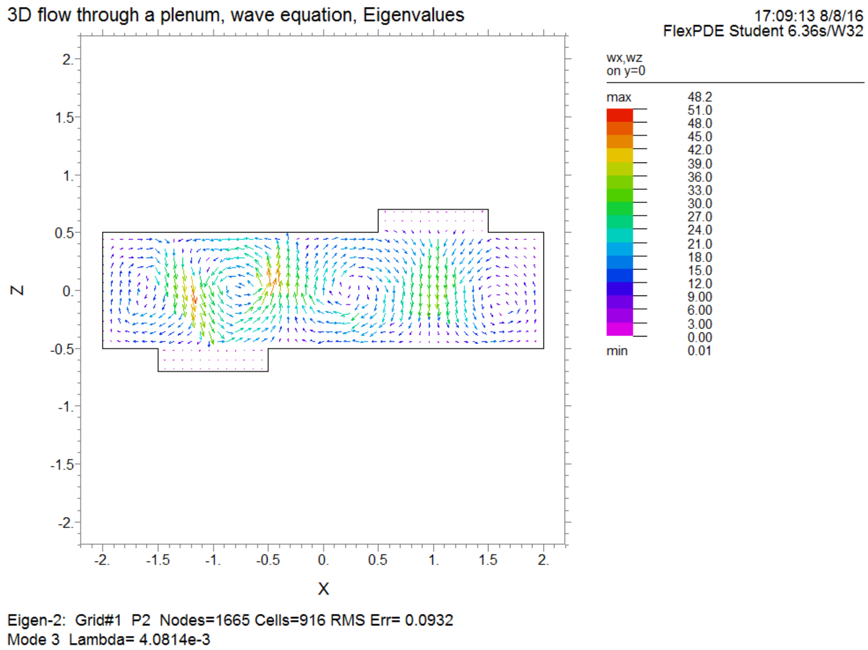

Figure 11: Vorticity of Eq. (40) on Y=0, eigen state 3.

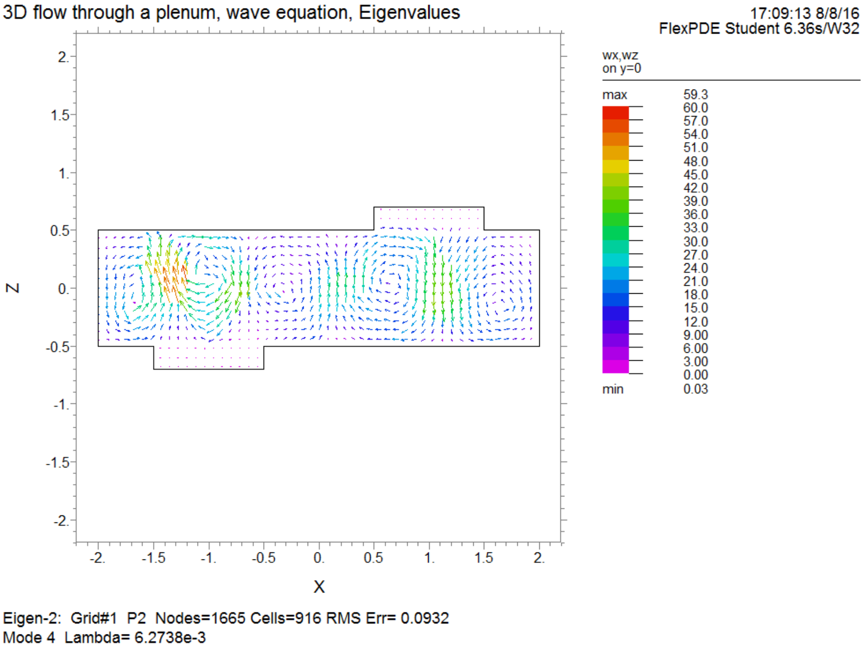

Figure 12: Vorticity of Eq. (40) on Y=0, eigen state 4.

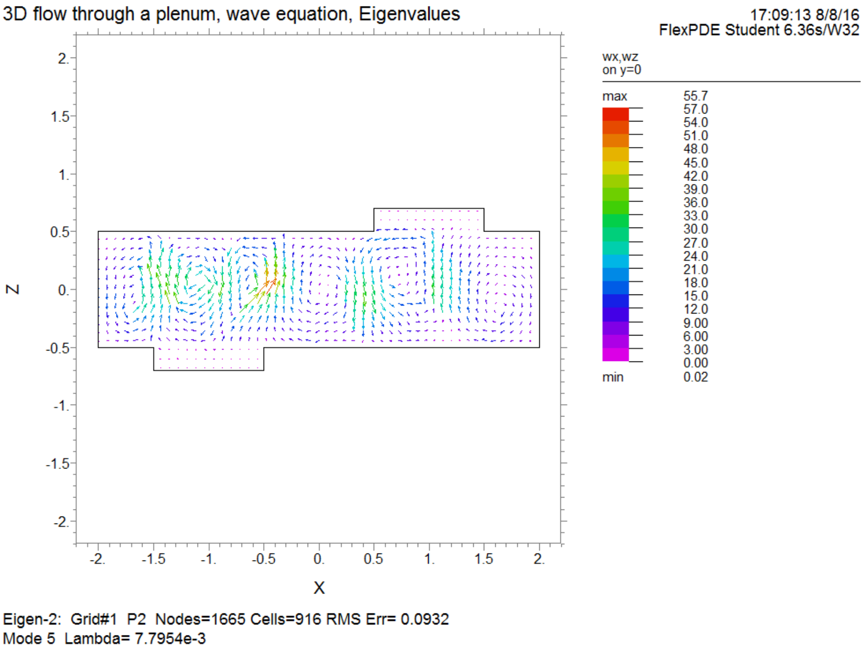

Figure 13: Vorticity of Eq. (40) on Y=0, eigen state 5.

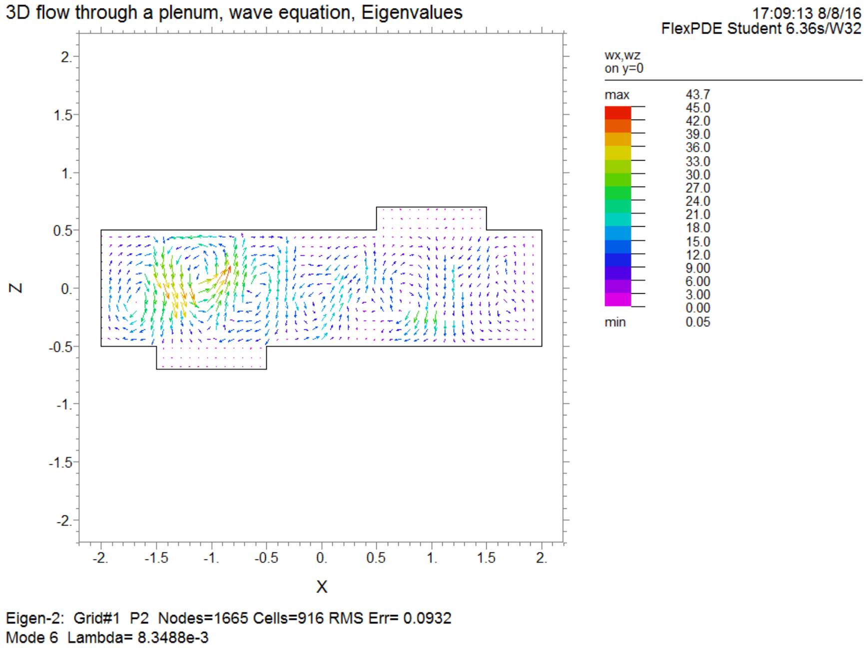

Figure 14: Vorticity of Eq. (40) on Y=0, eigen state 6.

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