HAMILTON JACOBI FORMALISM

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ABSTRACT

The Hamilton Jacobi (HJ) formulation of ECE2 theory is developed in preparation for a HJ development of m theory. The action is found by integration of the HJ equation. An extensive computer analysis is given of the Hamilton equations in special relativity.

Keywords: ECE2 theory, Hamilton Jacobi equations and Hamilton equations.
1. INTRODUCTION

In recent papers of this series {1 - 41} The Euler Lagrange and Hamilton dynamics have been applied to m theory, thus giving information that is not available when considering the Euler Lagrange equations alone. A fourth complete system of dynamics has been developed recently, the Evans Eckardt dynamics, based simply on the fact that the hamiltonian and angular momentum are constants of motion on any level, classical, special relativistic, m theory and in quantized dynamics. The three complete systems of dynamics were hitherto thought to be the Euler Lagrange, Hamilton and Hamilton Jacobi equations.

This paper is a short synopsis of detailed calculations found in the notes that accompany UFT426 on www.aias.us and www.upitec.org. These notes are an intrinsic part of the paper and should be read with the paper itself. Note 426(1) gives the fundamental equations of the Euler Lagrange Hamilton dynamics and derives a new equation of motion of the Hamilton dynamics. Note 426(2) uses the new equation to show that the $m_{(r1)}$ function of m theory has no dependence on the Newtonian velocity $\sqrt{\mathbf{N}}$. Notes 426(3) and 426(4) review the Hamilton Jacobi system of dynamics. Note 426(5) gives a new formulation of the Hamilton Jacobi equation and Note 426(6) computes the action of the Hamilton Jacobi equation applied to ECE2 theory, which develops the equations of special relativity in a space with finite curvature and torsion. These results are a preparation for the application of the Hamilton Jacobi system of dynamics to m theory in future work.

Section 2 computes the action of the Hamilton Jacobi formulation of ECE2 and Section 3 gives an extensive computer analysis of the Hamilton equations.

2. HAMILTON AND HAMILTON JACOBI DEVELOPMENT OF ECE2 THEORY

Define the ECE hamiltonian as:
where $p$ is the relativistic linear momentum:

$$p = \gamma m v_n,$$

and $m$ is the mass of an object that orbits $M$. The Newton constant is $G$ and $r$ is the distance between $m$ and $M$. The canonically conjugate generalized coordinates $p$ and $q$ of the Hamilton dynamics are chosen to be:

$$p_r = \gamma m v_r, \quad q_r = r - (3)$$

and

$$p_\phi = L, \quad q_\phi = \phi - (4)$$

where $L$ is the angular momentum, a constant of motion and where $\phi$ is defined by the plane polar coordinates $(r, \phi)$. The Evans Eckardt equations of the system are:

$$\frac{dH}{dt} = 0 - (5)$$

and

$$\frac{dL}{dt} = 0. - (6)$$

The first Hamilton equation gives:

$$\dot{p} = -\frac{dH}{dr} = -\frac{m M G}{r^2} - (7)$$

i.e.:
The same result is given by the Euler Lagrange system of dynamics as shown in previous papers of the UFT series. This is a successful demonstration of the rigorous self consistency of the UFT series. The second Hamilton equation is:

\[
\dot{r} = \frac{\partial H}{\partial p} = \frac{pc^2}{(p^2c^4 + m^2c^4)^{1/2}} = \frac{pc}{\gamma mc} = \frac{p}{\gamma m} - (9)
\]

where use has been made of:

\[
E = \gamma mc^2 = (p^2c^4 + m^2c^4)^{1/2} - (10)
\]

where the Lorentz factor is:

\[
\gamma = \left(1 - \frac{v_N^2}{c^2}\right)^{-1/2} - (11)
\]

It follows that the second Hamilton equation gives:

\[
\dot{r} = \frac{p}{\gamma m} - (12)
\]

i. e.

\[
p = \gamma m \dot{r} - (13)
\]

This is the relativistic momentum:

\[
p = \gamma m v_N - (14)
\]

provided that:

\[
v_N = \dot{r} - (15)
\]
Q. E. D.

To extend to plane polar coordinates use:
\[ p = \gamma m \dot{r} = \gamma m \left( r \ddot{r} + r \dot{\phi}^2 \right) \] - (16)

It follows that:
\[ p^2 = p_r^2 + \frac{p_\phi^2}{r^2} \] - (17)

The angular generalized coordinates are:
\[ p_\phi = L, \quad \dot{\phi} = \phi \] - (18)

where the angular momentum is:
\[ L = \gamma m r^2 \dot{\phi} \] - (19)

The first Hamilton equation gives:
\[ \dot{L} = -\frac{\partial H}{\partial \phi} = 0 \] - (20)

so \( L \) is a constant of motion:
\[ \frac{dL}{dt} = 0 \] - (21)

This is the second Evans Eckardt equation.

The second Hamilton equation gives:
\[ \dot{\phi} = \dot{\phi} = \frac{\partial H}{\partial L} \] - (22)

where:
It follows that:

\[ \Phi = \frac{Lc^2}{\gamma mc^2 r^2} = \frac{L}{mvr^2} \]  

or:

\[ L = \gamma vr^2 \Phi \]  

which is the constant angular momentum, Q. E. D. Again, the same result is given by the Euler Lagrange analysis of ECE2 theory \{1 - 41\}, another successful check of the rigorous self consistency of ECE2 theory.

With reference to the background notes accompanying UFT426 on www.aias.us and www.upitec.org the Hamilton Jacobi system of dynamics defines:

\[ p_r = \frac{dS}{dr}, \quad p_\phi = \frac{dS}{d\phi} \]  

where \( S \) is the total action:

\[ S = S_r + S_\phi. \]  

The quantum of action is \( \hbar \), the reduced Planck constant. Therefore Eq. (23) gives the two Hamilton Jacobi equations:

\[ E = \left( c^2 \left( \left( \frac{dS_r}{dr} \right)^2 + \frac{L^2}{r^2} \right) + m^2 c^4 \right)^{1/2} - \frac{mMc}{r} \]  

and

\[ L = \frac{dS_\phi}{d\phi} \]
where $E$ is the total energy:

$$H = E.$$  \(30\)

This equation quantizes to the Schroedinger equation:

$$H\psi = E\psi.$$ \(31\)

where $\psi$ is the wave function.

These equations pave the way for the application of the Hamilton Jacobi formalism to m theory and its eventual quantization. This will be the subject of future work.

The Hamilton Jacobi equation (28) is integrated using Maxima in Section 3, by co author Horst Eckardt, giving interestingly original results described in Section 3. The latter also gives an extensive numerical analysis of the Hamilton equations applied to ECE2 theory and special relativity.

The two Evans Eckardt equations:

$$\frac{dH}{dt} = 0, \quad \frac{dL}{dt} = 0.$$ \(32\)

can be used with the Hamilton Jacobi equations, and the Euler Lagrange system of dynamics can be combined with the Hamilton Jacobi system of dynamics. The essence of the HJ system is to find constants of motion and to find the action. The lagrangian is the integral of the action and the Hamilton Principle of Least Action minimizes the action to find essentially all of classical physics.

3. NUMERICAL RESULTS AND GRAPHICS.

Section by Dr. Horst Eckardt
Hamilton Jacobi formalism

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3 Numerical results and graphics

3.1 Hamilton equations of central motion

We worked out the Hamilton equations of central motion in a plane polar coordinate system. The general form of the Hamilton equations is

\[ \dot{q}_i = \frac{\partial H}{\partial p_i}, \]
\[ \dot{p}_i = -\frac{\partial H}{\partial q_i}, \]

where \( q_i \) are the canonical or generalized coordinates and \( p_i \) are the conjugate canonical momenta. The index \( i \) refers to the coordinate components. In spherical polar coordinates we have

\[ q_1 = r, \]
\[ q_2 = \phi, \]
\[ p_1 = m \dot{q}_1, \]
\[ p_2 = m q_1^2 \dot{q}_2, \]

where \( p_2 \) is to be augmented by a \( \gamma \) factor in the relativistic case as discussed in section 2. The Hamilton equations for the non-relativistic case are listed in Table 1, first for an inertial system, then for a system with two dimensions \((r, \phi)\). The inertial system has only one coordinate \( q_r = r \). The orbital motion seen from an external observer is added in the plane polar system. By rewriting (35-38), the equations of motion are the same as obtained from the Euler-Lagrange equations which are of second order:

\[ \ddot{r} = \dot{\phi}^2 r - \frac{GM}{r^2}, \]
\[ \ddot{\phi} = -\frac{2\dot{r}\dot{\phi}}{r}. \]

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The Hamilton equations of special relativity are seldom used. The Hamiltonian can be written in two forms, see Tables 2 and 3. The first form uses a $\gamma$ factor

$$\gamma = \left(1 - \frac{p_r^2 + \frac{p^2}{m^2c^2}}{1 - \frac{p^2}{m^2c^2}}\right)^{-\frac{1}{2}}. \quad (41)$$

This $\gamma$ factor differs from standard special relativity where it is written by the velocities:

$$\gamma = \left(1 - \frac{q_r^2 + \frac{q^2}{c^2}}{1 - \frac{q^2}{c^2}}\right)^{-\frac{1}{2}}. \quad (42)$$

We have evaluated the Hamilton equations with both forms of $\gamma$ factor and found that the second form (42) does not give results that pass into the non-relativistic case when considering the limit $\gamma \to 1$. Therefore we used the form (41) containing the generalized momenta. From Table 2 can be seen that there is an augmentation by $\gamma$ in the inertial system which gives the values of Table 1 in the non-relativistic limit. For the two-dimensional case the equations become more complicated, there are additional terms in proportion to $1/c^2$, a relativistic correction.

The relativistic Hamiltonian can be written in an alternate form as in Table 3. Here no $\gamma$ factor appears. One can define a factor $\epsilon_1$ as listed there. Then the Hamiltonian can be written simply as

$$H = \frac{1}{\epsilon_1} - \frac{GMm}{q_r} \quad (43)$$

and the resulting Hamilton equations can be written in a quite simple form using $\epsilon_1$, see Table 3. It has been shown by computer algebra that theses equations are identical to those obtained in Table 2. However the relativistic corrections are not visible so directly as in the form of Table 2.

For a numerical solution the Hamilton equations are well suited because they are of first time order only. Since the right hand sides in Table 2 do not depend on the time derivatives, the equations can be programmed directly in this way. We used a model system of an orbiting mass as in previous papers. In the non-relativistic case we obtain an ellipse graphed in Fig. 1. The relativistic version leads to a precession, see Fig. 2. This is the same behaviour as for the solution of the Euler-Lagrange equations as expected. However, for identical parameters, the ellipses are equal in radial extension but the relativistic effects (precession) are much greater for the Hamilton equations. The reason could be that the $\gamma$ factor is defined by momenta here, while it is defined by velocities in the Euler-Lagrange equations. Since $\gamma$ is implicitly contained in the momenta, this could be the reason for a significantly different solution with greater effects. We had to increase the velocity of light from 20 to 50 units (i.e. decrease relativistic effects) to obtain roughly the same precession by the Hamilton equations.

### 3.2 Self-consistency of the $\gamma$ factor

We tried to resolve the problem that the $\gamma$ factor depends on a function that depends on $\gamma$ itself:

$$\gamma = f(\gamma, p_\theta) \quad (44)$$
<table>
<thead>
<tr>
<th>system</th>
<th>Hamiltonian</th>
<th>Hamilton equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>inertial</td>
<td>$H = \frac{p^2}{2m} - \frac{GMm}{q_r}$</td>
<td>$\dot{q}<em>r = \frac{p_r}{m}$, $\dot{q}</em>\phi = 0$, $\dot{p}<em>r = \frac{GMm}{q_r}$, $\dot{p}</em>\phi = 0$</td>
</tr>
<tr>
<td>polar coord.</td>
<td>$H = \frac{1}{2m} \left( p_r^2 + \frac{p_\phi^2}{q_r^2} \right) - \frac{GMm}{q_r}$</td>
<td>$\dot{q}<em>r = \frac{p_r}{m}$, $\dot{q}</em>\phi = \frac{m p_\phi}{q_r^3}$, $\dot{p}<em>r = \frac{p</em>\phi^2}{q_r^3} - \frac{GMm}{q_r}$, $\dot{p}_\phi = 0$</td>
</tr>
</tbody>
</table>

Table 1: Non-relativistic Hamilton equations in inertial frame and plane polar coordinates.

<table>
<thead>
<tr>
<th>system</th>
<th>$\gamma$ and Hamiltonian</th>
<th>Hamilton equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>inertial</td>
<td>$\gamma = \left( 1 - \frac{p_r^2}{m^2 c^2} \right)^{-1/2}$</td>
<td>$\dot{q}<em>r = \left( \frac{\dot{\gamma} - \gamma}{m} \right) \frac{p_r}{\gamma c^2 m^3}$, $\dot{q}</em>\phi = 0$, $\dot{p}<em>r = \frac{GMm}{q_r}$, $\dot{p}</em>\phi = 0$</td>
</tr>
<tr>
<td>polar coord.</td>
<td>$\gamma = \left( 1 - \frac{p_r^2 + p_\phi^2/q_r^2}{m^2 c^2} \right)^{-1/2}$</td>
<td>$\dot{q}<em>r = \left( \frac{\dot{\gamma} - \gamma}{m} \right) \frac{p_r}{\gamma c^2 m^3}$, $\dot{q}</em>\phi = \left( \frac{\dot{\gamma} - \gamma}{m} \right) \frac{p_\phi}{\gamma c^3 m^3 q_r}$, $\dot{p}<em>r = \frac{GMm}{q_r}$, $\dot{p}</em>\phi = 0$</td>
</tr>
</tbody>
</table>

Table 2: Relativistic Hamilton equations in inertial frame and plane polar coordinates.

<table>
<thead>
<tr>
<th>system</th>
<th>$\epsilon_1$ and Hamiltonian</th>
<th>Hamilton equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>inertial</td>
<td>$H = \sqrt{c^2 p_r^2 + m^2 c^4} - \frac{GMm}{q_r}$</td>
<td>$\dot{q}<em>r = \frac{c^2 p_r}{\sqrt{c^2 p_r^2 + m^2 c^4}}$, $\dot{q}</em>\phi = 0$, $\dot{p}<em>r = \frac{GMm}{q_r}$, $\dot{p}</em>\phi = 0$</td>
</tr>
<tr>
<td>polar coord.</td>
<td>$\epsilon_1 = \left( \frac{c^2 p_r^2}{q_r^2} + p_\phi^2 \right)^{-1/2}$</td>
<td>$\dot{q}<em>r = \epsilon_1 c^2 p_r$, $\dot{q}</em>\phi = \epsilon_1 c^2 p_\phi$, $\dot{p}<em>r = \frac{GMm}{q_r}$, $\dot{p}</em>\phi = 0$</td>
</tr>
</tbody>
</table>

Table 3: Relativistic Hamilton equations in inertial frame and plane polar coordinates, alternative form.
with
\[ p_\phi = \gamma m q^2 \dot{q}_\phi. \] (45)

For simplicity we write
\[ p = \gamma m v^2 \] (46)

with the modulus of velocity \( v \). This then gives
\[ \gamma = \left( 1 - \frac{\gamma v^2}{c^2} \right)^{-\frac{1}{2}}. \] (47)

It is possible to resolve this equation for \( \gamma \), giving two solutions
\[ \gamma_{1,2} = \frac{v^2}{2\gamma^2} \left( 1 \pm \sqrt{1 - \frac{4v^2}{c^2}} \right). \] (48)

Only the solution with the minus sign gives the correct limit
\[ \gamma \rightarrow 1 \quad \text{as } v \rightarrow 0. \] (49)

Unfortunately this solution has a pole of \( \gamma \) for \( v = c/2 \). This can be avoided by redefining
\[ \gamma = \left( 1 - \frac{v^2}{4c^2} \right)^{-\frac{1}{2}}, \] (50)

giving the solutions
\[ \gamma_{1,2} = \frac{2c^2}{v^2} \left( 1 \pm \sqrt{1 - \frac{v^2}{c^2}} \right). \] (51)

Here the correct \( \gamma \) factor re-appears but there is no infinite limit for \( v \rightarrow c \), instead we find
\[ \gamma \rightarrow \sqrt{2} \quad \text{as } v \rightarrow c. \] (52)

Interestingly the same limit was found for a \( \gamma \) factor describing the gravitational light deflection of photons correctly as described in earlier papers:
\[ \gamma_{\text{photon}} = \left( 1 - \frac{v^2}{2c^2} \right)^{-\frac{1}{2}}, \] (53)

which has the same limit for \( v \rightarrow c \). Both \( \gamma \) factors are compared in Fig. 3, together with the usual \( \gamma \) definition
\[ \gamma = \left( 1 - \frac{v^2}{c^2} \right)^{-\frac{1}{2}}. \] (54)

Obviously the usual \( \gamma \) function rises much more steeply than the others. The function from the self-consistent calculation rises most slowly and meets the photon \( \gamma \) at \( v = c \) with \( \gamma = \sqrt{2} \) as described. This is a remarkable result, bringing together different paths of ECE development. The difference of the three \( \gamma \) factors has to be investigated further.
3.3 Computation of the function of action $S_r$

The radial function of action $S_r$ is part of the Hamilton-Jacobi equation (28) for central motion. This is a differential equation for $S_r$. It is possible to find a solution of this function by computer algebra. First, equation (28) has to be resolved for $(\partial S_r / \partial r)^2$. This gives a quartic equation for $\partial S_r / \partial r$. Solving this gives four similar looking differential equations containing the parameters of (28):

$$\frac{\partial S_r(r)}{\partial r} = \pm \sqrt{(E^2 - c^4 m^2) r^2 \pm 2EGMmr + G^2 M^2 m^2 - L^2 c^2}. \quad (55)$$

The solutions are analytical and highly depend on ratios between the parameters. We therefore put in the parameters of the numerical model calculations. The total energy including the term $mc^2$ has to be used. One obtains four complex functions. The real parts have been graphed in Fig. 4. It can be seen that these are (besides a null function) exactly two inverse functions, probably describing the two possible directions of motion of the orbiting mass. The functions are non-constant exactly in the physical range of $r$ which in this example is $0.3 < r < 1$. This is obviously the first time the relativistic action function $S_r$ was determined for the central motion problem.
Figure 1: Non-relativistic orbit of central motion from Hamilton equations.

Figure 2: Relativistic orbit of central motion from Hamilton equations.
Figure 3: $\gamma$ functions from three models.

Figure 4: Four solutions for the relativistic action $S_r(r)$. 
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