

# ANALYTICAL CALCULATION OF PRECESSION FROM ECE2 RELATIVITY.

by

M .W. Evans and H. Eckardt

Civil List and AIAS / UPITEC

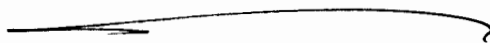
([www.aias.us](http://www.aias.us), [www.upitec.org](http://www.upitec.org), [www.archive.org](http://www.archive.org), [www.webarchive.org](http://www.webarchive.org), , [www.et3m.net](http://www.et3m.net))

## ABSTRACT

An analytical method is developed to show that the hamiltonian of ECE2 relativity produces a differential orbital function whose difference from the non relativistic theory can be calculated directly. The differential function can be compared directly with experimental data. By a comparison of UFT363 and UFT372 it is shown that the methods of fluid dynamics also produce a precessing orbit.

Keywords: ECE2 relativity and fluid dynamics, orbital precession.

UFT 373



## 1. INTRODUCTION

In the immediately preceding paper of this series {1 - 12} (UFT372), a numerical method was used to solve the lagrangian of ECE2 relativity to give a precessing orbit. This important result shows that the incorrect Einsteinian general relativity (EGR) is also redundant by Ockham's Razor, in that a simpler theory can produce precession. The result of UFT371 also confirms the method used in UFT328, simultaneous numerical solution of the lagrangian and hamiltonian. The theory of ECE2 fluid dynamics also produces a precessing orbit as shown in this paper by comparison with UFT372. The main result of Section 2 is a differential orbital function which can be calculated analytically from ECE2 relativity and compared with the same function from the non relativistic theory of planar orbits. The differential function can also be observed experimentally. The difference is known from UFT372 to be due to a precessing orbit, which can therefore be calculated analytically.

This paper is a synopsis of extensive calculations in the notes accompanying UFT373 on [www.aias.us](http://www.aias.us). Note 373(1) is a comparison of the orbital precession produced by ECE2 fluid dynamics (UFT363) and the ECE2 lagrangian (UFT372). It is important and significant that both theories produce precession of a planar orbit. Notes 373(2) to 373(5) are preparatory attempts at an analytical solution. Note 373(6) calculates an orbital differential function by simultaneous solution of the relativistic and non relativistic hamiltonians, and Note 373(7) calculates the same differential function from experimental data at the perihelion, so a comparison with theory and experiment is possible.

## 2. ANALYTICAL CALCULATION OF PRECESSION:

Consider the non relativistic orbital hamiltonian:

$$H = \frac{1}{2} m v^2 + U \quad - (1)$$

of an object of mass  $m$  orbiting a mass  $M$  with orbital velocity  $v$ . The gravitational potential energy is well known to be:

$$U = -\frac{mMg}{r} \quad - (2)$$

where  $G$  is Newton's constant and  $r$  the distance between  $m$  and  $M$ . The relativistic hamiltonian of ECE2 theory is {1 - 12}:

$$H_0 := H_1 - mc^2 = (\gamma - 1) mc^2 + U \quad - (3)$$

where the Lorentz factor is:

$$\gamma = \left( 1 - \frac{v^2}{c^2} \right)^{-1/2} \quad - (4)$$

The velocity appearing in the Lorentz factor is {1 - 12}:

$$v^2 = \left( \frac{L}{mr} \right)^2 \left( 1 + \frac{1}{r^2} \left( \frac{dr}{d\phi} \right)^2 \right) \quad - (5)$$

where  $L$  is the angular momentum of the system, a constant of motion defined by:

$$L = mr^2 \dot{\phi} \quad - (6)$$

where the angular velocity is:

$$\dot{\phi} = \frac{d\phi}{dt} \quad - (7)$$

in the plane polar coordinate system  $(r, \phi)$ .

Using Eqs. ( 1 ) and ( 3 ) the differential orbital function  $dr/d\phi$

can be calculated in terms of the constants of motion  $H_0 - H$  and  $L$ . This calculation is

carried out using computer algebra in Section 3. The non relativistic hamiltonian is given by:

$$H = -\frac{mMG}{2a} \quad - (8)$$

where  $a$  is the semi major axis of the non relativistic orbit:

$$r = \frac{d}{1 + \epsilon \cos \phi}, \quad a = \frac{d}{1 - \epsilon^2}, \quad - (9)$$

where  $d$  is the half right latitude and  $\epsilon$  the eccentricity. The relation between the non relativistic  $L$  and  $d$  is as follows:

$$L^2 = m^2 M b d. \quad - (10)$$

In the non relativistic limit:

$$v \ll c \quad - (11)$$

the differential orbital function reduces to:

$$\left(\frac{dr}{d\phi}\right)^2 = \frac{\epsilon^2 r^4 \sin^2 \phi}{L^2} = \frac{\epsilon^2 r^4}{L^2} \left(1 - \frac{1}{\epsilon^2} \left(\frac{dr}{r d\phi} - 1\right)^2\right) \quad - (12)$$

The numerical lagrangian analysis of UFT372 shows that the function  $dr/d\phi$  from Eqs. ( 1 ) and ( 3 ) is due to orbital precession. This is a major discovery that makes Einsteinian general relativity obsolete.

By astronomical observation it is claimed that the perihelion advance after  $2\pi$  radians is:

$$\phi = 2\pi \left(1 + \frac{3MG}{dc^2}\right). \quad - (13)$$

From the elliptical orbit ( 9 ), the orbital function is:

$$\frac{dr}{d\phi} = \frac{\epsilon r^2}{d} \sin \phi \quad - (14)$$

and it follows that:

$$r = \frac{d}{1 + \epsilon \cos \phi} \quad - (15)$$

The perihelion, or distance of closest approach of M to m is defined by:

$$\phi = 2\pi \quad - (16)$$

because M is situated at one focus of the ellipse. Under the condition ( 16 ):

$$r_{\min} = \frac{d}{1 + \epsilon} \quad - (17)$$

and therefore at the perihelion:

$$\frac{dr}{d\phi} = 0 \quad - (18)$$

for the static elliptical orbit ( 15 ) of the non relativistic theory.

However, by observation, the perihelion advances every orbit by:

$$\phi = 2\pi \left( 1 + \frac{3MG}{dc^2} \right) \quad - (19)$$

so using this value of  $\phi$  in Eq. ( 14 ) produces:

$$\frac{dr}{d\phi} = \frac{\epsilon d}{(1 + \epsilon)^2} \sin \left( 2\pi \left( 1 + \frac{3MG}{dc^2} \right) \right) \quad - (20)$$

Now use:

$$\sin(A+B) = \sin A \cos B + \cos A \sin B - (21)$$

to find that:

$$\sin\left(2\pi\left(1 + \frac{3MG}{dc^2}\right)\right) = \sin\left(\frac{6\pi MG}{dc^2}\right) \sim \frac{6\pi MG}{dc^2} - (22)$$

using the small angle approximation:

$$\sin x \sim x. - (23)$$

Therefore the precession of the perihelion produces the change:

$$\Delta\left(\frac{dr}{d\phi}\right) = \frac{6\pi MG}{c^2} \frac{\epsilon}{(1+\epsilon)^2} - (24)$$

in the differential orbital function  $dr/d\phi$ . Using:

$$r_0 = \frac{2MG}{c^2} = 2,950 \text{ metres} - (25)$$

for the mass M of the sun, and using the Earth's eccentricity:

$$\epsilon = 0.0167 - (26)$$

it is found that the experimental change in  $dr/d\phi$  is:

$$\Delta\left(\frac{dr}{d\phi}\right) = 299.46 \text{ metres} - (27)$$

and this is produced analytically by ECE2 relativity. The change in the orbital differential function can be calculated as in Section 3. The experimental value ( 27 ) is found by adjusting the relativistic hamiltonian  $H_0$ .

(UFT363) and also from the lagrangian theory of UFT372. These are important confirmations of both theories, because the astronomically observed orbit is a precessing orbit. From fluid dynamics:

$$\ddot{r} = \frac{r}{(1 + \Omega'_{01})^2} \dot{\phi}^2 - \frac{mG}{(1 + \Omega'_{01})^2} \frac{1}{r^2} \quad - (28)$$

where  $\Omega'_{01}$  is a spin connection, and from the lagrangian theory:

$$\ddot{r} = r \dot{\phi}^2 - \frac{mG}{r^2} \left( \frac{1}{\gamma} \left( 1 - \frac{\dot{r}^2}{c^2} \right) \right) \quad - (29)$$

These two expressions can be compared as in Note 373(1). In the limit:

$$\Omega'_{01} \ll 1 \quad - (30)$$

Eq. ( 28 ) reduces to:

$$\ddot{r} \sim r \dot{\phi}^2 - \frac{mG}{(1 + \Omega'_{01})^2} \frac{1}{r^2} \quad - (31)$$

so:

$$\frac{1}{(1 + \Omega'_{01})^2} \sim \frac{1}{\gamma} \left( 1 - \frac{\dot{r}^2}{c^2} \right) \quad - (32)$$

and:

$$\Omega'_{01} \sim \left( \left( 1 - \frac{v^2}{c^2} \right) \left( 1 - \frac{\dot{r}^2}{c^2} \right) \right)^{-1/2} - 1 \quad - (33)$$

For small precessions as in the solar system, the experimental precession can be modelled by:

$$r = \frac{d}{1 + \epsilon \cos(x\phi)}, \quad - (34)$$

$$x = 1 + \frac{3mG}{dc^2} \quad - (35)$$

in the first approximation. Note carefully that Eq. ( 34 ) is not the true orbit. The latter must be calculated numerically and analytically. In Eq. ( 34 ):

$$\sin^2(x\phi) + \cos^2(x\phi) = 1. \quad - (36)$$

Therefore the experimental differential orbital function is modelled to be:

$$\frac{dr}{d\phi} = \frac{x \epsilon r^2}{d} \sin(x\phi). \quad - (37)$$

This result can be reproduced theoretically by finding  $\dot{r}$  and  $\dot{\phi}$  numerically as in UFT372, so:

$$\frac{dr}{d\phi} = \frac{dr}{dt} \frac{dt}{d\phi} = \frac{\dot{r}}{\dot{\phi}}. \quad - (38)$$

From the fluid gravitational theory, Eq. ( 28 ), Eq. ( 37 ) is found by adjusting the spin connection to the experimental result. From the lagrangian theory ( 29 ) is found directly by expressing  $\dot{r}$  in terms of  $\dot{\phi}$  :

$$r\dot{\phi} = A - B\dot{r}^2 \quad - (39)$$

where:

$$A = \ddot{r} + \frac{mG}{\gamma r^2} \quad - (40)$$

and:

$$B = \frac{mG}{\gamma r^2 c^2}. \quad - (41)$$

So

$$\left( \frac{\dot{\phi}}{\dot{r}} \right)^2 = \frac{1}{r} \left( \frac{A}{\dot{r}^2} - B \right) = \left( \frac{d}{x \epsilon r^2 \sin(x\phi)} \right)^2. \quad - (42)$$



The experimentally observed  $x$  is therefore given by:

$$\left(\frac{d}{x\epsilon}\right)^2 = r^3 \left(\frac{A}{r^2} - B\right) \left(1 - \frac{1}{\epsilon^2} \left(\frac{d}{r} - 1\right)^2\right) - (43)$$

and an exact match obtained between experiment and theory.

As shown in Note 373(1), the kinetic energy used in the non relativistic UFT363 is:

$$T = \frac{1}{2} m \left( \left(1 + \Omega'_{01}\right)^2 \dot{r}^2 + \dot{\phi}^2 r^2 \right) - (44)$$

so the differential function found by comparing Eqs. ( 1 ) and ( 3 ) can be expressed in terms of the spin connection. This is carried out with computer algebra in Section 3.

In conclusion, the lagrangian and relevant Euler Lagrange equations of ECE2 relativity produce a precessing orbit, and the hamiltonian analysis of this Section develops and confirms the result of UFT372.

### 3. COMPUTER ALGEBRA AND GRAPHICAL RESULTS

Section by Dr. Horst Eckardt

# Analytical calculation of precession from the ECE2 relativity

M. W. Evans\*<sup>‡</sup>, H. Eckardt<sup>†</sup>  
Civil List, A.I.A.S. and UPITEC

([www.webarchive.org.uk](http://www.webarchive.org.uk), [www.aias.us](http://www.aias.us),  
[www.atomicprecision.com](http://www.atomicprecision.com), [www.upitec.org](http://www.upitec.org))

## 3 Computer algebra and graphical results

We present some methods of showing the relativistic shifts of the orbital ellipse. First we want to get an impression on the Newtonian orbit  $r(\phi)$  and the orbital derivatives  $dr/d\phi$  and  $(dr/d\phi)^2$ , see Fig. 1.  $r(\phi)$  oscillates between perihelion and aphelion, both derivative functions have zeros at these positions. A precession means shift of these zero crossings. For the subsequent calculations we have to express the major axis  $a$ , the angular momentum  $L$  and the non-relativistic Hamiltonian  $H$  in terms of orbital parameters:

$$a = \frac{\alpha}{1 - \epsilon^2}, \quad (45)$$

$$L = m\sqrt{\alpha MG}, \quad (46)$$

$$H = -\frac{mMG}{2a}. \quad (47)$$

In note 373(5) The function  $(dr/d\phi)^2$  was separated from the Hamiltonian of Newtonian and relativistic theory. Computer algebra gives for this function from Newtonian theory:

$$\left(\frac{dr}{d\phi}\right)_N^2 = \frac{\alpha^2 \epsilon^2 \sin^2(\phi)}{(\epsilon \cos(\phi) + 1)^4}, \quad (48)$$

from the relativistic theory (with precessing orbit):

$$\begin{aligned} \left(\frac{dr}{d\phi}\right)_{rel}^2 &= \frac{\alpha^2 \epsilon^2 \sin^2(\phi)}{(\epsilon \cos(\phi) + 1)^4} \\ &\quad - \frac{3}{mc^2} \left( \frac{GM (\epsilon \cos(\phi) + 1)}{\alpha} - \frac{GM (1 - \epsilon^2)}{2\alpha} \right)^2. \end{aligned} \quad (49)$$

---

\*email: [emyrone@aol.com](mailto:emyrone@aol.com)

†email: [mail@horst-eckardt.de](mailto:mail@horst-eckardt.de)

There is an additional term subtracted from  $(dr/d\phi)^2$  of the Newtonian orbit to obtain the function for the precessing orbit. Both functions and their difference are graphed in Fig. 2. Both functions should be positive because they are squares but the relativistic function shows up negative values. There is a region of imaginary values for  $dr/d\phi$  near to  $\phi = 0$  and  $\phi = 2\pi$ . This seems not to be very satisfactory but the function for precession crosses zero at other values than for the Newtonian orbit. In so far the effect of precession is visible.

Instead of computing  $(dr/d\phi)^2$  in dependence of  $\phi$ , Eqs. (48-49) can be re-arranged to obtain the dependence of radius  $r$ :

$$\left(\frac{dr}{d\phi}\right)_N^2 = \frac{(\epsilon^2 - 1)r^4 + 2\alpha r^3}{\alpha^2} - r^2, \quad (50)$$

$$\left(\frac{dr}{d\phi}\right)_{rel}^2 = \frac{c^2}{GM\alpha} \left( 1 - \frac{1}{\left(\frac{1}{mc^2} \left(\frac{GMm}{r} - \frac{GM(1-\epsilon^2)m}{2\alpha}\right) + 1\right)^2} \right) r^4 - r^2. \quad (51)$$

Then the results of Fig. 3 are obtained. It can be seen that both the perihelion and aphelion (represented by zero crossings) are shifted by relativistic effects. The strength of these effects is modeled in Fig. 4 by using different values of  $c$ . It can clearly be seen that the deviation from the Newtonian orbit is increased for smaller  $c$ , i.e. stronger relativistic effects.

In note 373(6) the difference between the Hamiltonians (1) and (3),  $H_0 - H$ , has been investigated. From the result an expression for  $(dr/d\phi)^2$  can be computed. For this, the equation has to be resolved for  $v^2$  first. This gives a highly complicated equation with four solutions. Two solutions are complex, one is  $v = c$  and the fourth is real-valued. We used the fourth solution and inserted Eq. (5). Then a highly complicated expression for  $(dr/d\phi)^2$  follows. Unfortunately it is complex. The real and imaginary part are plotted in Fig. 5. The result depends on the choice of constant  $H_0$ . With  $H = -3.75$  and  $H_0 = -3.70$  the real part in Fig. 5 starts to become positive at the minimum radius of the ellipse. In so far this end behaves correctly, there is no zero crossing of  $(dr/d\phi)^2$  at the other end ( $r \approx 2$ ). The impact of  $H_0$  on the real part of the solution is graphed in Fig. 6 for three different values of  $H_0$  with  $H = -3.75$  each. Increase of  $H_0$  leads to a shift of zero crossing to lower radii.

Alternatively we did the following: We resolved both the Newtonian and relativistic Hamiltonian (1) and (3) separately according to  $v^2$ . From  $H$  follows the non-relativistic form, Eq. (5). Equating both solutions for  $v^2$  gives an equation for  $(dr/d\phi)^2$ , containing  $H_0$  as a parameter. The calculation has the benefit of not leading to complex-valued results (although the formula is complicated). The result is plotted in Fig. 7. The positive part now is on the left hand side of the zero crossing, i.e. the relativistic effect of the aphelion is modeled. The radial range is shifted to higher radii by the relativistic effects ( $H_0 > H$ ) but the result is less sensitive than in Fig. 6.

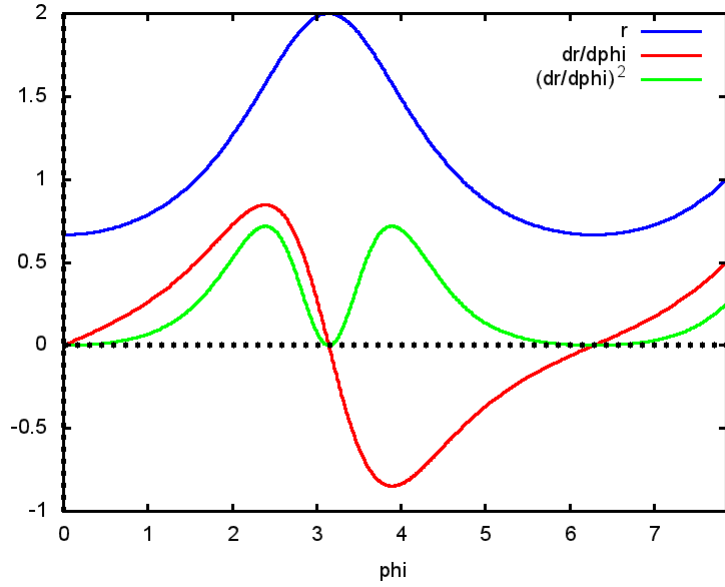


Figure 1:  $r(\phi)$ ,  $dr/d\phi$  and  $(dr/d\phi)^2$  for a Newtonian elliptic orbit.

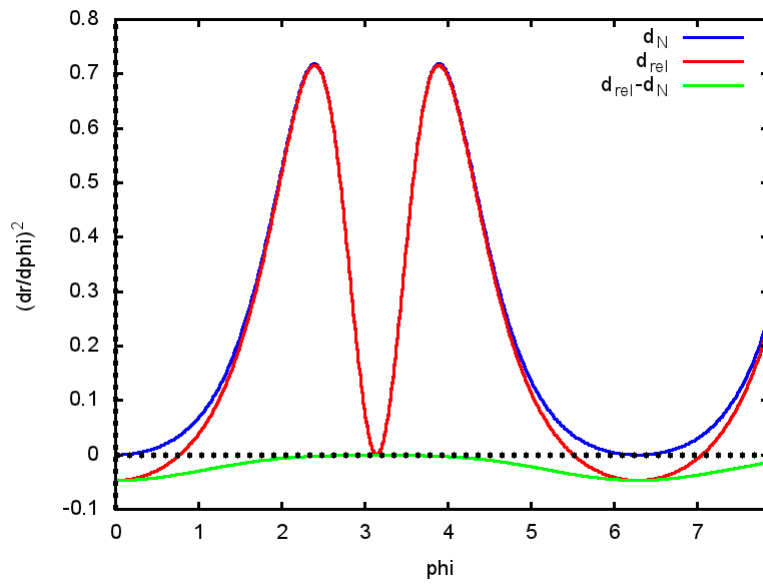


Figure 2: Angular dependence of  $(dr/d\phi)^2$  from note 373(5).

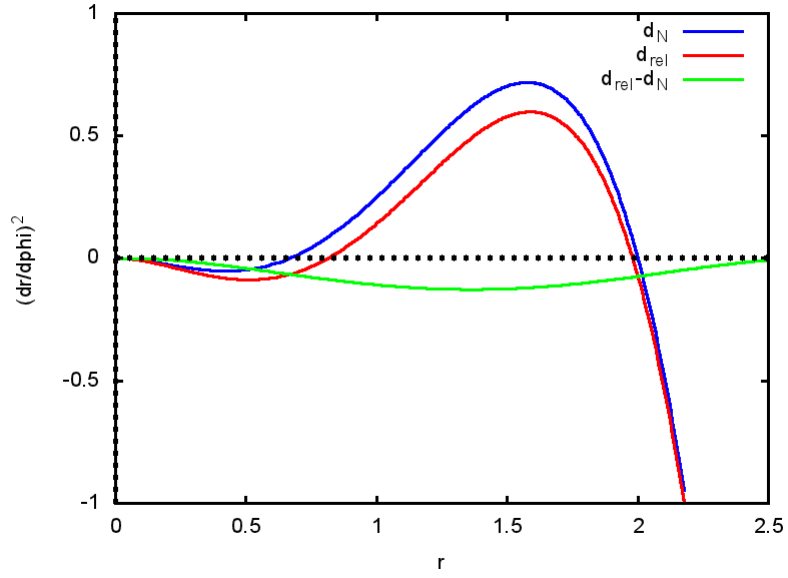


Figure 3: Radial dependence of  $(dr/d\phi)^2$  from note 373(5).

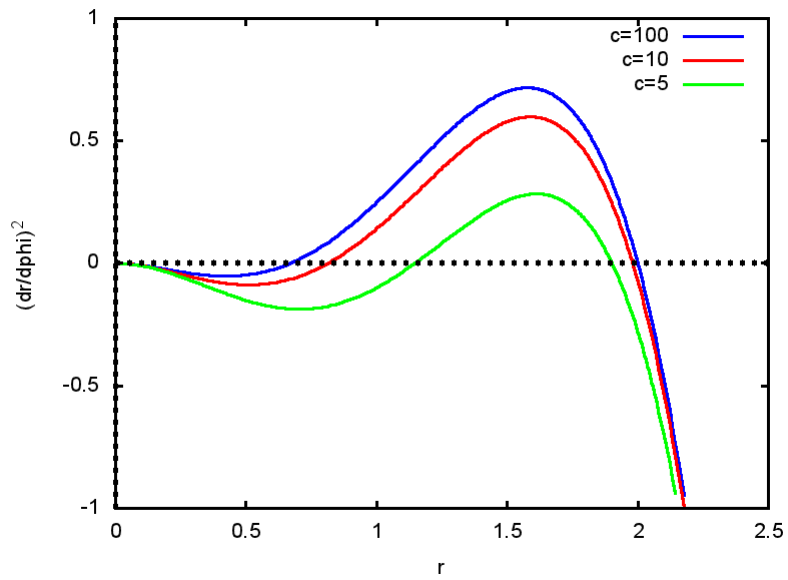


Figure 4: Strength of relativistic effects in Fig. 3, described by varying  $c$ .

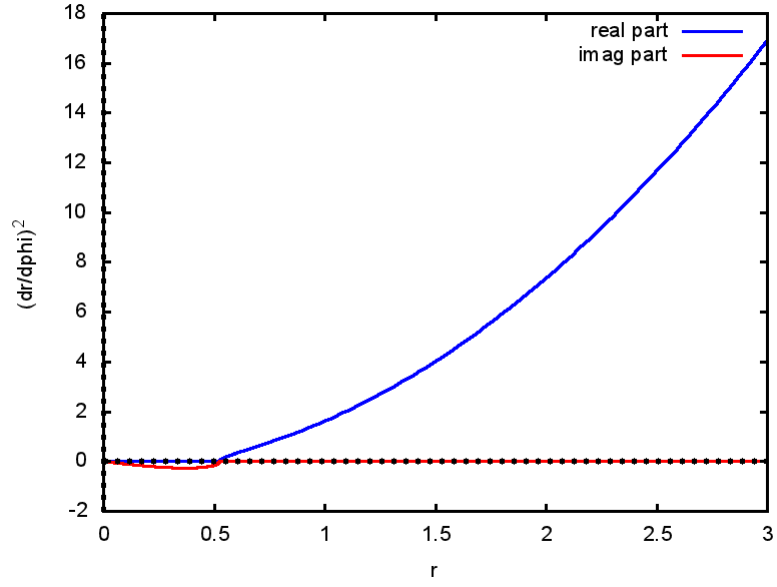


Figure 5: Real and imaginary part of  $(dr/d\phi)^2$  from note 373(6).

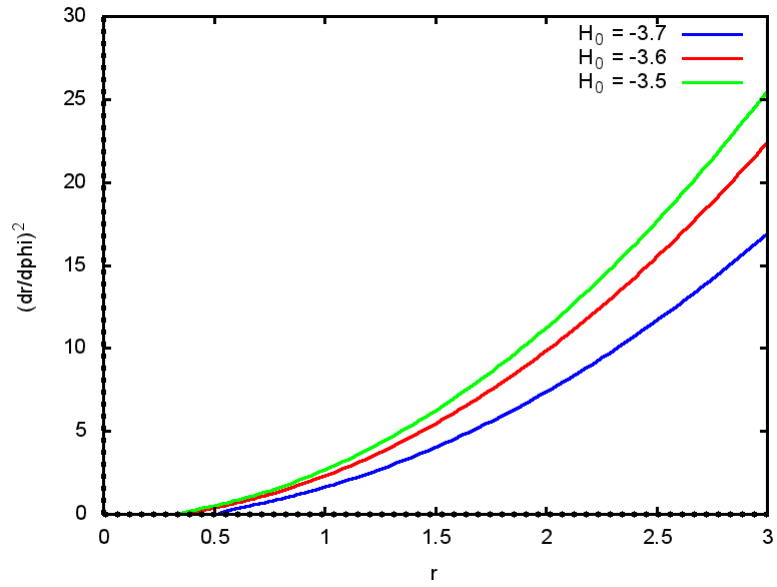


Figure 6: Strength of relativistic effects in Fig. 5, described by varying  $H_0$ .

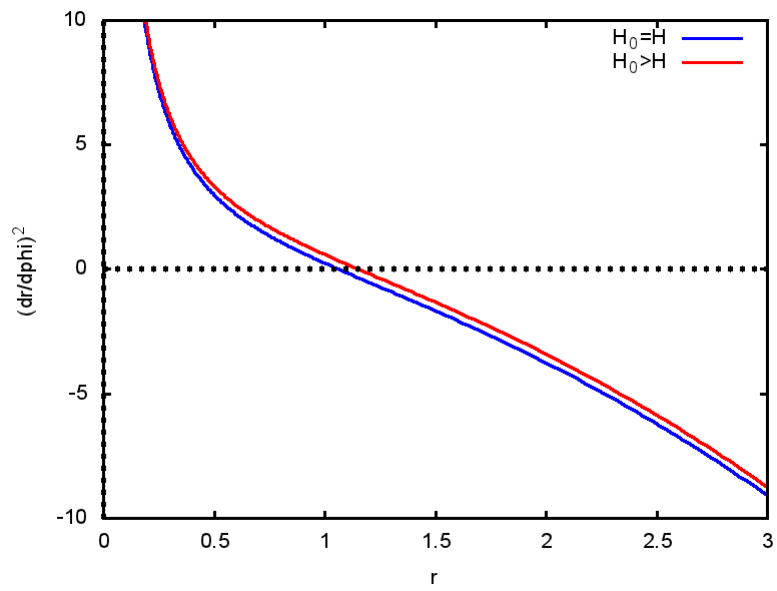


Figure 7:  $(dr/d\phi)^2$  from equating the Newtonian and relativistic velocity terms.

## ACKNOWLEDGMENTS

The British Government is thanked for a Civil List Pension and the staff of AIAS and others for many interesting discussions. Dave Burleigh, CEO of Annexa Inc., is thanked for hosting [www.aias.us](http://www.aias.us), site maintenance and feedback software and hardware maintenance. Alex Hill is thanked for translation and broadcasting, and Robert Cheshire for broadcasting.

## REFERENCES

- {1} M. W. Evans, H. Eckardt, D. W. Lindstrom and S. J. Crothers, "ECE2 : The Second Paradigm Shift" (open access on combined sites [www.aias.us](http://www.aias.us) and [www.upitec.com](http://www.upitec.com) as UFT366 and ePubli in prep., translation by Alex Hill)
- {2} M. W. Evans, H. Eckardt, D. W. Lindstrom and S. J. Crothers, "The Principles of ECE" (open access as UFT350 and Spanish section, ePubli, Berlin 2016, hardback, New Generation, London, softback, translation by Alex Hill, Spanish section).
- {3} M. W. Evans, S. J. Crothers, H. Eckardt and K. Pendergast. "Criticisms of the Einstein Field Equation" (open access as UFT301, Cambridge International, 2010).
- {4} M. W. Evans, H. Eckardt and D. W. Lindstrom, "Generally Covariant Unified Field Theory" (Abramis 2005 - 2011, in seven volumes softback, open access in relevant UFT papers, combined sites).
- {5} L. Felker, "The Evans Equations of Unified Field Theory" (Abramis 2007, open access as UFT302, Spanish translation by Alex Hill).
- {6} H. Eckardt, "The ECE Engineering Model" (Open access as UFT303, collected equations).
- {7} M. W. Evans, "Collected Scientometrics (Open access as UFT307, New Generation 2015).



{8} M. W. Evans and L. B Crowell, "Classical and Quantum Electrodynamics and the B(3) Field" (World Scientific 2001, Open Access Omnia Opera Section of [www.aias.us](http://www.aias.us)).

{9} M. W. Evans and S. Kielich (eds.), "Modern Nonlinear Optics" (Wiley Interscience, New York, 1992, 1993, 1997, 2001) in two editions and six volumes.

{10} M. W. Evans and J. - P. Vigi er, "The Enigmatic Photon", (Kluwer, 1994 to 2002, in five volumes hardback and softback, open access Omnia Opera Section of [www.aias.us](http://www.aias.us) ).

{11} M. W. Evans, Ed., "Definitive Refutations of the Einsteinian General Relativity" (Cambridge International 2012, open access on combined sites).

{12} M .W. Evans and A. A. Hasanein, "The Photomagnetron in Quantum Field Theory" (World Scientific, 1994).