Chapter 11

Self Consistent Derivation Of The Evans Lemma And Application To The Generally Covariant Dirac Equation

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Abstract

The self consistency of two derivations of the Evans Lemma is demonstrated rigorously and the Evans wave equation derived therefrom. The wave equation is reduced to the Dirac equation in the appropriate limit and the meaning discussed of the generally covariant Dirac and Pauli spinors. The effect of gravitation on particle physics may be investigated with the Evans equation.

Key words : Evans unified field theory, lemma and wave equation; generally covariant Dirac equation; effect of gravitation on particle physics.

11.1 Introduction

Recently a generally covariant unified field theory has been developed [1]–[14]which gives a plausible description of all radiated and matter fields in terms of the tetrad. The latter is the fundamental field of the Palatini variation of general relativity [15]– [17]. The Evans lemma [3, 4] is an identity of Cartan geometry which is the subsidiary proposition to the Evans wave equation. The latter unifies general relativity and quantum mechanics and is a wave equation of causal and objective physics [3, 4]. It has been demonstrated experimentally [11,12] beyond reasonable doubt that the Heisenberg uncertainty principle is an intellectual aberration, so should be abandoned in favor of a causal and generally covariant interpretation [1]– [14] of quantum mechanics based on Cartan geometry. The lemma and wave equation are therefore fundamentally important to a consistent interpretation of natural philosophy as an objective subject in which every event has a cause. It is therefore necessary to demonstrate the lemma rigorously in more than one way, and to demonstrate its geometrical self consistency.

In Section 11.2 the self consistency of the Lemma is demonstrated with two independent methods. In Section 11.3 the Lemma is transformed into a wave equation using the index contracted form of the Einstein field equation, and the resulting equation reduced to the single particle Dirac equation in the appropriate limit, proving that the origin of the Dirac four spinor is Cartan geometry. The effect of gravitation on the single particle Dirac equation can therefore be calculated from the generally covariant single particle Dirac equation derived directly from the Evans wave equation.

11.2 Geometrical Self Consistency Of The Evans Lemma

The lemma is an identity which is derived from the standard tetrad postulate of Cartan geometry [18]:

$$D_\mu q^a{}_\nu = 0. \tag{11.1}$$

Covariant differentiation of Eq.(11.1) gives the identity:

$$D^\mu \left(D_\mu q^a{}_\nu \right) := 0. \tag{11.2}$$

The covariant derivative is defined [18] such that:

$$D^\mu \left(\phi \right) = \partial^\mu \left(\phi \right) \tag{11.3}$$

where ϕ is a scalar. Thus for every scalar element defined in Eq.(11.1), Eq.(11.3) applies. It follows that:

$$\partial^\mu \left(D_\mu q^a{}_\nu \right) := 0. \tag{11.4}$$

The tetrad postulate is expanded out next as follows [18]:

$$D_\mu q^a{}_\lambda = \partial_\mu q^a{}_\lambda + \omega^a{}_{\mu b} q^b{}_\lambda - \Gamma^\nu{}_{\mu\lambda} q^a{}_\nu = 0 \tag{11.5}$$

where $\omega^a{}_{\mu b}$ is the spin connection and where $\Gamma^\nu{}_{\mu\lambda}$ is the gamma connection for a spacetime both with curvature and torsion. Using the inverse tetrad relation:

$$q^a{}_\mu q^\mu{}_a = 1 \tag{11.6}$$

it follows directly from Eq.(11.4) that:

$$\Box q^a{}_\mu = R q^a{}_\mu \tag{11.7}$$

where

$$R = q^\lambda{}_a \partial^\mu \left(\Gamma^\nu{}_{\mu\lambda} q^a{}_\nu - \omega^a{}_{\mu b} q^b{}_\lambda \right) \tag{11.8}$$

and where the d'Alembertian operator is defined by:

$$\Box = \partial^\mu \partial_\mu . \tag{11.9}$$

Eq.(11.7) is the lemma, or subsidiary geometrical proposition, that leads to the Evans wave equation. It is a simple identity of Cartan geometry and is the structure that leads directly to the generally covariant, causal and thus objective wave equations of physics. From the tetrad postulate (11.1):

$$\Gamma^\nu{}_{\mu\lambda} q^a{}_\nu - \omega^a{}_{\mu b} q^b{}_\lambda = \partial_\mu q^a{}_\lambda \tag{11.10}$$

and using Eq.(11.10) in Eq.(11.8) it is found self-consistently that:

$$R = q^\lambda{}_a \partial^\mu \left(\partial_\mu q^a{}_\lambda \right) = q^\lambda{}_a \Box q^a{}_\lambda \tag{11.11}$$

which leads back self consistently to Eq. (11.7) upon use of Eq.(11.6). It has therefore been shown that the most basic structure of Cartan geometry is the wave equation (11.7). This wave equation of geometry is the source of quantum mechanics in physics. The importance of the lemma is therefore clear, it indicates that all of physics is derived from Cartan geometry. Geometry is transformed into physics using:

$$R = -kT. \tag{11.12}$$

Eq.(11.12) is the most fundamental equation of relativity, and is the simplest way in which geometry can be translated into physics via the scalar energy-momentum density T and the Einstein constant k. Here R is the scalar curvature in inverse square meters. Eq.(11.12) applies to all radiated and matter fields as intended originally by Einstein himself [19]. Not only can we recover the Einstein Hilbert field equation from Eq.(11.12) but also a number of other field equations [1]– [14]. The Einstein Hilbert field equation is derived from the second Bianchi identity of Riemann geometry on the assumption [18] of the Christoffel connection which is symmetric in its lower two indices. This assumption implies that the torsion tensor is zero. Therefore the Einstein Hilbert field theory assumes that there is spacetime curvature but no spacetime torsion. In some circumstances this assumption is perfectly adequate, for example for the sun (Cassini experiments at NASA, 2002 to present), but in other circumstances it is well known that there are cosmological anomalies [20], some of them appear to be very large anomalies. So the Einstein Hilbert field equation appears to be only partially successful in a cosmological context when we take all the data into account.

In the Evans field theory on the other hand curvature and torsion are both present in general [1]– [14] and the connection is the spin connection of the Palatini variation of general relativity in which the fundamental field is the tetrad and not the symmetric metric of the EinsteinHilbert field theory. The symmetric metric is the dot product of two tetrads, as is well known [18]:

$$g_{\mu\nu} = q^a{}_\mu q^b{}_\nu \eta_{ab} \tag{11.13}$$

where η_{ab} is the Minkowski metric of the tangent spacetime. It follows immediately that there always exists an antisymmetric metric - the wedge product of two tetrads:

$$g^c{}_{\mu\nu} = q^a{}_\mu \wedge q^b{}_\nu . \tag{11.14}$$

The antisymmetric metric is a vector valued two-form of differential geometry. The most general metric is the outer product of two tetrads [1]– [14]. The outer product is a matrix, and therefore can always be written [21] as the sum of a symmetric and antisymmetric matrix. The trace of the symmetric matrix is essentially the dot product and the antisymmetric traceless part is essentially the cross product. A simple example is vector analysis in three dimensional Euclidean space. If the dot product $\mathbf{A}\cdot\mathbf{B}$ is defined of two vectors, we can always define a cross product $\mathbf{A}\times\mathbf{B}$. This rule can be generalized to n dimensional non-Euclidean geometry through the use of tetrads. The dot product is generalized to Eq.(11.13) and the cross product is generalized to Eq.(11.14).

The antisymmetric metric is missing from the Einstein Hilbert field theory of gravitation, but is a special case of the Evans field theory when the spin connection is dual to the tetrad [3, 4]. In this special case the wedge product of the spin connection and the tetrad that appears in the first Cartan structure equation:

$$T^a{}_{\mu\nu} = (d \wedge q^a)_{\mu\nu} + \omega^a{}_{\mu b} \wedge q^b{}_\nu \tag{11.15}$$

reduces to the antisymmetric metric within a factor κ with the dimensions of wavenumber. This duality condition:

$$\omega^a{}_{\mu b} = -\frac{1}{2}\kappa\epsilon^a{}_{bc} q^c{}_\mu \tag{11.16}$$

defines free space electromagnetic radiation [1]– [14] decoupled from gravitation - a special case of the general Evans unified field theory. $T^a{}_{\mu\nu}$ is the torsion form (a vector valued two-form) $d\wedge$ denotes the exterior derivative, and $\omega^a{}_{\mu b}$ denotes the spin connection of the well known Palatini variation [15]– [17] of relativity theory. Therefore the Einstein-Hilbert field theory of gravitation, although well known and well used, is severely constrained by its fundamental geometrical assumption of a Christoffel (symmetric) connection:

$$\Gamma^\kappa{}_{\mu\nu} = \Gamma^\kappa{}_{\nu\mu} . \tag{11.17}$$

This could well be the source of the well observed [20] anomalies of cosmology and the Evans field theory should be used to address these anomalies. If relativity theory is abandoned, objective physics is abandoned, leaving essentially no physics at all. The use of the Christoffel connection means that:

$$R_{\rho\sigma\mu\nu} + R_{\rho\mu\nu\sigma} + R_{\rho\nu\sigma\mu} = 0 \tag{11.18}$$

whereas more generally [1]– [14] Eq.(11.18) is the first Bianchi identity of Cartan geometry:

$$(d \wedge T^a)_{\mu\nu\sigma} + \omega^a{}_{\mu b} \wedge T^b{}_{\nu\sigma} := R^a{}_{b\mu\nu} \wedge q^b{}_\sigma . \tag{11.19}$$

It has been shown [1]– [14] that Eq.(11.19) is the same as the following identity of Riemann geometry:

$$\begin{aligned}
&\partial_\mu \Gamma^\lambda{}_{\mu\rho} - \partial_\nu \Gamma^\lambda{}_{\mu\rho} + \Gamma^\lambda{}_{\mu\sigma}\Gamma^\sigma{}_{\nu\rho} - \Gamma^\lambda{}_{\nu\sigma}\Gamma^\sigma{}_{\mu\rho} \\
&+\partial_\nu \Gamma^\lambda{}_{\rho\mu} - \partial_\rho \Gamma^\lambda{}_{\nu\mu} + \Gamma^\lambda{}_{\nu\sigma}\Gamma^\sigma{}_{\rho\mu} - \Gamma^\lambda{}_{\rho\sigma}\Gamma^\sigma{}_{\nu\mu} \\
&+\partial_\rho \Gamma^\lambda{}_{\mu\nu} - \partial_\mu \Gamma^\lambda{}_{\rho\nu} + \Gamma^\lambda{}_{\rho\sigma}\Gamma^\sigma{}_{\mu\nu} - \Gamma^\lambda{}_{\mu\sigma}\Gamma^\sigma{}_{\rho\nu} \\
&:= R^\lambda{}_{\rho\mu\nu} + R^\lambda{}_{\mu\nu\rho} + R^\lambda{}_{\nu\rho\mu}
\end{aligned} \tag{11.20}$$

using both the Riemann and torsion tensors, both being non-zero in general. In general the Riemann form of Cartan geometry is [1]– [14]:

$$R^a{}_{b\mu\nu} = q^a{}_\sigma q^\tau{}_b R^\sigma{}_{\tau\mu\nu} \tag{11.21}$$

where $R^\sigma{}_{\tau\mu\nu}$ is the Riemann tensor of Riemann geometry. The symmetries:

$$R^a{}_{b\mu\nu} = -R^a{}_{b\nu\mu} \tag{11.22}$$

$$R^\sigma{}_{\tau\mu\nu} = -R^\sigma{}_{\tau\nu\mu} \tag{11.23}$$

are always true, but in general the Riemann form and Riemann tensor are asymmetric in their first two indices. The Riemann tensor becomes antisymmetric in its first two indices if and only if Eq.(11.18) is true [18]. This is another illustration of the rather severe geometrical constraints on the Einstein Hilbert field theory. In the Evans field theory these constraints are lifted and a lot of new physics awaits exploration.

A simple example of a new field equation from Eq.(11.12) is:

$$R q^a{}_\mu = -kT q^a{}_\mu \tag{11.24}$$

which is a classical field equation closely similar to the Evans wave equation of generally covariant quantum mechanics:

$$(\Box + kT)\, q^a{}_\mu = 0 \tag{11.25}$$

obtained from Eqs.(11.7) and (11.12). Therefore the Evans lemma of geometry translates into physics using Eq.(11.12). To solve Eq.(11.25) it is possible for example to first define T and then derive the eigenfunctions $q^a{}_\mu$ if possible analytically or otherwise computationally. There are various model situations that may be used for T. One of the simplest is the single particle special relativistic limit where:

$$T \longrightarrow \frac{m}{V_0} \tag{11.26}$$

in the particle rest frame. Here m is the mass of an elementary particle and V_0 is its rest volume, a new fundamental concept introduced by the Evans field theory [1]– [14]. The correspondence principle states essentially that general relativity reduces to special relativity under well defined conditions. The wave equation of special relativistic quantum mechanics is the experimentally well tested Dirac equation:

$$\left(\Box + \frac{m^2c^2}{\hbar^2}\right) q^a{}_\mu = 0 \tag{11.27}$$

where c is the speed of light in vacuo and $\hbar$ the reduced Planck constant. It is deduced therefore that the Dirac four spinor, the wavefunction of the Dirac equation, is a tetrad. The latter is the fundamental field of the Palatini variation of general relativity and as such must remain the fundamental field in special relativity. This important conclusion is a direct consequence of the correspondence principle.

Using this argument and comparing Eqs.(11.25) and (11.27) it follows that the fundamental rest volume is defined by:

$$V_0 = \frac{\hbar^2 k}{mc^2} := \frac{\hbar^2 k}{En_0} \tag{11.28}$$

for all elementary particles, including the photon, neutrinos, gravitons and gravitinos. This is one of the major discoveries of the Evans field theory because it removes the necessity for Feynman calculus and renormalization in quantum electrodynamics and quantum chromodynamics. It also removes the unphysical infinities of classical electrodynamics, infinities which originate in the notion of point electron without volume. From Eq.(11.28) it is deduced from general relativity that there are no point particles in nature, and that every elementary particle must have mass. From this deduction it follows that the Higgs mechanism must be abandoned and that theories based on the Higgs mechanism, such as the GWS theory, must be modified. The first steps towards such a modification have been taken [1]– [14].

By deriving the Dirac equation from Cartan geometry spin has been introduced into general relativity, and this is a key step towards towards the unification of gravitational theory with electromagnetic theory and the theory of the weak and strong fields. Spin enters into consideration through the tetrad. The four fundamental fields of physics presently thought to exist: gravitation, electromagnetic, the weak and strong, are all mathematical representations of the fundamental tetrad field.

By use of the Leibnitz Theorem [1]– [14, 18] we also obtain from the tetrad postulate:

$$\left(D^\mu \partial_\mu\right) q^a{}_\lambda + \left(D^\mu \omega^a{}_{\mu b}\right) q^b{}_\lambda - \left(D^\mu \Gamma^\nu{}_{\mu\lambda}\right) q^a{}_\nu = 0. \tag{11.29}$$

Eqs.(11.4) and (11.29) may be used to cross check the derivation of the Evans Lemma as follows. In Eq.(11.29) use the results:

$$D_\mu \partial^\mu = \Box + \Gamma^\mu{}_{\mu\lambda} \partial^\lambda \tag{11.30}$$

$$D^{\mu} = g^{\mu\nu} D_{\nu} \tag{11.31}$$

$$\partial_{\mu} = g_{\mu\nu} \partial^{\nu} \tag{11.32}$$

implying that:

$$D^{\mu}\partial_{\mu} = g^{\mu\nu} D_{\nu} g_{\mu\nu} \partial^{\nu} = 4 D_{\mu} \partial^{\mu}. \tag{11.33}$$

From Eq.(11.33) in Eq.(11.29):

$$4 \left(D^{\mu}\partial_{\mu}\right) q^{a}{}_{\lambda} + \left(D^{\mu}\omega^{a}{}_{\mu b}\right) q^{b}{}_{\lambda} - \left(D^{\mu}\Gamma^{\nu}{}_{\mu\lambda}\right) q^{a}{}_{\nu} = 0. \tag{11.34}$$

Using the Leibnitz rule:

$$\left(D_{\mu}\partial^{\mu}\right) q^{a}{}_{\lambda} = D_{\mu}\left(\partial^{\mu} q^{a}{}_{\lambda}\right) + \partial^{\mu}\left(D_{\mu} q^{a}{}_{\lambda}\right), \tag{11.35}$$

therefore in Eq.(11.34)

$$4 \left(D_{\mu}\left(\partial^{\mu} q^{a}{}_{\lambda}\right) + \partial^{\mu}\left(D_{\mu} q^{a}{}_{\lambda}\right)\right) + \left(D^{\mu}\omega^{a}{}_{\mu b}\right) q^{b}{}_{\lambda} - \left(D^{\mu}\Gamma^{\nu}{}_{\mu\lambda}\right) q^{a}{}_{\nu} = 0. \tag{11.36}$$

By comparison of Eq.(11.36) and (11.4):

$$4 D_{\mu}\left(\partial^{\mu} q^{a}{}_{\lambda}\right) + D^{\mu}\omega^{a}{}_{\mu b} - \left(D^{\mu}\Gamma^{\nu}{}_{\mu\lambda}\right) q^{a}{}_{\nu} = 0 \tag{11.37}$$

i.e.

$$D^{\mu}\left(\partial_{\nu} q^{a}{}_{\lambda} + \omega^{a}{}_{\mu b} q^{b}{}_{\lambda} - \Gamma^{\nu}{}_{\mu\lambda} q^{a}{}_{\nu}\right) = 0 \tag{11.38}$$

which is

$$D^{\mu}\left(D_{\mu} q^{a}{}_{\lambda}\right) = 0. \tag{11.39}$$

Eq.(11.2) is recovered self consistently, Q.E.D., thus proving the self consistency and correctness of both derivations of the Evans Lemma.

Therefore the set of equations to solve in Cartan geometry is as follows:

$$\Box q^{a}{}_{\mu} = R q^{a}{}_{\lambda} \tag{11.40}$$

$$T^{a} = d \wedge q^{a} + \omega^{a}{}_{b} \wedge q^{b} \tag{11.41}$$

$$d \wedge T^{a} = R^{a}{}_{b} \wedge q^{b} - \omega^{a}{}_{b} \wedge T^{b} \tag{11.42}$$

$$R^{a}{}_{b} = d \wedge \omega^{a}{}_{b} + \omega^{a}{}_{c} \wedge \omega^{c}{}_{b} \tag{11.43}$$

$$D \wedge R^{a}{}_{b} = 0. \tag{11.44}$$

There are five equations, and the unknowns are R, q^{a} and $\omega^{a}{}_{b}$. Eqs.(11.41) and (11.42) give a relation between q^{a} and $\omega^{a}{}_{b}$:

$$\begin{aligned} & d \wedge \left(d \wedge q^{a} + \omega^{a}{}_{b} \wedge q^{b}\right) \\ & + \omega^{a}{}_{b} \wedge \left(d \wedge q^{b} + \omega^{b}{}_{c} \wedge q^{c}\right) \\ & = \left(d \wedge \omega^{a}{}_{b} + \omega^{a}{}_{c} \wedge \omega^{c}{}_{b}\right) \wedge q^{b}. \end{aligned} \tag{11.45}$$

Eq.(11.44) is an equation in $\omega^{a}{}_{b}$:

$$D \wedge \left(d \wedge \omega^{a}{}_{b} + \omega^{a}{}_{c} \wedge \omega^{c}{}_{b}\right) = 0. \tag{11.46}$$

If for a simple model, $\omega^a{}_b$ is found from Eq.(11.46), the second Bianchi identity, then q^a may be found from Eq.(11.45). Given this q^a, R and T may be found as eigenvalues of Eq.(11.40). In order to translate this Cartan geometry into electrodynamics we use the Evans Ansatz:

$$A^a{}_\mu = A^{(0)} q^a{}_\mu, \tag{11.47}$$

$$F^a{}_{\mu\nu} = A^{(0)} T^a{}_{\mu\nu}\,, \tag{11.48}$$

where $A^{(0)}$ is the scalar magnitude of the vector potential of electrodynamics.

11.3 Structure Of The Dirac Equation

The wave equation (11.25) becomes the generally covariant Dirac equation when the appropriate representation space is used. In n dimensional non-Euclidean geometry the tetrad is defined in general [18] by:

$$V^a = q^a{}_\mu V^\mu \tag{11.49}$$

where V^a is a vector in the tangent spacetime, and V^μ is a vector in the base manifold. These vectors are in general n dimensional. The tetrad appropriate to the Dirac equation is given by $n = 2$. Thus V^a and V^μ are two-vectors and the tetrad is a 2×2 matrix. For electrodynamics on the other hand $n = 4$, and V^a and V^μ are four-vectors. The potential field of generally covariant electrodynamics is:

$$A^a{}_\mu = A^{(0)} q^a{}_\mu. \tag{11.50}$$

The intrinsic spin of the electromagnetic field can be described by the three space-like components of $A^a{}_\mu$ and so we can restrict attention to $n = 3$ for this illustrative purpose. More generally, $A^a{}_\mu$ always has a fourth, time-like dimension which defines the scalar potential.

The three dimensional ($n = 3$) representation space can be characterized by two sets of basis vectors, each with $O(3)$ symmetry in contrast with the $SU(2)$ symmetry of the $n = 2$ representation space of the Dirac equation. The first basis of the $n = 3$ space is the Cartesian (X, Y, Z) and the second is the complex circular [1]– [14] $((1),(2),(3))$. The intrinsic spin of electrodynamics is therefore described by assigning:

$$a = (1),(2),(3), \tag{11.51}$$

$$\mu = X, Y, Z, \tag{11.52}$$

and the electromagnetic potential is, within $A^{(0)}$, the tetrad constructed from Eqs.(11.51) and (11.52), with components such as:

$$A_X{}^{(1)}, \ldots, A_Z{}^{(3)}. \tag{11.53}$$

By introduction of the electromagnetic phase factor $e^{i\phi}$, one frame spins and translates with respect to the other. This is electromagnetism - spinning spacetime. Gravitation is curving spacetime. For electromagnetism, the tangent spacetime labeled a is a static Minkowski spacetime and the base manifold labeled μ is a spacetime that spins and translates with respect to the Minkowski spacetime. For gravitation the tangent spacetime is the same, but the base manifold curves with respect to the tangent spacetime. For the unified field the base manifold, spins, translates and curves with respect to the tangent spacetime.

The experimentally observed circular polarization of electromagnetism is described by the following type of tetrad in vector notation [1]– [14]:

$$\mathbf{A}^{(1)} = \frac{A^{(0)}}{\sqrt{2}} (\mathbf{i} - i\mathbf{j})\, e^{i\phi}, \tag{11.54}$$

$$\mathbf{A}^{(2)} = \frac{A^{(0)}}{\sqrt{2}} (\mathbf{i} + i\mathbf{j})\, e^{-i\phi}, \tag{11.55}$$

together with:

$$\mathbf{A}^{(3)} = A^{(0)}\mathbf{k}. \tag{11.56}$$

Here $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are Cartesian unit vectors defined by the $O(3)$ symmetry rule:

$$\mathbf{i} \times \mathbf{j} = \mathbf{k}. \tag{11.57}$$

These are related to the unit vectors of the complex circular basis by:

$$\mathbf{e}^{(1)} = \frac{1}{\sqrt{2}} (\mathbf{i} - i\mathbf{j}) \tag{11.58}$$

$$\mathbf{e}^{(2)} = \frac{1}{\sqrt{2}} (\mathbf{i} + i\mathbf{j}) \tag{11.59}$$

$$\mathbf{e}^{(3)} = \mathbf{k}. \tag{11.60}$$

implying the $O(3)$ symmetry rule:

$$\mathbf{e}^{(1)} \times \mathbf{e}^{(2)} = i\mathbf{e}^{(3)*} \tag{11.61}$$

The tetrads in vector notation are therefore:

$$\mathbf{q}^{(1)} = \mathbf{e}^{(1)} e^{i\phi}, \tag{11.62}$$

$$\mathbf{q}^{(2)} = \mathbf{e}^{(2)} e^{-i\phi}, \tag{11.63}$$

$$\mathbf{q}^{(3)} = \mathbf{e}^{(3)}, \tag{11.64}$$

and obey the $O(3)$ symmetry rule:

$$\mathbf{q}^{(1)} \times \mathbf{q}^{(2)} = i\mathbf{q}^{(3)*}. \tag{11.65}$$

The electromagnetic phase ϕ is in general the generally covariant Evans phase [1]– [14]. Thus the tetrad can be used straightforwardly for electromagnetism as well as for gravitation. This is the key to the unified field theory.

These considerations can now be applied to the single particle Dirac equation, in which $n = 2$. As for the intrinsic spin of the electromagnetic field, $q^a{}_\mu$ in the Dirac equation represents a spinning and translating of spacetime, the helicity of the elementary fermion. The photon of the electromagnetic field is a boson. In the generally covariant Dirac equation the spin is superimposed on a curving of the base manifold (the Evans spacetime [1]– [14]), and this curving is gravitation. The generally covariant Dirac equation therefore describes the effect of gravitation on the fermion, i.e. the influence of gravitation on elementary particles and anti-particles that are fermions. The famous Dirac equation of special relativity is recovered as discussed already in this paper, i.e. in the limit defined by Eq.(11.26). As for electrodynamics the spin is introduced through a phase factor $e^{i\phi}$ and as for electrodynamics there are right and left handed helicities. For the electromagnetic potential right and left handed senses of circular polarization are defined by:

$$\mathbf{A}_R^{(1)} = \frac{A^{(0)}}{\sqrt{2}} \left(\mathbf{i} - i\mathbf{j}\right) e^{i\phi}, \tag{11.66}$$

$$\mathbf{A}_L^{(2)} = \frac{A^{(0)}}{\sqrt{2}} \left(\mathbf{i} + i\mathbf{j}\right) e^{i\phi}. \tag{11.67}$$

For the same phase factor $e^{i\phi}$, the right basis vector is $\left(\mathbf{i} - i\mathbf{j}\right)/\sqrt{2}$ and the left basis vector is $\left(\mathbf{i} + i\mathbf{j}\right)/\sqrt{2}$. These are complex conjugate basis vectors for the same phase factor $e^{i\phi}$, and therefore for a given phase factor a right left basis may be defined being the complex conjugate basis defined by $\left(\mathbf{i} - i\mathbf{j}\right)/\sqrt{2}$ and $\left(\mathbf{i} + i\mathbf{j}\right)/\sqrt{2}$

It follows that such a basis can also be defined for the Dirac equation, and that there exists a two-dimensional column vector in the tangent spacetime with complex conjugate components:

$$V^a = \begin{bmatrix} V^R \\ V^L \end{bmatrix}. \tag{11.68}$$

In the base manifold there exists a column vector:

$$V^\mu = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \tag{11.69}$$

which is spinning and translating with respect to V^a. The column vectors are linked by:

$$\begin{bmatrix} V^R \\ V^L \end{bmatrix} = \begin{bmatrix} q^R{}_1 & q^R{}_2 \\ q^L{}_1 & q^L{}_2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \tag{11.70}$$

where the tetrad is:

$$q^a{}_\mu = \begin{bmatrix} q^R{}_1 & q^R{}_2 \\ q^L{}_1 & q^L{}_2 \end{bmatrix} \tag{11.71}$$

This is a matrix consisting of a row vector $\begin{bmatrix} q^R{}_1 & q^R{}_2 \end{bmatrix}$ superimposed on a row vector $\begin{bmatrix} q^L{}_1 & q^L{}_2 \end{bmatrix}$. Transposition of the two row vectors gives the left and right

Pauli spinors in the limit defined by Eq.(11.26):

$$\xi^R = \begin{bmatrix} q^R{}_1 \\ q^R{}_2 \end{bmatrix}, \xi^L = \begin{bmatrix} q^L{}_1 \\ q^L{}_2 \end{bmatrix} \tag{11.72}$$

and the Dirac spinor is:

$$\psi = \begin{bmatrix} \xi^R \\ \xi^L \end{bmatrix}. \tag{11.73}$$

Since:

$$\left(\Box + \frac{m^2c^2}{\hbar^2}\right) \begin{bmatrix} q^R{}_1 & q^R{}_2 \\ q^L{}_1 & q^L{}_2 \end{bmatrix} = 0 \tag{11.74}$$

it follows that:

$$\left(\Box + \frac{m^2c^2}{\hbar^2}\right) \begin{bmatrix} q^R{}_1 \\ q^R{}_2 \\ q^L{}_1 \\ q^L{}_2 \end{bmatrix} = 0 \tag{11.75}$$

and this is the single particle Dirac equation. This equation is generalized in the Evans field theory to:

$$(\Box + kT)\,\psi = 0 \tag{11.76}$$

The whole of Dirac algebra may be recovered from Cartan geometry as exemplified by:

$$\overline{\psi}\psi = \xi^{L+}\xi^R + \xi^{R+}\xi^L \tag{11.77}$$

where $\overline{\psi}$ is the adjoint of the Dirac spinor ψ. In order to investigate the effect of gravitation on the single particle Dirac equation a model is chosen for T and Eq.(11.76) solved for the eigenfunctions, which are components of the Dirac spinor. The Dirac equation in general relativity is a matter field equation and reduces in the non-relativistic limit to the Schrodinger equation and in the classical limit to the Newton equation of motion. Through the Poisson equation we recover simultaneously the Newton inverse square law in the appropriate weak field and non-relativistic limit. This shows why gravitational and inertial acceleration are identical, both are derived from Cartan geometry.

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