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ECE2 DYNAMICS WITH THE LAGRANGE DERIVATIVE

by

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ABSTRACT

Classical dynamics is developed into ECE dynamics, in which the background spacetime is a fluid governed by the equations of fluid dynamics. This development is a logical consequence of the unification of gravitation and fluid dynamics by ECE theory. To exemplify ECE2 dynamics the acceleration is evaluated with the Lagrange derivative, which is shown to be an example of the covariant derivative of Cartan

Keywords: ECE2 unified field theory, ECE dynamics, acceleration with the Lagrange derivative.

UFT 361

1. INTRODUCTION

In recent papers of this series, gravitation has been unified with fluid dynamics {1 -12} so that spacetime or the vacuum or aether is considered to be a fluid. In this paper, ECE2 dynamics are developed with the Lagrange derivative of fluid dynamics. ECE2 dynamics is a generally covariant theory and is therefore a theory of general relativity. It contains more information than classical dynamics, to which it reduces in well defined limits. ECE2 dynamics does not use any of the concepts proposed by Einstein, and is an example of van der Merwe's post Einsteinian paradigm shift. The generally covariant ECE2 dynamics is based on Cartan geometry, and it is shown in Section 2 that the Lagrange (or material, or convective) derivative is an example of the Cartan derivative, which is defined through a spin connection matrix. The presence of the latter indicates that the theory is generally covariant, and is part of a unified field theory. The Hooke / Newton / Leibnitz orbital theory of the seventeenth century is Galilean covariant and does not contain a spin connection. Obviously the seventeenth century theory was not part of a unified field theory.

This paper is a condensed synopsis of the main results contained in extensive calculations in the notes accompanying UFT361 on www.aias.us and www.upitec.org. These notes give full details, most of which are missing from the usual textbooks and which are difficult to find. In Note 361(1) the acceleration is defined as the Lagrange derivative of velocity, which becomes a velocity field as usually defined in fluid dynamics. The spin connection is defined in Cartesian coordinates. In notes 361(2) and 361(3) the Lagrange derivative is developed in cylindrical polar coordinates from first principles. The development in these notes is given in all detail and results in the discovery of new fundamental accelerations not inferred by Coriolis in 1835. The Coriolis accelerations are recovered in well defined limits. This shows that the use of the Lagrange derivative generalizes classical dynamics. The result is named "ECE2 dynamics". In notes 361(4) and 361(5) the results of ECE2 dynamics are expressed in terms of a spin connection matrix in cylindrical coordinates and plane polar coordinates. The overall conclusion is that the usual cylindrical or plane polar coordinate system is a limiting case of a more general coordinate system. As in any theory of general relativity, the dynamics become those of the coordinate system itself. This inference applies both to material dynamics, such as those of particles, and also to the spacetime, or vacuum or aether, because the spacetime is the frame of reference itself. The presence of a covariant derivative means that the frame of reference is a dynamic quantity. In the Hooke / Newton Leibnitz dynamics, the Cartesian frame of reference does not move, and the vacuum is a structureless nothingness, an early anthropomorphic concept.

2. DETAILS OF DYNAMICS

In cylindrical coordinates the velocity field of ECE2 dynamics is the following function:

$$\underline{v} = \underline{v}(t, r(t), \theta(t), Z(t)). \quad -(1)$$

In classical dynamics the velocity is a function of time:

$$\underline{v} = \underline{v}(t). \quad -(2)$$

As shown from first principles in the background notes, the definition (1) means that the derivative of velocity must be the Lagrange derivative:

$$\frac{D\underline{v}}{Dt} = \frac{\partial \underline{v}}{\partial t} + (\underline{v}\cdot\underline{\nabla})\underline{v} = \frac{\partial \underline{v}}{\partial t} + \left(v_r\frac{\partial}{\partial r} + \frac{v_\theta}{r}\frac{\partial}{\partial \theta} + v_Z\frac{\partial}{\partial Z}\right)\left(v_r\underline{e}_r + v_\theta\underline{e}_\theta + v_Z\underline{k}\right)$$

$$= v_r\frac{\partial}{\partial r}(v_r\underline{e}_r) + \frac{v_\theta}{r}\frac{\partial}{\partial \theta}(v_r\underline{e}_r) + v_Z\frac{\partial}{\partial Z}(v_r\underline{e}_r)$$

$$+ v_r\frac{\partial}{\partial r}(v_\theta\underline{e}_\theta) + \frac{v_\theta}{r}\frac{\partial}{\partial \theta}(v_\theta\underline{e}_\theta) + v_Z\frac{\partial}{\partial Z}(v_\theta\underline{e}_\theta)$$

$$+ v_r \frac{\partial}{\partial r}\left(v_Z \underline{k}\right) + \frac{v_\theta}{r}\frac{\partial}{\partial \theta}\left(v_Z \underline{k}\right) + v_Z \frac{\partial}{\partial Z}\left(v_Z \underline{k}\right)$$

$$+ \frac{\partial \underline{v}}{\partial t}. \qquad -(3)$$

In general the derivatives of the quantities inside the brackets must be evaluated with the

Leibnitz theorem. In the cylindrical polar system:

$$\frac{\partial \underline{e}_r}{\partial r} = \frac{\partial \underline{e}_r}{\partial \theta} = \frac{\partial \underline{e}_r}{\partial Z} = \frac{\partial \underline{e}_\theta}{\partial Z} = \frac{\partial \underline{k}}{\partial r} = \frac{\partial \underline{k}}{\partial \theta} = \frac{\partial \underline{k}}{\partial Z} = \underline{0}$$

$$-(4)$$

and:

$$\partial r / \partial \theta = 0. \qquad -(5)$$

By construction:

$$\frac{\partial \underline{e}_r}{\partial \theta} = \underline{e}_\theta , \quad \frac{\partial \underline{e}_\theta}{\partial \theta} = -\underline{e}_r . \qquad -(6)$$

This means that the Lagrange derivative is:

$$\frac{D\underline{v}}{Dt} = \frac{\partial \underline{v}}{\partial t} + \left(v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} + v_Z \frac{\partial v_r}{\partial Z}\right)\underline{e}_r + \frac{v_\theta v_r}{r}\frac{\partial \underline{e}_r}{\partial \theta}$$

$$+ \left(v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + v_Z \frac{\partial v_\theta}{\partial Z}\right)\underline{e}_\theta - \frac{v_\theta^2}{r}\frac{\partial \underline{e}_\theta}{\partial r}$$

$$+ \left(v_r \frac{\partial v_Z}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_Z}{\partial \theta} + v_Z \frac{\partial v_Z}{\partial Z}\right)\underline{k}$$

$$= \frac{\partial}{\partial t}\begin{bmatrix} v_r \\ v_\theta \\ v_Z \end{bmatrix} + \left(\begin{bmatrix} \frac{\partial v_r}{\partial r} & \frac{1}{r}\frac{\partial v_r}{\partial \theta} & \frac{\partial v_r}{\partial Z} \\ \frac{\partial v_\theta}{\partial r} & \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} & \frac{\partial v_\theta}{\partial Z} \\ \frac{\partial v_Z}{\partial r} & \frac{\partial v_Z}{\partial \theta} & \frac{\partial v_Z}{\partial Z} \end{bmatrix} + \begin{bmatrix} 0 & -\frac{v_\theta}{r} & 0 \\ \frac{v_\theta}{r} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\right)\begin{bmatrix} v_r \\ v_\theta \\ v_Z \end{bmatrix}$$

$$-(7)$$

in cylindrical polar coordinates. The second matrix has the antisymmetric structure of a rotation generator and is the matrix form of the angular velocity of the rotating frame of the coordinate system.

The Lagrange derivative of ECE2 dynamics, exemplified in Eq. (7), is the Cartan derivative of ECE2 generally covariant unified field theory:

$$\frac{Dv^a}{Dt} = \frac{\partial v^a}{\partial t} + \omega^a{}_{ob} v^b \quad -(8)$$

in which the spin connection matrix is:

$$\omega^a{}_{ob} = \begin{bmatrix} \frac{\partial v_r}{\partial r} & \frac{1}{r}\frac{\partial v_r}{\partial \theta} \\ \frac{\partial v_\theta}{\partial r} & \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} \end{bmatrix} + \begin{bmatrix} 0 & -\frac{v_\theta}{r} \\ \frac{v_\theta}{r} & 0 \end{bmatrix} \quad -(9)$$

In plane polar coordinates appropriate to a planar orbit, Eq. (8) means:

$$\frac{D}{Dt}\begin{bmatrix} v_r \\ v_\theta \end{bmatrix} = \frac{\partial}{\partial t}\begin{bmatrix} v_r \\ v_\theta \end{bmatrix} + \left(\begin{bmatrix} \frac{\partial v_r}{\partial r} & \frac{1}{r}\frac{\partial v_r}{\partial \theta} \\ \frac{\partial v_\theta}{\partial r} & \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} \end{bmatrix} + \begin{bmatrix} 0 & -\frac{v_\theta}{r} \\ \frac{v_\theta}{r} & 0 \end{bmatrix} \right) \begin{bmatrix} v_r \\ v_\theta \end{bmatrix} \quad -(10)$$

The velocity vector in plane polar coordinates is:

$$\underline{v} = \dot{r}\,\underline{e}_r + r\dot{\theta}\,\underline{e}_\theta \quad -(11)$$

so:

$$v_r = \dot{r}, \quad v_\theta = r\dot{\theta} \quad -(12)$$

and

$$\begin{bmatrix} 0 & -\frac{v_\theta}{r} \\ \frac{v_\theta}{r} & 0 \end{bmatrix} = \begin{bmatrix} 0 & -\omega \\ \omega & 0 \end{bmatrix} \quad -(13)$$

where the angular velocity of the rotating frame of the plane polar system of coordinates is:

$$\omega = \frac{d\theta}{dt} = \dot{\theta}. \quad -(14)$$

The plane polar coordinates system is therefore a moving frame of reference of general relativity, and its spin connection components are:

$$\omega^1{}_{01} = \partial\dot{r} / \partial r \quad -(15)$$

$$\omega^1{}_{02} = \frac{1}{r}\frac{\partial \dot{r}}{\partial \theta} - \dot{\theta} \quad -(16)$$

$$\omega^2{}_{01} = \partial(r\dot{\theta}) / \partial r + \dot{\theta} \quad -(17)$$

$$\omega^2{}_{02} = \frac{1}{r}\frac{\partial(r\dot{\theta})}{\partial \theta} \quad -(18)$$

The Cartan / Lagrange derivative of velocity can be expressed as:

$$\frac{D\underline{v}}{Dt} = \frac{\partial v_r}{\partial t}\underline{e}_r + \frac{\partial v_\theta}{\partial t}\underline{e}_\theta + \left(v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} \right)\underline{e}_r + \left(v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_\theta v_r}{r} \right)\underline{e}_\theta \quad -(19)$$

in terms of the unit vectors of the plane polar system. Eq. (19) is the following covariant derivative of Cartan geometry:

$$\underline{a} = \frac{D\underline{v}}{Dt} = \frac{Dv_r}{Dt}\underline{e}_r + \frac{Dv_\theta}{Dt}\underline{e}_\theta . \quad -(20)$$

This is a new and original definition of any acceleration. It follows that:

$$\underline{a} = \frac{\partial \underline{v}}{\partial t} + (\underline{v}\cdot\underline{\nabla})\underline{v} = \frac{Dv_r}{Dt}\underline{e}_r + \frac{Dv_\theta}{Dt}\underline{e}_\theta . \quad -(21)$$

The individual covariant derivatives are:

$$\frac{Dv_r}{Dt} = \frac{\partial v_r}{\partial t} + \left(v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} \right)$$

$$= \frac{\partial v_r}{\partial t} + \dot{r}\frac{\partial \dot{r}}{\partial r} + \dot{\theta}\frac{\partial \dot{r}}{\partial \theta} - r\dot{\theta}^2 \quad -(22)$$

and

$$\frac{Dv_\theta}{Dt} = \frac{\partial v_\theta}{\partial t} + \left(v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_\theta v_r}{r} \right)$$

$$= \frac{\partial v_\theta}{\partial t} + r\ddot{\theta} + 2\dot{r}\dot{\theta} + r\dot{r}\frac{\partial \dot{\theta}}{\partial r} + \dot{\theta}^2\frac{\partial r}{\partial \theta} \quad -(23)$$

Eqs. (22) and (23) are equivalent to the matrix equation (10). Therefore:

$$v_r \frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r} = \dot{r}\frac{\partial \dot{r}}{\partial r} + \dot{\theta}\frac{\partial \dot{r}}{\partial \theta} - r\dot{\theta}^2$$

$$= \left(\begin{bmatrix} \frac{\partial v_r}{\partial r} & \frac{1}{r}\frac{\partial v_r}{\partial \theta} \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & -\frac{v_\theta}{r} \\ 0 & 0 \end{bmatrix} \right) \begin{bmatrix} v_r \\ v_\theta \end{bmatrix} \quad -(24)$$

and

$$v_r \frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_\theta v_r}{r} = r\ddot{\theta} + 2\dot{r}\dot{\theta} + r\dot{r}\frac{\partial \dot{\theta}}{\partial r} + \dot{\theta}^2\frac{\partial r}{\partial \theta}$$

$$= \begin{bmatrix} 0 & 0 \\ \frac{\partial v_\theta}{\partial r} & \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} \end{bmatrix} \begin{bmatrix} v_r \\ v_\theta \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \frac{v_\theta}{r} & 0 \end{bmatrix} \begin{bmatrix} v_r \\ v_\theta \end{bmatrix} \quad -(25)$$

The Cartan / Lagrange definition of the acceleration leads to new accelerations:

$$\underline{a}_1 = \left(\dot{r}\frac{\partial \dot{r}}{\partial r} + \dot{\theta}\frac{\partial \dot{r}}{\partial \theta} \right)\underline{e}_r + \left(r\dot{r}\frac{\partial \dot{\theta}}{\partial r} + \dot{\theta}^2\frac{\partial r}{\partial \theta} \right)\underline{e}_\theta \quad -(26)$$

which are absent form the usual textbook definition of acceleration in plane polar coordinates:

$$\underline{a} = \frac{d\underline{v}}{dt} = \frac{d}{dt}\left(v_r \underline{e}_r \right) + \frac{d}{dt}\left(r\dot{\theta}\,\underline{e}_\theta \right)$$

$$= \left(\ddot{r} - r\dot{\theta}^2 \right) \underline{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta} \right) \underline{e}_\theta . \quad -(27)$$

On the right hand side of Eq. (27) appear the Newtonian acceleration:

$$\underline{a}_N = \ddot{r}\,\underline{e}_r \quad -(28)$$

the centrifugal acceleration:

$$\underline{a}_{cent} = -r\dot{\theta}^2\,\underline{e}_r \quad -(29)$$

and the Coriolis accelerations:

$$\underline{a}_{Coriolis} = \left(r\ddot{\theta} + 2\dot{r}\dot{\theta} \right) \underline{e}_\theta \quad -(30)$$

as is well known (Coriolis, 1835).

The Cartan / Lagrange derivative leads to the discovery of new accelerations:

$$\underline{a}_1 = \left(\dot{r}\frac{\partial \dot{r}}{\partial r} + \dot{\theta}\frac{\partial \dot{r}}{\partial \theta} \right) \underline{e}_r + \left(r\dot{r}\frac{\partial \dot{\theta}}{\partial r} + \dot{\theta}^2\frac{\partial r}{\partial \theta} \right) \underline{e}_\theta \quad -(31)$$

not inferred by Coriolis and which are unknown in classical dynamics. They are a fundamental result of ECE unified field theory and they are the result of the velocity field:

$$\underline{v} = \underline{v}\left(t, r(t), \theta(t)\right) \quad -(32)$$

which generalizes

$$\underline{v} = \underline{v}(t) \quad -(33)$$

of classical dynamics.

In the usual development of the plane polar coordinate system:

$$\frac{\partial r}{\partial \theta} = 0 \quad -(34)$$

because r and θ are the independent variables of the coordinate system (r, θ).

Similarly in the Cartesian system:

$$\frac{\partial X}{\partial Y} = \frac{\partial X}{\partial Z} = \frac{\partial Y}{\partial Z} = 0 . \quad -(35)$$

However, if r and θ become functions of time, and if $\underline{v}$ depends on r and θ as in Eq. (32), the plane polar coordinate system is generalized and becomes a moving frame, because the time derivative of $\underline{v}$ must be worked out with the chain rule of differentiation as in Note 361(3). The velocity field (32) results in the new accelerations (31).

Considering the components of the new acceleration, there are results such as:

$$\frac{\partial \dot{\theta}}{\partial r} = \frac{\partial}{\partial r}\left(\frac{d\theta}{dt}\right) = \frac{\partial \omega}{\partial r} . \quad -(36)$$

If the angular velocity of the rotating frame is independent of r, then:

$$\frac{\partial \dot{\theta}}{\partial r} = 0 \quad -(37)$$

because of Eq. (34). The new acceleration (31) is then a radially directed acceleration which augments the centrifugal acceleration. In Eq. (31):

$$\frac{\partial \dot{r}}{\partial \theta} = \frac{\partial}{\partial \theta}\left(\frac{dr}{dt}\right) = \frac{\partial v_r}{\partial \theta} \quad -(38)$$

and

$$\frac{\partial \dot{r}}{\partial r} = \frac{\partial}{\partial r}\left(\frac{dr}{dt}\right) = \frac{\partial v_r}{\partial r} . \quad -(39)$$

In the usual development that leads to Eq. (27):

$$v_r = \dot{r} = \frac{dr}{dt} = \frac{dr(t)}{dt} \quad -(40)$$

so v_r has no functional dependence on r or θ. In consequence, in the usual development:

$$\frac{\partial v_r}{\partial r} = \frac{\partial v_r}{\partial \theta} = 0 \quad -(41)$$

and

$$\underline{a}_1 = \underline{0}. \quad -(42)$$

In this case:

$$\begin{aligned}\underline{a} &= \frac{D\underline{v}}{Dt} = \frac{\partial \underline{v}}{\partial t} + (\underline{v}\cdot\underline{\nabla})\underline{v} \\ &= \frac{Dv_r}{Dt}\underline{e}_r + \frac{Dv_\theta}{Dt}\underline{e}_\theta \\ &= (\ddot{r} - r\dot{\theta}^2)\underline{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\underline{e}_\theta\end{aligned} \quad -(43)$$

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However, Eq. (43) is a very limited result that depends on:

$$\underline{v} = \underline{v}(t, r(t), \theta(t)) \longrightarrow \underline{v}(t). \quad -(44)$$

3. SAMPLE GRAPHICS

Section by Dr. Horst Eckardt

ECE2 dynamics with the Lagrange derivative

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3 Sample graphics

First we do some calculations on the accelerations in ECE2 fluid dynamics as described in section 2. The accelerations in plane polar coordinates are

$$\mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\mathbf{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{e}_\theta, \tag{45}$$

$$\mathbf{a}_1 = \left(\dot{r}\frac{\partial \dot{r}}{\partial r} + \dot{\theta}\frac{\partial \dot{r}}{\partial \theta}\right)\mathbf{e}_r + \left(r\dot{r}\frac{\partial \dot{\theta}}{\partial r} + \dot{\theta}^2\frac{\partial r}{\partial \theta}\right)\mathbf{e}_\theta. \tag{46}$$

Now use

$$\frac{\partial \dot{\theta}}{\partial \theta} = \frac{\partial \dot{\theta}}{\partial t}\frac{dt}{d\theta} = \frac{\ddot{\theta}}{\dot{\theta}}, \tag{47}$$

and similarly

$$\frac{\partial \dot{\theta}}{\partial r} = \frac{\ddot{\theta}}{\dot{r}}, \qquad \frac{\partial \dot{r}}{\partial \theta} = \frac{\ddot{r}}{\dot{\theta}}, \tag{48}$$

$$\frac{\partial \dot{r}}{\partial r} = \frac{\ddot{r}}{\dot{r}}, \qquad \frac{\partial r}{\partial \theta} = \frac{\dot{r}}{\dot{\theta}}\ . \tag{49}$$

Inserting this into (46) gives

$$\mathbf{a}_1 = 2\ \ddot{r}\ \mathbf{e}_r + \left(r\ddot{\theta} + \dot{r}\dot{\theta}\right)\mathbf{e}_\theta. \tag{50}$$

In total, the acceleration now is

$$\mathbf{a}_{\mathrm{tot}} = \mathbf{a} + \mathbf{a}_1 = (3\ddot{r} - r\dot{\theta}^2)\mathbf{e}_r + (2r\ddot{\theta} + 3\dot{r}\dot{\theta})\mathbf{e}_\theta. \tag{51}$$

This is a massive modification of (45). The latter is valid for mass point dynamics only with $\mathbf{v} = \mathbf{v}(t)$.

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For evaluation of examples, the time dependence shall be eliminated. From conservation of angular momentum L_0 in Z direction follows

$$\dot{\theta} = \frac{L_0}{mr^2} \tag{52}$$

and

$$\ddot{\theta} = \frac{d\dot{\theta}}{dt} = \frac{dr}{dt}\frac{\partial}{\partial r}\left(\frac{L_0}{mr^2}\right) = -2\,\dot{r}\frac{L_0}{mr^3}. \tag{53}$$

Similarly:

$$\dot{r} = \frac{\partial r}{\partial \theta}\dot{\theta}, \tag{54}$$

$$\begin{aligned}\ddot{r} &= \frac{d}{dt}\left(\frac{\partial r}{\partial \theta}\dot{\theta}\right) = \frac{d}{dt}\left(\frac{\partial r}{\partial \theta}\right)\dot{\theta} + \frac{\partial r}{\partial \theta}\ddot{\theta} = \frac{d\theta}{dt}\frac{\partial}{\partial \theta}\left(\frac{\partial r}{\partial \theta}\right)\dot{\theta} + \frac{\partial r}{\partial \theta}\ddot{\theta} \\ &= \frac{\partial^2 r}{\partial \theta^2}\dot{\theta}^2 + \frac{\partial r}{\partial \theta}\ddot{\theta}.\end{aligned} \tag{55}$$

Thus all quantities depend on θ and r only. For cylindrical coordinates it follows correspondingly:

$$\dot{Z} = \frac{\partial Z}{\partial \theta}\dot{\theta}, \tag{56}$$

$$\ddot{Z} = \frac{\partial^2 Z}{\partial \theta^2}\dot{\theta}^2 + \frac{\partial Z}{\partial \theta}\ddot{\theta}, \tag{57}$$

$$\frac{\partial \dot{Z}}{\partial \theta} = \frac{\partial^2 Z}{\partial \theta^2}\dot{\theta}. \tag{58}$$

All time derivatives have been brought into a form depending on θ and $\dot{\theta}$ which is given by (52).

As a non-trivial example we consider a three-dimensional vortex field called Torkado [1], see Fig. 1. This could also be a description for the dynamics of the plasma model of galaxies. We concentrate on a streamline in the middle of the structure which may be described by the analytical approach in cylindrical coordinates (r, θ, Z):

$$r(\theta) = 0.05 + \cos\left(\frac{\theta}{10}\right)^2, \tag{59}$$

$$Z(\theta) = 2\sin\left(\frac{\theta}{5}\right), \tag{60}$$

see Fig. 2. For a plot in cartesian coordinates, the plane polar part is to be transformed by

$$X = r\cos(\theta), \tag{61}$$

$$Y = r\sin(\theta). \tag{62}$$

In order to make the analysis not too complicated, we restrict it to the plane polar parts of the acceleration as given in Eqs. (45, 46, 51). The orbital quantities $r(\theta)$, $Z(\theta)$, $\partial r/\partial\theta$ and $\partial Z/\partial\theta$ are graphed in Fig. 3 in dependence of θ.

These are oscillatory as to be expected from (59, 60). The time derivatives of r, θ and Z, calculated with aid of (53-58), are essential where r is small due to conservation of angular momentum (Fig. 4). The radial acceleration parts a_r, a_{1r} and its sum $a_r + a_{1r}$ are presented in Fig. 5, showing that the signs of a_r and a_{1r} are different, leading to zero crossings in the sum of both. The angular part of $\mathbf{a}_1$ reflects the well known fact that for a plane polar system $a_\theta = 0$, i.e. there is no angular force component. This does not hold for $a_{1\theta}$.

The correct handling requires use of Eq. (33) of UFT paper 356 for describing the acceleration in 3D cylindrical coordinates. The result from computer algebra is:

$$\frac{D\mathbf{v}}{Dt} = \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \boldsymbol{\nabla})\,\mathbf{v} = \begin{bmatrix} \ddot{r} - r\dot{\theta}^2 + \frac{\partial \dot{r}}{\partial \theta}\dot{\theta} \\ r\ddot{\theta} + (r\frac{\partial \dot{\theta}}{\partial \theta} + \frac{\partial r}{\partial \theta}\dot{\theta})\dot{\theta} + 3\dot{r}\dot{\theta} \\ \ddot{Z} + \frac{\partial \dot{Z}}{\partial \theta}\dot{\theta} \end{bmatrix}. \tag{63}$$

This can be re-expressed by

$$\begin{aligned} \frac{D\mathbf{v}}{Dt} &= \begin{bmatrix} \ddot{r} - r\dot{\theta}^2 + \frac{\ddot{r}}{\dot{\theta}}\dot{\theta} \\ r\ddot{\theta} + (r\frac{\ddot{\theta}}{\dot{\theta}} + \frac{\partial r}{\partial \theta}\dot{\theta})\dot{\theta} + 3\dot{r}\dot{\theta} \\ \frac{\partial^2 Z}{\partial \theta^2}\dot{\theta}^2 + \frac{\partial Z}{\partial \theta}\ddot{\theta} + \frac{\partial^2 Z}{\partial \theta^2}\ddot{\theta} \end{bmatrix} \\ &= \begin{bmatrix} 2\ddot{r} - r\dot{\theta}^2 \\ 2r\ddot{\theta} + 4\dot{r}\dot{\theta} \\ \frac{\partial^2 Z}{\partial \theta^2}\dot{\theta}^2 + (\frac{\partial Z}{\partial \theta} + \frac{\partial^2 Z}{\partial \theta^2})\ddot{\theta} \end{bmatrix} \end{aligned} \tag{64}$$

where the derivatives of Z can be calculated from (60). This result is different from the plane polar case as expected. The three components are graphed in Fig. 7. There is qualitative similarity to Figs. 5 for the radial component, it reflects both extremal points. The θ component surprisingly vanishes again as for the plane polar system. Obviously there is no coupling to the Z component that would prevent this. The Z component is antisymmetric to the two radial peaks, indicating the lower and upper turning points of the orbit.

References

[1] Gabriele Müller, Viva Vortex, ISBN 9783741287213, 2016, and http://www.viva-vortex.de/.

see images in

www.torkado.de

and

http://www.viva-vortex.de/texte/img/Torkado_RautBlau.jpg

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Figure 1: Torkado vortex after [1].

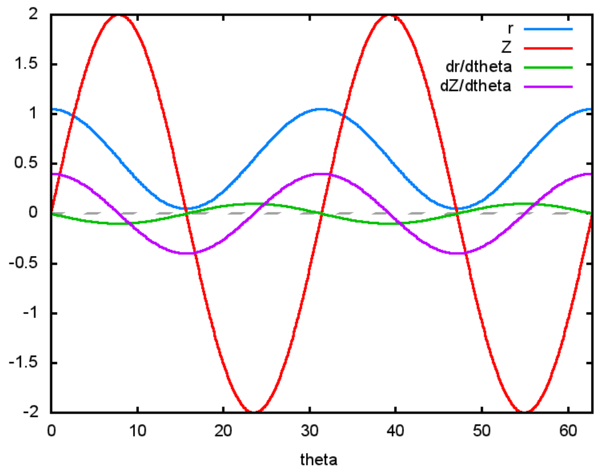

Figure 2: Structure of Torkado 3D orbit.

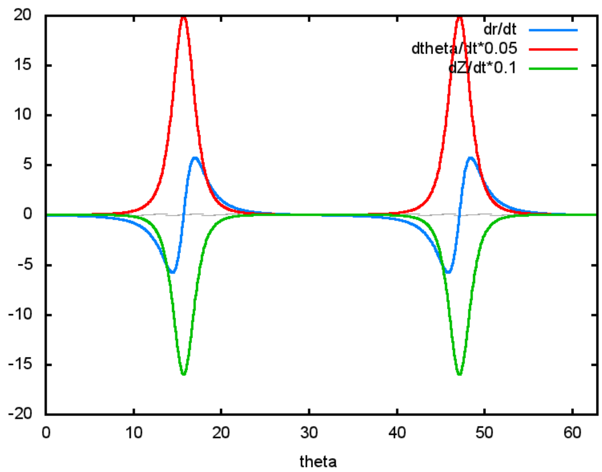

Figure 3: Angular dependence of $r, Z, \partial r/\partial\theta, \partial r/\partial Z$.

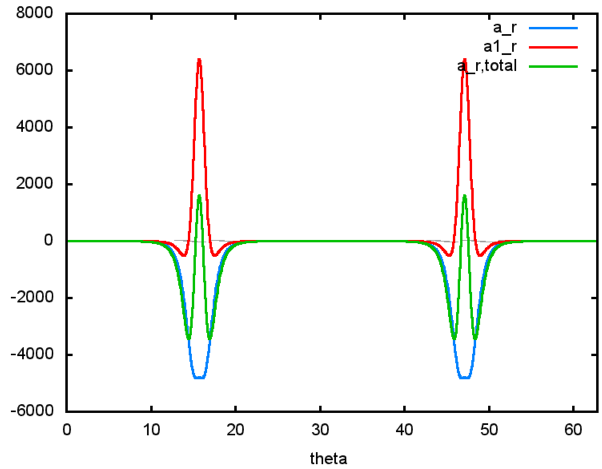

Figure 4: Scaled angular dependence of $dr/dt, d\theta/dt, dZ/dt$.

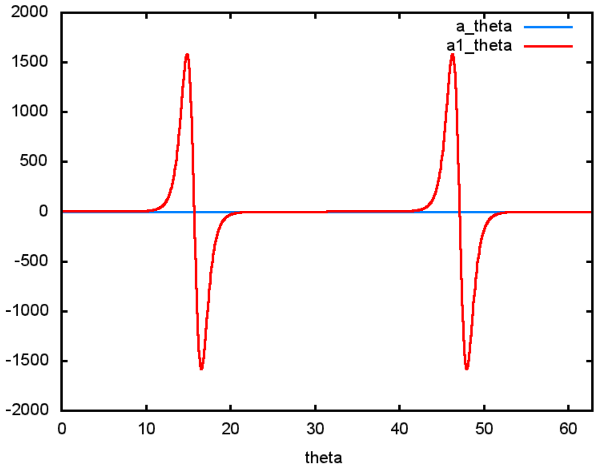

Figure 5: Angular dependence of radial accelerations.

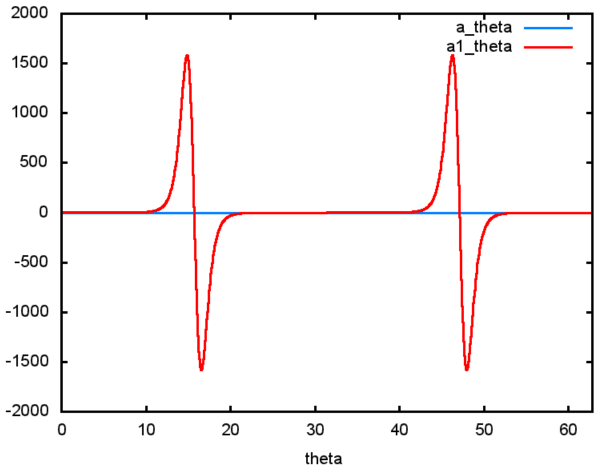

Figure 6: Angular dependence of angular accelerations.

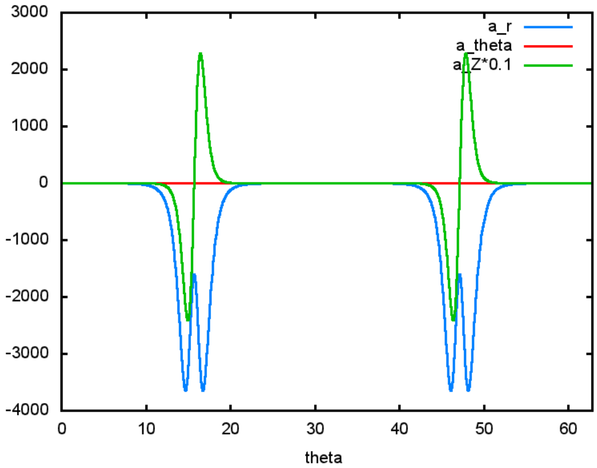

Figure 7: Angular dependence of acceleration components for full cylindrical coordinates.

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