FORWARD AND RETROGRADE PRECESSION FROM THE ECE2 LAGRANGIAN.

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ABSTRACT

It is shown that the ECE2 covariant lagrangian gives both forward and retrograde precessions, whereas Einsteinian general relativity (EGR) produces only forward precessions. The relevant force equation is the relativistic Newton force equation. This is combined with the ECE2 covariant gravitational field equations for gravitostatics to give a precisely self consistent theory and to define the relevant spin connections.

Keywords: ECE2 unified field theory, forward and retrograde precessions, spin connections.

UFT 377
1. INTRODUCTION

In recent papers of this series {1 - 12}, the theory of precession has been developed with various force equations and lagrangians. In this paper it is shown that the ECE2 covariant lagrangian can give both forward and retrograde precession, depending on how it is solved. The retrograde precession is given by the ECE2 lagrangian corresponding to the relativistic Newton force law and a vector Euler Lagrange equation, and the forward precession by the use of the same lagrangian and two scalar Euler Lagrange equations. The solution is combined with the ECE2 field equations to calculate the relativistic spin connections uniquely. The spin connections are determined completely by the orbit.

This paper is a short synopsis of detailed calculations in the background notes posted with UFT377 on combined sites www.aias.us and www.upitec.org. In note 377(1), the orbital equation is defined using the relativistic Newton law. In Note 377(2) the spin connection vector is introduced and retrograde precession discussed. Note 377(3) gives a summary of calculations. Notes 377(4) to 377(7) show that the forward and retrograde precessions can be the same if and only if the orbit is the Newtonian ellipse and if and only if the precessions vanish. Note 377(8) uses the gravitostatic limit of the field equations of ECE2 together with the force equation to define the relevant spin connections uniquely.

2. FORWARD AND RETROGRADE PRECESSION FROM THE SAME LAGRANGIAN

Consider the ECE2 lagrangian in the Cartesian format {1 - 12}:

$$L = -mc^2 \left(1 - \frac{x^2 + y^2}{c^2} \right)^{1/2} + \frac{\mu M G}{(x^2 + y^2)^{1/2}} \cdot - (1)$$

The Lorentz factor is:
\[
\chi = \left(1 - \frac{x^2 + y^2}{c^2}\right)^{-1/2} - (2)
\]

in which:
\[
L^2 = x^2 + y^2 - (3)
\]

The potential energy is:
\[
V = -\frac{mMg}{(x^2 + y^2)^{1/2}} - (4)
\]

This lagrangian is a description of a mass m orbiting a mass M in a plane, a distance r apart.

The proper Lagrange variables are X and Y, and there are two Euler Lagrange equations:
\[
\frac{\partial L}{\partial X} = \frac{d}{dt} \frac{\partial L}{\partial \dot{X}} - (5)
\]

and
\[
\frac{\partial L}{\partial Y} = \frac{d}{dt} \frac{\partial L}{\partial \dot{Y}} - (6)
\]

They are developed with computer algebra as in Section 3 to give:
\[
\ddot{X} = \frac{mL}{\gamma (x^2 + y^2)^{3/2}} \left(\frac{x\ddot{Y}Y + xx^2}{c^2} - X\right) - (7)
\]

and
\[
\ddot{Y} = \frac{mL}{\gamma (x^2 + y^2)^{3/2}} \left(\frac{y\ddot{X}X + yy^2}{c^2} - Y\right) - (8)
\]

These equations are integrated by computer algebra as discussed in Section 3. They give an
orbit in which forward precession occurs in a plane. This is precession in the same direction as the motion of m around M. For an initial condition of:

\[ v(0) = 7.7529 \times 10^6 \text{ m s}^{-1} \]  

the precession is

\[ \Delta \phi = 5.903 \times 10^{-4} \text{ rad} \]  

Now consider the same lagrangian \( \frac{1}{2} \) written as:

\[ L = -mc^2 \left( 1 - \frac{\vec{\dot{r}} \cdot \vec{\dot{r}}}{c^2} \right)^{1/2} + nM\delta \]  

where

\[ \vec{r} = X_i + Y_j \]  

The proper Lagrange variable is \( r \) and the Euler Lagrange equation is:

\[ \frac{dL}{dt} \frac{\partial}{\partial \dot{r}} = \frac{d}{dt} \frac{\partial}{\partial \dot{r}} \]  

in which the relativistic momentum is:

\[ p = \frac{d\vec{r}}{dt} = Y \frac{ds}{dt} = YmV_o = \frac{dL}{d\dot{r}} \]  

where \( v \) is the Newtonian or non-relativistic velocity:

\[ v_o = \dot{r} = \frac{ds}{dt} \]  

It can be shown as in Note 377(1) that:

\[ \frac{dp}{dt} = \frac{d}{dt} \frac{\partial L}{\partial \dot{r}} = \chi^3 m \dddot{r} \]  

Furthermore:
\[ \frac{dL}{dt} = -mMg \frac{5}{r^3} \quad -(17) \]

so the Euler Lagrange equation (13) becomes:
\[ F = \gamma^3 m \frac{\ddot{r}}{r^3} = -mMg \frac{5}{r^3} \quad -(18) \]

This is the orbital equation with the relativistic Newtonian force:
\[ F = \gamma^3 m \ddot{r} \quad -(19) \]

As shown in Note 377(1) this force is consistent with the Einstein energy equation:
\[ E = \gamma mc^2 \quad -(20) \]

The Cartesian component equations of Eq. (18) are:
\[ \dot{x} = -m \frac{\dot{y}}{\gamma^3 (x^2 + y^2)^{3/2}} x \quad -(21) \]

and
\[ \dot{y} = -m \frac{\dot{x}}{\gamma^3 (x^2 + y^2)^{3/2}} y \quad -(22) \]

These are integrated by computer algebra in Section 3 and give a negative or retrograde precession of:
\[ \Delta \phi = -1.7697 \times 10^{-3} \text{ radians} \quad -(23) \]

for an initial condition of the S2 star system \{1 - 12\} of:
\[ v(0) = 7.7529 \times 10^6 \text{ m/s}^{-1} \quad -(24) \]
The experimentally observed precession for the S2 star system is between 

\[ -0.017 \text{ and } 0.035 \ \text{ radians.} \]

The theoretical results are:

\[ \Delta \phi = 5.903 \times 10^{-4} \ \text{rad}, \ - (25) \]

\[ \Delta \phi = -1.7697 \times 10^{-3} \ \text{rad} \ - (26) \]

from Eqs. (7) and (8), and from Eq. (13) respectively. Therefore the theoretical results are in the middle of the experimental range. Einsteinian general relativity (EGR) can give only forward precession, so ECE2 relativity is preferred to EGR in yet another way.

During the course of development of ECE2, the EGR has been refuted in at least eighty three ways (“Eighty Three Refutations of EGR” on www.aias.us).

In the non relativistic limit:

\[ g \rightarrow 1 \ - (27) \]

the lagrangian (1) becomes:

\[ \mathcal{L} = \frac{1}{2} m \left( \dot{x}^2 + \dot{y}^2 \right) + \frac{mMg}{\left( x^2 + y^2 \right)^{1/2}} \ - (28) \]

and the Euler Lagrange equations (5) and (6) give an elliptical orbit via the equations:

\[ \ddot{x} = - m \frac{g x}{\left( x^2 + y^2 \right)^{3/2}} \ - (29) \]

and

\[ \ddot{y} = - m \frac{g y}{\left( x^2 + y^2 \right)^{3/2}} \ - (30) \]

In the same non relativistic limit, the non relativistic lagrangian can be written as:

\[ \mathcal{L} = \frac{1}{2} m \left( \dot{x}^2 + \dot{y}^2 \right) + \frac{mMg}{\left| \mathbf{v} \right|} \ - (31) \]
and the Euler Lagrange equation (13):

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = 0$$

which is the vector form of Eqs. (29) and (30).

The vector lagrangian is the same as the scalar lagrangian because:

$$\frac{\dot{r} \cdot \ddot{r}}{r} = \frac{\dot{x}^2 + \dot{y}^2}{r}$$

and:

$$\frac{1}{|\dot{r}|} = \frac{1}{r} \left( \frac{\dot{x}^2 + \dot{y}^2}{r^2} \right)^{1/2}$$

However, the vector Euler Lagrange equation is:

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{r}} - \frac{\partial L}{\partial r} = 0$$

Now note that:

$$\frac{\partial L}{\partial \dot{r}} = \frac{\partial L}{\partial \dot{x}} \frac{\dot{x}}{\partial (x + y \dot{j})} = \frac{1}{i} \frac{\partial L}{\partial \dot{x}}$$

so:

$$\frac{\partial L}{\partial \dot{r}} = \frac{1}{i} \frac{\partial L}{\partial \dot{x}}$$

Similarly:

$$\frac{\partial L}{\partial \dot{j}} = \frac{1}{i} \frac{\partial L}{\partial \dot{y}}$$
Also:
\[ \frac{\partial L}{\partial x} = i \cdot \frac{\partial L}{\partial (\dot{x}_i + \dot{y}_j)} \]  
- (40)

and:
\[ \frac{\partial L}{\partial y} = j \cdot \frac{\partial L}{\partial (\dot{x}_i + \dot{y}_j)} \]  
- (41)

Therefore:
\[ i \cdot \frac{\partial L}{\partial \dot{x}} = i \cdot \frac{d}{dt} \frac{\partial L}{\partial x} \]  
- (42)

gives:
\[ \frac{\partial L}{\partial x} = \frac{d}{dt} \frac{\partial L}{\partial x} \]  
- (43)

and:
\[ j \cdot \frac{\partial L}{\partial \dot{y}} = j \cdot \frac{d}{dt} \frac{\partial L}{\partial y} \]  
- (44)

gives:
\[ \frac{\partial L}{\partial y} = \frac{d}{dt} \frac{\partial L}{\partial y} \]  
- (45)

The scalar Euler Lagrange equations are the two components of the vector Euler Lagrange equation. The remarkable conclusion is reached that the same lagrangian can give forward and retrograde precession, depending on the method of solution.

As described in Notes 377(4) to 377(7), if it is assumed that:
\[ \dot{x} = \frac{m_6}{8 \left( x^2 + y^2 \right)^{3/2}} \left( \frac{\dot{y} y + x \dot{x}}{c^2} - x \right) = -\frac{m_6 x}{8 \left( x^2 + y^2 \right)^{3/2}} \]  
- (46)
This result is given by the equation of an ellipse:
\[
\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1
\]  
This cannot be described by EGR.

The spin connections for retrograde precession are found from the force equation (33) and two ECE2 gravitational field equations \{1 - 12\}:
\[
\nabla \cdot g = \kappa \cdot g = 4\pi G \rho_m - (50)
\]

\[
\nabla \times g = \kappa \times g = 0 - (51)
\]
in which \(\kappa\) is related to the spin connection as described in UFT318 and in which \(\rho_m\) is the mass density of the source of mass \(M\). It follows from the gravitational Coulomb law (50) that:
\[ \begin{align*}
\mathbf{X} \cdot \mathbf{\kappa}_x + \mathbf{Y} \cdot \mathbf{\kappa}_y &= -1 \quad - (52)
\end{align*} \]

(Note 377(8)). In the gravitostatic limit, the ECE2 Faraday law of induction becomes:

\[ \nabla \times \mathbf{g} = \mathbf{\kappa} \times \mathbf{g} \quad - (53) \]

so:

\[ \mathbf{\kappa} \parallel \mathbf{g} \quad - (54) \]

It follows that:

\[ \mathbf{\kappa}_x = -\frac{m_6 x}{\sqrt{v_0^2 (x^2 + y^2)^{3/2}}} \quad - (55) \]

and:

\[ \mathbf{\kappa}_y = -\frac{m_6 y}{\sqrt{v_0^2 (x^2 + y^2)^{3/2}}} \quad - (56) \]

where \( v_0 \) is a velocity to be defined. Using Eq. (52), the velocity is deduced to be:

\[ v_0^2 = \frac{m_6}{(x^2 + y^2)^{1/2}} - \frac{m_6}{r} \quad - (57) \]

and as shown in Note 377(8):

\[ \mathbf{\kappa}_x = -\frac{x}{x^2 + y^2} \quad - (58) \]

and

\[ \mathbf{\kappa}_y = -\frac{y}{x^2 + y^2} \quad - (59) \]

for the retrograde precession, and can be found from the numerical solution that gives \( X \) and \( Y \). These vector components of \( \mathbf{\kappa} \) are plotted in Section 3.
Forward and retrograde precession from the ECE2 Lagrangian

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3 Discussion of numerical results and graphics

3.1 Four theories of relativistic motion

We give some refinement of the equations for forward and retrograde precession. First we present the equations of four formulations of the relativistic equations of motion derived from the relativistic Lagrangian (1).

3.1.1 Relativistic Lagrangian model with t

With the Lagrange variables $X$ and $Y$ the following equations of motion are obtained from the Euler-Lagrange equations (5,6):

\[
\ddot{X} = MG \frac{\dot{X} \dot{Y} + X \dot{X}^2}{\gamma c^2 (Y^2 + X^2)^{3/2}} - \frac{MGX}{\gamma (X^2 + Y^2)^{3/2}},
\]

\[
\ddot{Y} = MG \frac{\dot{Y} \dot{X} + Y \dot{Y}^2}{\gamma c^2 (X^2 + Y^2)^{3/2}} - \frac{MGY}{\gamma (X^2 + Y^2)^{3/2}}.
\]

These can be combined in vector form as

\[
\ddot{\mathbf{r}} = \frac{MG}{\gamma r^3} \left( \frac{\dot{\mathbf{r}} \cdot \dot{\mathbf{r}}}{c^2} - \mathbf{r} \right)
\]

with

\[
r = (X^2 + Y^2)^{1/2}.
\]

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3.1.2 Relativistic Lagrangian model with τ

In standard relativistic Lagrange theory the time derivative in the Euler-Lagrange equations (5-6) is defined by the proper time \( τ \):

\[
\frac{\partial \mathcal{L}}{\partial X} = \frac{d}{dτ} \frac{\partial \mathcal{L}}{\partial \dot{X}},
\]
\[
\frac{\partial \mathcal{L}}{\partial Y} = \frac{d}{dτ} \frac{\partial \mathcal{L}}{\partial \dot{Y}}.
\]

This leads to an additional factor of \( 1/γ \) in Eqs. (60, 61):

\[
\ddot{X} = MG \frac{XY \dot{Y} + X \ddot{X}^2}{γ^2 c^2 (Y^2 + X^2)^{3/2}} - \frac{MGX}{γ^2 (X^2 + Y^2)^{3/2}},
\]
\[
\ddot{Y} = MG \frac{YX \dot{X} + Y \ddot{Y}^2}{γ^2 c^2 (X^2 + Y^2)^{3/2}} - \frac{MGY}{γ^2 (X^2 + Y^2)^{3/2}},
\]

and in vector form:

\[
\ddot{\mathbf{r}} = \frac{MG}{γ^2 r^3} \left( \frac{\dot{\mathbf{r}} (\dot{\mathbf{r}} \cdot \mathbf{r})}{c^2} - \mathbf{r} \right).
\]

3.1.3 Relativistic Newton equation

The relativistic Newton equation (21, 22), as derived in section 2 from the vector form (18) of the Euler Lagrange equations, is:

\[
\ddot{\mathbf{r}} = -\frac{MG}{γ^3 r^3} \mathbf{r}.
\]

Here a factor of \( γ^3 \) appears in the denominator, and there is no additional velocity dependence as in (62) and (68).

3.1.4 Minkowski force

The Minkowski force equations can be derived from Minkowski theory directly and contains one more \( γ \) factor in the denominator:

\[
\ddot{\mathbf{r}} = -\frac{MG}{γ^4 r^3} \mathbf{r}.
\]

3.1.5 Comparison of equations

All four equations have been solved numerically for the S2 star as described in UFT 375 in detail. Our first focus is on the angular momentum. In both Lagrange theories (62) and (68) the relativistic angular momentum is conserved by construction (Figs. 1 and 2). For the relativistic Newton and Minkowski force only the non-relativistic angular momentum is conserved (Figs. 3 and 4), giving an inconsistent result. There is no simple explanation available since the relativistic angular momentum was used in Eq. (14).

Precession is negative (or retrograde) for both the Minkowski and relativistic Newton force. From Lagrange theory equations the precession is positive.
However precession is extremely small in the \( \tau \) version of Lagrangian theory. It is barely above the numerical precision limit of \( 10^{-8} \) rad as determined in UFT 375. There is also a logical problem for the Lagrange theory based on proper time \( \tau \). All quantities are computed in the observer frame but the time derivative of \( \partial L / \partial \dot{r} \) is computed for the frame local to the orbiting mass. This seems to be inconsistent. Therefore the Lagrange theory based on observer time \( t \) seems to be the best choice for a consistent overall description of relativistic effects. A comparison of all four theory variants is made in Table 1 for the orbit of the S2 star. The differences in maximum radius and eccentricity are marginal.

### 3.2 Spin connection vector

The spin connection vector \( \kappa \) can be computed from the solution of orbit trajectories in several degrees of approximation. By Eq. (52) we have one equation for both components \( \kappa_X \) and \( \kappa_Y \). Under the assumption that the \( \kappa \)'s vary only slowly with time, we can take the time derivative of this equation as a second equation, obtaining

\[
\kappa_X X + \kappa_Y Y = -1, \\
\kappa_X \dot{X} + \kappa_Y \dot{Y} = 0.
\]

Solving these equation set, we obtain

\[
\kappa_X = \frac{\dot{Y}}{XY - X^2}, \\
\kappa_Y = -\frac{\dot{X}}{XY - X^2}.
\]

We see that the \( \kappa \)'s nevertheless depend on time since this is the case for the trajectories \( X(t), Y(t) \) and their derivatives. These trajectories are graphed in Figs. 5 and 6 for the S2 star. Due to the high ellipticity of the orbit, there are sharp peaks at periastron. The approximate solution (73, 74) is graphed in Fig. 7. There is high similarity to the derivatives \( \dot{X}, \dot{Y} \) of Fig. 6, with interchange of \( X \) and \( Y \) and the sign of one component.

Eq. (52) was derived from the Coulomb-like field equation (50). Instead of taking an additional time derivative, we can use the static Ampere law (51). In

<table>
<thead>
<tr>
<th></th>
<th>( T ) [yr]</th>
<th>( r_{\text{max}} ) [10^{14} \text{ m}]</th>
<th>( \epsilon )</th>
<th>( \Delta \phi ) [rad]</th>
<th>const. of motion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Euler-Lagr. ( t )</td>
<td>15.50</td>
<td>2.78609</td>
<td>0.88712</td>
<td>( 5.9033 \times 10^{-4} )</td>
<td>( L_{\text{rel}} )</td>
</tr>
<tr>
<td>Euler-Lagr. ( \tau )</td>
<td>15.57</td>
<td>2.79440</td>
<td>0.88746</td>
<td>( 7.5090 \times 10^{-7} )</td>
<td>( L_{\text{rel}} )</td>
</tr>
<tr>
<td>Rel. Newton</td>
<td>15.50</td>
<td>2.78621</td>
<td>0.88720</td>
<td>( -1.7697 \times 10^{-3} )</td>
<td>( L_{\text{non-rel}} )</td>
</tr>
<tr>
<td>Minkowski</td>
<td>15.06</td>
<td>2.79452</td>
<td>0.88753</td>
<td>( -2.3585 \times 10^{-3} )</td>
<td>( L_{\text{non-rel}} )</td>
</tr>
<tr>
<td>Experiment</td>
<td>15.56</td>
<td>2.68398</td>
<td>0.8831</td>
<td>-0.017...</td>
<td>( +0.035 )</td>
</tr>
</tbody>
</table>

Table 1: Parameters of S2 star orbit (\( v_0 = 7.7529648 \times 10^6 \) m/s, various calculations and experiment).
two dimensions there is only a $Z$ component of the curl operator, giving in total the equation set

$$\kappa_X X + \kappa_Y Y = -1, \quad (75)$$

$$\kappa_X Y - \kappa_Y X = 0. \quad (76)$$

This can be solved by computer algebra directly:

$$\kappa_X = -\frac{X}{X^2 + Y^2}, \quad (77)$$

$$\kappa_Y = \frac{Y}{X^2 + Y^2}. \quad (78)$$

The denominator is always positive now, leading to a smoother curve of the $\kappa$’s except at periastron, see Fig. 8. This fact should give further serious concern for Einsteinian general relativity where infinities (“black holes”) are built in by construction. As we can see from these examples, in nature there are no infinities, and improving the descriptive approach removes infinities. This stage has never been reached by the Einsteinian theory.

Instead of solving the field equations (75, 76) for $\kappa$, we can alternatively solve them for $X, Y$. This leads to

$$X = -\frac{\kappa_X}{\kappa_X^2 + \kappa_Y^2}, \quad (79)$$

$$Y = -\frac{\kappa_Y}{\kappa_X^2 + \kappa_Y^2}. \quad (80)$$

Obviously the orbit is entirely determined by the spin connection, a completely new result of relativistic gravitational physics. Spin connections and orbit coordinates are mutually symmetric, showing some sort of symmetry, which perhaps can be interpreted as a correspondence between configuration and momentum space.

This offers the capability of investigating what happens when the spin connections are slightly modified, a kind of spacetime or aether engineering. As an example we modify the $Y$ component of the spin connection vector by

$$\kappa_Y \rightarrow \kappa_Y - t \cdot 10^{-17}. \quad (81)$$

The $Y$ component is continuously decreased. The result is a retrograde precession of the orbit as shown in Fig. 9. So any kind of precession can - besides other methods - be evoked by aether engineering.
Figure 1: Angular momentum, Euler-Lagrange equations with $t$.

Figure 2: Angular momentum, Euler-Lagrange equations with $\tau$. 
Figure 3: Angular momentum, relativistic Newton Equation.

Figure 4: Angular momentum, Minkowski Equation.
Figure 5: Orbital trajectories for S2 motion.

Figure 6: Orbital time derivative trajectories for S2 motion.
Figure 7: Spin connection components for approximation (73, 74).

Figure 8: Spin connection components for exact solution (77, 78).
Figure 9: Retrograde precessing orbit evoked by aether engineering (modified spin connection).
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