COMPLETE SOLUTION OF THE ECE2 FIELD EQUATIONS: THE BIEFELD BROWN COUNTER GRAVITATIONAL EFFECT.

by

M. W. Evans and H. Eckardt

Civil List and AIAS / UPITEC


ABSTRACT

The homogeneous field equations of ECE2 field theory are used together with the antisymmetry laws, to solve for the spin connection four vector and vector potential three vector. Example solutions are given using hand calculations checked with computer algebra. It is possible to obtain the general solutions computationally using a package such as Mathematica. Some solutions can be obtained with Maxima. A straightforward explanation is given for the Biefeld Brown counter gravitational effect.

Keywords: ECE2 field equations, solutions for the spin connection and vector potential.
1. INTRODUCTION

In recent papers of this series {1 - 12}, the important discovery has been made that the ECE lagrangian produces both forward and retrograde precession in planar orbits, thus going beyond the Einsteinian general relativity (EGR). This work suggests that general solutions of the ECE2 field equations are needed, both for gravitation and electromagnetism. In this paper the homogeneous field equations of ECE2 gravitation and electromagnetism are solved together with the antisymmetry laws {1 - 12}. Some example solutions are described, and the general problem can be solved with a package such as Mathematica. The Maxima package can solve certain types of problem.

This paper is a brief synopsis of detailed calculations found in the notes accompanying UFT380 on www.aias.us and www.upitec.org. These notes are an intrinsic part of the paper. Note 380(1) gives a method of evaluating the vector potential three vector for gravitation (\( Q \)) and electromagnetism (\( A \)) and the spin connection four vector, (\( \omega^\mu \)), which is an intrinsic part of geometry and which is the same for gravitation and electromagnetism. The note defines the field tensor, the antisymmetry laws and the Gauss law for gravitation and electromagnetism. Note 380(2) combines electrodynamics and gravitation to give a straightforward explanation of the Biefeld Brown counter gravitational effect. Note 380(3) describes schemes of calculation for \( \omega^\mu \) and \( Q \). Note 380(4) gives Cartesian equations for the vector potential components and the components of the spin connection four vector. Note 350(5) introduces the Faraday law of induction for gravitation and electromagnetism and shows that the problem is exactly determined, seven equations in seven unknowns. This note is used as the basis for Section 2. Note 380(6) gives a hand calculated plane wave solution of the problem.
In Section 3 some example solutions and schemes of calculation and computation are solved with Maxima and graphed en route to the general solution that can be used throughout the physical sciences and engineering.

2. THE SEVEN EQUATIONS IN SEVEN UNKNOWNS

Consider the homogeneous field equations and antisymmetry laws of ECE2 gravitation. The homogeneous field equations are the Gauss law:

\[ \nabla \cdot \Omega = 0 \quad - (1) \]

and the Faraday law of induction:

\[ \nabla \times g + \frac{\partial \Omega}{\partial t} = 0 \quad - (2) \]

Here \( g \) is the gravitational field and \( \Omega \) the gravitomagnetic field, respectively defined by:

\[ g = - \frac{\partial Q}{\partial t} - \omega \times Q = - \nabla \Phi + \omega \times \Phi \quad - (3) \]

and

\[ \Omega = \nabla \times Q - \omega \times Q \quad - (4) \]

Here \( Q \) is the vector potential of gravitation, \( \Phi \) is the scalar potential, \( \omega \) is the spin connection vector, and \( \omega_0 \) is the timelike part of the spin connection four vector.

In terms of Cartesian components, Eq. (1) gives

\[ Q_x \left( \frac{\partial \omega_z}{\partial y} - \frac{\partial \omega_y}{\partial z} \right) + Q_y \left( \frac{\partial \omega_x}{\partial z} - \frac{\partial \omega_z}{\partial x} \right) + Q_z \left( \frac{\partial \omega_y}{\partial x} - \frac{\partial \omega_x}{\partial y} \right) \]

as described in the Notes. Eqs. (2) to (4) give:
\[
\frac{d}{dt} \left( \mathbf{\omega} \times \mathbf{Q} \right) + \nabla \times \left( \mathbf{\omega} \cdot \mathbf{Q} \right) = 0
\] - (6)

The antisymmetry laws from Eq. (4) are derived in Note 380(4), and are:

\[
\begin{align*}
\frac{1}{2} \left( \frac{\partial}{\partial x} - \frac{\partial}{\partial y} \right) Q_z &= - \left( \frac{\partial}{\partial x} - \frac{\partial}{\partial y} \right) Q_y = & (7) \\
\frac{1}{2} \left( \frac{\partial}{\partial y} - \frac{\partial}{\partial z} \right) Q_x &= - \left( \frac{\partial}{\partial y} - \frac{\partial}{\partial z} \right) Q_z = & (8) \\
\frac{1}{2} \left( \frac{\partial}{\partial z} - \frac{\partial}{\partial x} \right) Q_y &= - \left( \frac{\partial}{\partial z} - \frac{\partial}{\partial x} \right) Q_x = & (9)
\end{align*}
\]

These can be expressed as:

\[
\begin{align*}
\frac{\partial Q_z}{\partial y} + \frac{\partial Q_y}{\partial z} &= \omega_z Q_y + \omega_y Q_z = & (10) \\
\frac{\partial Q_x}{\partial y} + \frac{\partial Q_y}{\partial x} &= \omega_x Q_x + \omega_y Q_z = & (11) \\
\frac{\partial Q_y}{\partial x} + \frac{\partial Q_x}{\partial y} &= \omega_x Q_y + \omega_y Q_x. = & (12)
\end{align*}
\]

Eq. (6) gives three further equations:

\[
\begin{align*}
\frac{d}{dt} \left( \omega_x Q_z - \omega_z Q_x \right) + \omega_z \left( \frac{\partial Q_z}{\partial y} - \frac{\partial Q_y}{\partial z} \right) + Q_z \frac{\partial \omega_z}{\partial y} - Q_y \frac{\partial \omega_z}{\partial z} &= 0 \quad & (13) \\
\frac{d}{dt} \left( \omega_x Q_x - \omega_x Q_z \right) + \omega_x \left( \frac{\partial Q_x}{\partial y} - \frac{\partial Q_y}{\partial x} \right) + Q_x \frac{\partial \omega_x}{\partial y} - Q_y \frac{\partial \omega_x}{\partial x} &= 0 \quad & (14) \\
\frac{d}{dt} \left( \omega_y Q_y - \omega_y Q_x \right) + \omega_y \left( \frac{\partial Q_y}{\partial x} - \frac{\partial Q_x}{\partial y} \right) + Q_y \frac{\partial \omega_y}{\partial x} - Q_x \frac{\partial \omega_y}{\partial y} &= 0 \quad & (15)
\end{align*}
\]

so the problem is exactly determined, to solve Eqs. (5), (10) to (12) and (13) to (15) for the three components of \(Q\) and the four components of \(\mathbf{\omega}\).
In general this system can be solved by computer using a package such as Mathematica, and the package Maxima can solve some problems. The general solution can address many situations in the physical sciences and engineering.

Consider the approximation:

$$\Omega \ll \omega \rightarrow \omega$$ \hspace{1cm} (16)

which means that the gravitomagnetic field is much smaller than the gravitational field. The approximation (16) means that:

$$\sqrt{1} \times \mathbf{Q} = \omega \times \mathbf{Q}$$ \hspace{1cm} (17)

and as shown in Note 380(5), the antisymmetry laws (10) to (12) and Eq. (17) lead to:

$$\frac{\partial Q_x}{\partial z} = \omega_z Q_x$$ \hspace{1cm} (18)

and:

$$\frac{\partial Q_y}{\partial z} = \omega_z Q_y$$ \hspace{1cm} (19)

so:

$$Q_x = Q_y$$ \hspace{1cm} (20)

A solution of Eqs. (18) and (20) is the wave:

$$Q_x = Q_y = i Q_0 \exp\left(i (\omega t - k_z z)\right)$$ \hspace{1cm} (21)

where $\omega$ is its angular frequency at instant $t$, and $k_z$ is its wave vector at position $Z$.

The use of Maxima confirms that the general solution has the format of Eq. (21). From
Eq. (21):
\[ k_z = \omega_z \quad -(22) \]

so for a unidirectional wave in the Z axis, as shown in Note 380(5):
\[ Q_x = Q_y = Q_z = iQ_0 \exp \left( i(\omega t - k_z z) \right) \quad -(23) \]

If it is further assumed that:
\[ \frac{d\omega_0}{dx} = \frac{d\omega_0}{y} = \frac{d\omega_0}{z} = 0 \quad -(24) \]

then the solution of the seven relevant equations described already is:
\[ Q_x = Q_y = Q_z = iQ_0 \exp \left( i(\omega t - k_z z) \right) \quad -(25) \]

and
\[ \omega \mu = \left( \frac{\omega}{c}, \kappa \right) \quad -(26) \]

with:
\[ \Omega \sim 0 \quad -(27) \]

Therefore the spin connection is the wave four vector:
\[ \omega \mu = \kappa \mu \quad -(28) \]

The system quantizes to:
\[ p \mu = \hat{p} \kappa \mu = \hat{p} \omega \mu \quad -(29) \]

where \( p \) the energy momentum four vector:
The energy momentum of a graviton is the spin connection four vector within the reduced Planck constant $\hbar$.

For ECE2 electrodynamics the corresponding solution for the vector potential is:

$$\mathbf{A}_x = \mathbf{A}_y = \mathbf{A}_z = i \, A^{(0)} \exp \left( i \left( \omega t - \kappa \mathbf{r} \right) \right)$$

and the spin connection four vector is the same. Further details are given in Note 380(5).

Another possible solution is given in Note 380(6) by evaluating the spin connection four vector from the vector potential plane wave $[1-12]$:

$$A^{(1)} = A^{(0)} \frac{1}{\sqrt{2}} \left( i - i \hat{\mathbf{J}} \right) \exp \left( -i \left( \omega t - \kappa \mathbf{r} \right) \right)$$

for which the antisymmetry laws reduce to:

$$\frac{\partial A^{(i)}_x}{\partial \mathbf{r}} = \omega_z A^{(i)}_x - (33)$$

$$\frac{\partial A^{(i)}_y}{\partial \mathbf{r}} = \omega_z A^{(i)}_y - (34)$$

$$- \omega_x A^{(i)}_y = \omega_y A^{(i)}_x - (35)$$

from which:

$$\omega_z = 0 - (36)$$

and

$$i \omega_x = \omega_y - (37)$$

as shown in Note 380(6). Eqs. (36) and (37) are satisfied by the complex conjugate spin connection plane wave.
It follows as in Note 380(6) that

\[ \omega_0 = 0. \quad -(39) \]

Therefore if it is assumed that:

\[ \mathbf{A}(t) = A^{(0)}(i - j) \exp \left( -i (\omega t - k z) \right) \quad -(40) \]

then a possible solution is

\[ \omega(2) = \omega^{(0)}(i + j) \exp \left( i (\omega t - k z) \right) \quad -(41) \]

so if the vector potential is assumed to be a plane wave, the spin connection is spacelike and also a plane wave. This is a plane wave of spacetime or the aether. In Section 3 other solutions are given using Maxima and graphed.

Finally, the Biefeld Brown counter gravitation effect \{1 - 12\} can be given a simple explanation using the inhomogeneous Coulomb laws of electromagnetism and gravitation, respectively:

\[ -\nabla^2 \phi + \nabla \cdot (\phi \omega) = \frac{\rho}{\epsilon_0} \quad -(42) \]

and

\[ -\nabla^2 \Phi + \nabla \cdot (\Phi \omega) = 4\pi G \rho \quad -(43) \]

as in Note 380(2). The spin connection vector \[ \frac{\omega}{\Phi} \] is the same in both equations. Here \[ \phi \] is the electromagnetic scalar potential and \[ \Phi \] is the gravitational scalar potential.
\( \rho \) is the electric charge density, \( \rho_m \) is the mass density, \( \varepsilon_0 \) is the vacuum permittivity and \( G \) is Newton’s constant. For a particle with charge \( e \) and mass \( m \) the total potential energy in joules is:

\[
U = e \phi + m \Phi - (44)
\]

so:

\[
-\nabla^2 U + \nabla \cdot (U \mathbf{e}) = \frac{e}{\varepsilon_0} + 4\pi G m \frac{\rho}{m} - (45)
\]

and the total force on the particle is:

\[
\mathbf{F} = -\nabla U - (46)
\]

By adding Eqs. (44) and (45) it is found that there are cross effects such as:

\[
-\nabla^2 \Phi + \nabla \cdot (\Phi \mathbf{e}) = \frac{e}{m \varepsilon_0} + \ldots - (47)
\]

where an electric charge density (as in a Biefield Brown capacitor) can influence the gravitational potential and cause counter gravitation. Q. E. D..

This effect is developed and graphed in Section 3.

3. COMPUTATION AND GRAPHICS

By Dr. Horst Eckardt
Complete solution of the ECE2 field equations: The Biefeld Brown counter gravitational effect

M. W. Evans∗, H. Eckardt†
Civil List, A.I.A.S. and UPITEC


3 Computation and graphics

3.1 Special Q vectors
We consider some special cases of the vector potential (A, or Q, respectively). A general (real-valued) wave form expanding in Z direction is

\[
Q = \begin{bmatrix}
Q_X \\
Q_Y \\
Q_Z \\
\end{bmatrix} = \begin{bmatrix}
Q_1 \\
Q_2 \\
Q_3 \\
\end{bmatrix} \cos(\beta t - (\kappa_X X + \kappa_Y Y + \kappa_Z Z)).
\] (48)

Application of Eqs. (13-15) leads to equations like

\[
\beta Q_3 \omega_Y - \beta Q_2 \omega_Z = (Q_3 \kappa_Y - Q_2 \kappa_Z) \omega_0.
\] (49)

It can be seen that the choice

\[
\omega_0 = \beta
\] (50)

\[
\omega_X = \kappa_X
\] (51)

\[
\omega_Y = \kappa_Y
\] (52)

\[
\omega_Z = \kappa_Z
\] (53)

is a valid solution. However, from antisymmetry equations (10-12) follows:

\[-Q_3 \kappa_Y = Q_2 \kappa_Z, \]

\[-Q_1 \kappa_Z = Q_3 \kappa_X, \]

\[-Q_2 \kappa_X = Q_1 \kappa_Y. \]

(54)

(55)

(56)

This is an incompatible equation set which only has the solutions \(Q_i = 0\) or \(\kappa_i = 0\) for \(i = 1, 2, 3\) or \(X, Y, Z\), respectively. Similar results appear if \(Q=\text{const.}\) is assumed.

∗email: emyrone@aol.com
†email: mail@horst-eckardt.de
3.2 General Q vector, constant $\omega$ vector

A different situation occurs if $Q$ is left general but a special choice of $\omega$ is made, for example choosing a constant value:

$$\omega = \begin{bmatrix} 0 \\ 0 \\ \kappa \end{bmatrix}. \quad (57)$$

This reduces the equations significantly to

$$- \left( \frac{\partial}{\partial t} Q_Y \right) \kappa = - \left( \frac{\partial}{\partial Y} Q_Z - \frac{\partial}{\partial Z} Q_Y \right) \omega_0 \quad (58)$$

$$\left( \frac{\partial}{\partial t} Q_X \right) \kappa = - \left( \frac{\partial}{\partial Z} Q_X - \frac{\partial}{\partial X} Q_Z \right) \omega_0 \quad (59)$$

$$0 = - \left( \frac{\partial}{\partial X} Q_Y - \frac{\partial}{\partial Y} Q_X \right) \omega_0 \quad (60)$$

$$\frac{\partial}{\partial Y} Q_Z = Q_Y \kappa - \frac{\partial}{\partial Z} Q_Y \quad (61)$$

$$\frac{\partial}{\partial Z} Q_X - Q_X \kappa = - \frac{\partial}{\partial X} Q_Z \quad (62)$$

$$\frac{\partial}{\partial X} Q_Y = - \frac{\partial}{\partial Y} Q_X \quad (63)$$

$$0 = \left( \frac{\partial}{\partial X} Q_Y - \frac{\partial}{\partial Y} Q_X \right) \kappa \quad (64)$$

3.3 Design of a counter gravitation experiment

Basing on the above result, we will try to find configurations where antigravity effects are visible. According to Eq. (3), the gravitational acceleration is defined by

$$g = - \nabla \Phi + \omega \Phi \quad (65)$$

where $\Phi$ is the gravitational potential and $\omega$ is the vector spin connection. The idea of manipulating gravity is to change the vector spin connection artificially. As explained in section 2 for the Biefeld-Brown effect, $\omega$ can be changed by electromagnetism. If there is an inferred change $\Delta \omega$, this is connected with a change of gravity

$$\Delta g = \Delta \omega \cdot \Phi(r_E) \quad (66)$$

where $r_E$ is the radius of the earth and

$$\Phi(r_E) = - \frac{M_E G}{r_E} \quad (67)$$

is the gravitational potential at the earth surface with mass of earth $M_E$. Best would be to produce a constant vector spin connection, for example in $Z$ direction as defined in Eq. (57). So, in order to achieve this, we have to find an electromagnetic vector potential $A$ that produces exactly this spin connection.
Since a vector potential cannot be created directly by design, we have to use a magnetic field in a second step that realizes exactly this vector potential.

We will present two tentative solutions to this problem. In the first solution we define a vector potential of the special form

\[
\mathbf{A} = \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} q e^{\beta t} \\ q e^{\beta t} \\ a(t)X + b(t)Y \end{bmatrix}
\]

with a constant \(q\) and time-dependent functions \(a\) and \(b\) which will be defined later. \(\beta\) may be a positive or negative constant. Solving the equation set (58-64), now with \(\mathbf{Q} = \mathbf{A}\), leads to the equations:

\[
\begin{align*}
-\beta q e^{\beta t} &= -\omega_0 b(t) \\
\beta q e^{\beta t} &= \omega_0 a(t) \\
0 &= 0 \\
b(t) &= \kappa q e^{\beta t} \\
-\kappa q e^{\beta t} &= -a(t) \\
0 &= 0
\end{align*}
\]

These are compatible if the unspecified functions are

\[
\omega_0 = \beta, \quad a(t) = b(t) = \kappa q e^{\beta t}.
\]

From this solution the generating magnetic field is

\[
\mathbf{B} = \nabla \times \mathbf{A} - \mathbf{\omega} \times \mathbf{A} = 2 \kappa q e^{\beta t} \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}.
\]

The resulting \(Z\) component of the vector potential (68) has been plotted in Fig. 1 for three discrete time values.

A second solution is presented by using an oscillating vector potential. However, because only the first time derivative appears in Eqs. (13-15), it is difficult to find a definition of \(\mathbf{A}\) that does not lead to contradictory equations. One possibility is using a vector potential with only one time-oscillating component:

\[
\mathbf{A} = \begin{bmatrix} q e^{i\beta t} \\ 0 \\ a(t)X \end{bmatrix}.
\]

Then the set of field and antisymmetry equations is

\[
\begin{align*}
0 &= 0 \\
i\beta q e^{i\beta t} &= \omega_0 a(t) \\
0 &= 0 \\
0 &= 0 \\
-\kappa q e^{i\beta t} &= -a(t) \\
0 &= 0 \\
0 &= 0
\end{align*}
\]
From these equations follows
\[ \omega_0 = i\beta, \]  
\[ a(t) = \kappa q e^{i\beta t}, \]  
\hfill (87)  
\hfill (88)
i.e. the scalar spin connection \( \omega_0 \) is imaginary. It is not possible to use real-valued time functions because then the equations become contradictory because of the time derivative problem. The generating magnetic field is
\[ B = \begin{bmatrix} 0 & -2\kappa q e^{i\beta t} \\ 0 & 0 \end{bmatrix}. \]  
\hfill (89)
and the real part of \( B \) is
\[ \text{Real}(B) = \begin{bmatrix} 0 & -2\kappa q \cos(\beta t) \\ 0 & 0 \end{bmatrix}. \]  
\hfill (90)
Here \( \beta \) is the frequency of the generating magnetic field. The resulting \( Z \) component of the vector potential (79) has been plotted in Fig. 1 for three discrete time values.

A quantitative assessment of \( \kappa \) and its impact on gravitation can be made as follows. For the amplitudes we have
\[ B_0 = -2\kappa q = -2\kappa A_0 \]  
\hfill (91)
where \( A_0 \) is the amplitude of the vector potential generating \( B_0 \). Approximating
\[ B_0 = \frac{A_0}{l} \]  
\hfill (92)
with a characteristic length dimension of the magnetic field of \( l = 0.1 \) m and assuming a field of \( B = 1 \) T, it follows
\[ \kappa = -\frac{B_0}{2A_0} = -0.05/\text{m}. \]  
\hfill (93)
The gravitational field at the earth radius \( r_E \) will be modified by
\[ \Delta g = \Delta \omega \Phi(r_E) = \kappa \Phi(r_E) \approx -0.05/\text{m} \cdot \Phi(r_E), \]  
\hfill (94)
where the full gravitational potential at the earth surface is
\[ \Phi(r_E) = -\frac{M_E G}{r_E} \approx -6.26 \cdot 10^7 \text{ SI units}. \]  
\hfill (95)
Because \( \Phi(r_E) \) is negative, the correction (94) is positive which has the right sign. There is however a problem with orders of magnitude: If we insert the value of (95) into (94), we obtain a huge correction which is six orders of magnitude greater than the gravitational field itself. In order to have a reasonable effect, \( \omega(r_E) \) has to be quite small:
\[ \omega(r_E) < \frac{g}{\Phi(r_E)} \approx -2.57 \cdot 10^{-7}/\text{m} \]  
\hfill (96)
Currently there is no solution to this mismatch.
Figure 1: Component $A_Z$ of Eq. (68), $t = 0, 1, 2$ for colours blue, green, red. All constants set to unity.

Figure 2: Component $A_Z$ of Eq. (79), $t = 0, 1, 2$ for colours blue, green, red. All constants set to unity.
ACKNOWLEDGMENTS

The British Government is thanked for a Civil List Pension and the staff of AIAS and others for many interesting discussions. Dave Burleigh, CEO of Annexa Inc., is thanked for hosting www.aias.us, site maintenance and feedback software and hardware maintenance. Alex Hill is thanked for translation and broadcasting, and Robert Cheshire for broadcasting.

REFERENCES


{6} H. Eckardt, “The ECE Engineering Model” (Open access as UFT303, collected equations).


