HAMILTON EQUATIONS OF \( m \) THEORY

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ABSTRACT

The recently inferred \( m \) theory, an inference of ECE unified field theory, is developed using the Hamilton equations of motion and the Hamilton Jacobi equations, two complete formalisms of classical dynamics. It is shown that the Hamilton equations give the same vacuum force as the Euler Lagrange equations used in UFT417, giving a rigorous cross check of concepts and technique. The Hamilton Jacobi dynamics are used to calculate and compute the action for \( m \) theory in readiness for quantization.

Keywords: ECE unified field theory, \( m \) theory, Hamilton equations.
1. INTRODUCTION

In recent papers of this series {1 - 41} the Hamilton and Hamilton Jacobi dynamics have been developed and applied on the classical level and on the ECE2 level. In Section 2 the Hamilton and Hamilton Jacobi equations are applied to m theory and it is shown that the vacuum force can be defined as the force due to the most general spherically symmetric spacetime (m space). It is shown that the force due to m space calculated with the Hamilton equations is the same as the force due to m space calculated with the Euler Lagrange equations in UFT417. This is another rigorous demonstration of the self consistency of concepts and calculations. The Hamilton Jacobi formalism is used to calculate the action function in preparation for quantization. In section 3 the calculations are checked with computer algebra and the main results presented graphically.

This paper is a short synopsis of Note 427(1) posted with UFT427. Notes 427(2) onwards are used for UFT428 on the quantization of m theory.

2. HAMILTON AND HAMILTON JACOBI EQUATIONS

Consider the relativistic momentum of m theory:

\[ \mathbf{p}_1 = \gamma m \mathbf{v}_1 \]  \hspace{1cm} (1)

in frame (\( r_1, \phi \)), where \{1 - 41\}:

\[ \mathbf{r}_1 = \frac{c}{m(r_1)^{1/2}} \]  \hspace{1cm} (2)

and

\[ \mathbf{v}_1 = \frac{\mathbf{v}_N}{m(r_1)^{1/2}} \]  \hspace{1cm} (3)

where \( \mathbf{v}_N \) is the Newtonian velocity.
It follows that:

\[ p_r^2 c^2 = \gamma^2 m^2 v_r^2 c^2 \] \hspace{1cm} (4)

where the generalized Lorentz factor of m theory is:

\[ \gamma = \left( m(r_i) - \frac{v_r^2}{c^2} \right)^{-1/2} \] \hspace{1cm} (5)

Therefore:

\[ p_r^2 c^2 = \gamma^2 m^2 c^4 \left( m(r_i) - \frac{1}{\gamma^2} \right) = \frac{E^2}{m(r_i)} - m^2 c^4 \] \hspace{1cm} (6)

It follows that the Einstein energy equation in m theory is:

\[ E^2 = m(r_i) \left( p_r^2 c^2 + m^2 c^4 \right) \] \hspace{1cm} (7)

The Hamiltonian of m theory is:

\[ H = m(r_i) \frac{\gamma}{r_i} \sqrt{m^2 c^2} = m(r_i) \frac{\sqrt{m^2 c^2}}{r_i} \] \hspace{1cm} (8)

Now define Hamilton's canonically conjugate generalized coordinates to be:

\[ p_r = p_r, \quad q_r = r_i \] \hspace{1cm} (9)

and

\[ p_\phi = L_1, \quad q_\phi = \phi \] \hspace{1cm} (10)

The Hamilton equations are:

\[ \dot{p}_r = - \frac{\partial H}{\partial q_r} \] \hspace{1cm} (11)
\[ \dot{\mathbf{r}} = \frac{\partial \mathbf{H}}{\partial \mathbf{p}} = -(12) \]
\[ \dot{p}_\phi = -\frac{\partial \mathbf{H}}{\partial \mathbf{q}_\phi} = -(13) \]
\[ \dot{\mathbf{q}}_\phi = \frac{\partial \mathbf{H}}{\partial \mathbf{p}_\phi} = -(14) \]

From Eq. (11):
\[ \dot{p}_1 = -\frac{nm_6}{r_1^3} - \frac{d}{dr_1} \left( \frac{m(r_1)}{r_1^2} \left( p_1^2 c + m_4^2 c^4 \right) \right)^{1/5} = -nm_6 - \frac{E}{m(r_1)} \frac{d}{dr_1} \left( m(r_1)^{1/5} \right) = -(15) \]

From the Lagrangian analysis of UFT417:
\[ \dot{p}_1 = -\frac{nm_6}{r_1^3} - \frac{ymc^2}{2} \frac{dm(r_1)}{dr_1} = -(16) \]

Now use:
\[ E = m(r_1) \gamma mc^2 = -(17) \]

and compare Eqs. (15) and (16)
\[ E \frac{d}{dr_1} \left( m^{1/2}(r_1) \right) = E \frac{dm(r_1)}{dr_1} = -(18) \]
\[ \frac{dm(r_1)}{dr_1} \]

to obtain:
\[ \frac{1}{2m^{1/2}(r_1)} \frac{dm(r_1)}{dr_1} = -(19) \]

Let
\[ f = m(r_1)^{1/2} = -(20) \]
to find that:
\[
\frac{\partial \dot{r}_1}{\partial \dot{m}_i} = \frac{\partial}{\partial m_i} \left( \frac{\partial m_i}{\partial \dot{r}_1} \right) = \frac{1}{2m_{ii}(r)} \frac{\partial m_i}{\partial \dot{r}_1}. \tag{21}
\]

Therefore Eq. (19) is confirmed, showing that the force from m space is the same in the Hamilton and Euler Lagrange system of dynamics, Q.E.D. The force magnitude is:
\[
F = -mc^2 \sum \frac{\partial m_i}{\partial \dot{r}_1} = -E \frac{1}{m(r)} \frac{\partial}{\partial \dot{r}_1} \left( m^{1/2}(r) \right). \tag{22}
\]

Therefore the concepts and calculations are rigorously self consistent, and show that m space produces a force and energy which is missing from Minkowski spacetime and from classical dynamics on the Newtonian level.

The Hamilton equation (12) gives:
\[
\dot{r}_1 = \frac{\partial H}{\partial \dot{p}_1} \tag{23}
\]

where:
\[
H = \left( m(r) \left( p_1 c^2 + mc^4 \right) \right)^{1/2} - \frac{mM}{r_1} \tag{24}
\]

Therefore:
\[
\frac{\partial H}{\partial \dot{p}_1} = \frac{m(r) p_1 c}{m(r) \gamma mc^2} = \frac{p_1}{m \gamma} \tag{25}
\]

where we have used:
\[
E = m(r) \gamma mc^2 = \left( m(r) \left( p_1 c^2 + mc^4 \right) \right)^{1/2} \tag{26}
\]

Therefore:
\[
p_1 = \gamma m \dot{r}_1 \tag{27}
\]
which is the correct definition of relativistic momentum, Q. E. D. The Hamilton equations applied to m theory are rigorously self consistent, Q. E. D.

To extend the theory to plane polar coordinates use:

$$p_1 = \gamma m \dot{r} - \gamma m \left( \frac{\dot{r}}{r} + \frac{\dot{\phi}}{\phi} \right) - \left( 28 \right)$$

so:

$$p_1^2 = p_r^2 + \frac{p_\phi^2}{r^2} = p_r^2 + \frac{L_1^2}{r^2} - \left( 29 \right)$$

where:

$$L_1 = p_\phi - \left( 30 \right)$$

is the conserved angular momentum. The second Evans Eckardt equation is:

$$\dot{L}_1 = \frac{dL_1}{dt} = 0. - \left( 31 \right)$$

The Hamilton equation \((\mathbf{14})\) gives:

$$\dot{\phi} = \frac{dH}{dL_1} - \left( 32 \right)$$

where the hamiltonian is:

$$H = \frac{m_0 (r_1) \left( \left( \frac{p_r^2 + L_1^2}{r_1^2} \right) c^2 + m_0 c^4 \right)^{1/2}}{r_1} - \frac{nm_e \gamma m_0 - \left( 33 \right)}{r_1}$$

It follows that:

$$\dot{\phi} = \frac{dH}{dL_1} = \frac{m_0 (r_1) L_1 c^2}{r_1^2 m_0 (r_1) \gamma m_e} = \frac{L_1}{\gamma m_0 r_1} - \left( 34 \right)$$

i.e.

$$L_1 = \gamma m_0 r_1^2 \dot{\phi} - \left( 35 \right)$$
which is the angular momentum of m theory found by an Euler Lagrange analysis in UFT417, Q. E. D.

The m theory in Hamilton dynamics and Euler Lagrange dynamics is rigorously self consistent, both theories give the same results. This is a rigorous check of the self consistency of concepts and calculations.

The Hamilton Jacobi equations of m theory are found from Eq. (33) using:

\[ p_r = \frac{\partial S}{\partial r}, \quad p_\phi = \frac{\partial S}{\partial \phi} \]  

where \( S \) is the action function:

\[ S = S_r + S_\phi \]  

Therefore the Hamilton Jacobi equations of m theory are:

\[ E_1 = \left( m(r) \left( \left( \frac{\partial S_r}{\partial r}\right)^2 + \frac{L_1^2}{r^2} \right) + c^2 m(r) v^2 \right)^{1/2} - \frac{\pi M r}{r_1} \]  

and

\[ L_1 = \frac{\partial S_\phi}{\partial \phi} \]  

These equations are integrated by computer in Section 3 to give the action functions \( S_r \) and \( S_\phi \). This is a route towards the quantization of m theory because the quantum of action is \( h \) and is the very basis of quantum mechanics.
3 Additional calculations and graphics

3.1 Hamilton equations of central motion in m theory

We have evaluated the Hamilton equations for m theory in two formulations of the Hamiltonian. This is done in analogy to UFT 426, Tables 2 and 3. We worked out the Hamilton equations of central motion in an inertial frame and in a plane polar coordinate system. The general form of the Hamilton equations is

\[ \dot{q}_i = \frac{\partial H}{\partial p_i} \]  
\[ \dot{p}_i = -\frac{\partial H}{\partial q_i} \]  

(40)

(41)

where \( q_i \) are the canonical or generalized coordinates and \( p_i \) are the conjugate canonical momenta. The index \( i \) refers to the coordinate components. The radial coordinate \( r \) has to be replaced by \( r_1 \) in m theory as defined in Eq. (2). In polar polar coordinates we have

\[ q_1 = r_1 = \frac{r}{\sqrt{m(r_1)}}, \]  
\[ q_2 = \phi, \]  
\[ p_1 = m \dot{q}_1, \]  
\[ p_2 = \gamma m q_1^2 \dot{q}_2, \]  

(42)

(43)

(44)

(45)

with the generalized \( \gamma \) factor

\[ \gamma = \left( m(q_1) - \frac{p_1^2 + \frac{p_2}{m_2 c^2}}{m^2 c^2} \right)^{-\frac{1}{2}}. \]  

(46)

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The resulting Hamilton equations written with the $\gamma$ factor are listed in Table 1. Compared to the inertial case, the angular coordinate gives additional terms for $\dot{q}_2$ and $\dot{p}_1$. We find the typical terms of $m$ theory with factors of $m(q_1)$ and $dm(q_1)/dq_1$. Similar effects are found for the alternative Hamiltonian of Table 2 where the characteristic function $\epsilon_1$ appears instead of the $\gamma$ factor as already discussed in UFT 426.

The equations of Table 1 have been solved numerically for a model system. Fig. 1 shows the orbit for $m=1$. There is a negative precession for the parameters used. The same parameters and initial conditions were taken in the calculation graphed in Fig. 2, using an exponential $m$ function as used in previous papers. The type of orbit changes to strong positive precession. As observed earlier, the deviations from Newtonian theory manifest themselves by several kinds of precession.

\[
\begin{align*}
\text{inertial} & \quad \gamma = \left( m(q_1) - \frac{\mu_1^2}{m_1 c^2} \right)^{-1/2} \\
H & = m(q_1) \gamma mc^2 - \frac{GMm}{q_1} \\
\dot{q}_1 & = m(q_1) \gamma^3 \frac{p_1}{m} \\
\dot{q}_2 & = 0 \\
\dot{p}_1 & = \frac{d m(q_1)}{dq_1} \gamma mc^2 \left( m(q_1) \frac{p_1^2}{2} - 1 \right) - \frac{GMm}{q_1^2} \\
\dot{p}_2 & = 0
\end{align*}
\]

\[
\begin{align*}
\text{polar coord.} & \quad \gamma = \left( m(q_1) - \frac{\mu_1^2 + \mu_2^2/(q_1^2)}{m_1 c^2} \right)^{-1/2} \\
H & = m(q_1) \gamma mc^2 - \frac{GMm}{q_1} \\
\dot{q}_1 & = m(q_1) \gamma^3 \frac{p_1}{m} \\
\dot{q}_2 & = m(q_1) \gamma^3 \frac{p_2}{m q_1} \\
\dot{p}_1 & = \frac{d m(q_1)}{dq_1} \gamma mc^2 \left( m(q_1) \frac{p_1^2}{2} - 1 \right) + m(q_1) \gamma^3 \frac{p_2^2}{m q_1} - \frac{GMm}{q_1^2} \\
\dot{p}_2 & = 0
\end{align*}
\]

Table 1: Hamilton equations of $m$ theory in inertial frame and plane polar coordinates.

\[
\begin{align*}
\text{inertial} & \quad \epsilon_1 = \sqrt{\frac{m(q_1)}{c^2 p_1^2 + m_2 c^4}} - \frac{GMm}{q_1} \\
H & = \sqrt{\frac{m(q_1)}{c^2 \left( \frac{p_1^2}{q_1^2} + p_1^2 \right) + m_2 c^4}} - \frac{GMm}{q_1} \\
\dot{q}_1 & = \frac{\sqrt{m(q_1)}}{c^2 p_1^2 + m_2 c^4} c^2 p_1 \\
\dot{q}_2 & = 0 \\
\dot{p}_1 & = \frac{d m(q_1)}{dq_1} \sqrt{\frac{c^2 p_1^2 + m_2 c^4}{m(q_1)}} - \frac{GMm}{q_1^2} \\
\dot{p}_2 & = 0
\end{align*}
\]

\[
\begin{align*}
\text{polar coord.} & \quad \epsilon_1 = \left( c^2 \left( \frac{p_1^2}{q_1^2} + p_1^2 \right) + m_2 c^4 \right)^{-1/2} \\
H & = \sqrt{\frac{m(q_1)}{c^2 \left( \frac{p_1^2}{q_1^2} + p_1^2 \right) + m_2 c^4}} - \frac{GMm}{q_1} \\
\dot{q}_1 & = \sqrt{m(q_1)} \epsilon_1 c^2 p_1 \\
\dot{q}_2 & = \sqrt{m(q_1)} \frac{\epsilon_1 c^2 p_2}{q_1} \\
\dot{p}_1 & = -\frac{d m(q_1)}{dq_1} \frac{2 \epsilon_1 \sqrt{m(q_1)}}{q_1} + \sqrt{\frac{m(q_1)}{c^2 p_1^2 + m_2 c^4}} - \frac{GMm}{q_1^2} \\
\dot{p}_2 & = 0
\end{align*}
\]

Table 2: Hamilton equations of $m$ theory in inertial frame and plane polar coordinates, alternative form.
3.2 Computation of the function of action $S_r$ in m theory

The Hamilton-Jacobi equation (38) of m theory has been solved similarly as described in UFT 426. By m theory only a factor of $m(q_1)$ is multiply added to the expressions derived in the preceding paper. We could simplify the solution method and avoid a quartic equation. The resulting differential equation is

$$\frac{\partial S_r(q_1)}{\partial q_1} = \pm \sqrt{-\left(c^4 m^2 q_1^2 + L^2 c^2\right) m(q_1) + E^2 q_1^2 + 2EGm q_1 + G^2 M^2 m^2} m(q_1) + E q_1 \sqrt{m(q_1)}.$$  (47)

This equation is not analytically solvable with a general $m(q_1)$. We graphed two solutions with constant $m(q_1) = 1$ and $m(q_1) = 0.99$, see Fig. 1. For $m(q_1) = 1$ the same result as found for UFT 426 comes out. For $m(q_1) = 0.99$ the results differ significantly, the orbital radius nearly doubles. We find as before that the results depend very sensitively on the form of $m(q_1)$. The effects are similar as in chaos theory where very small changes of the equations of state effect large deviations of the resulting trajectories.

![Figure 1: Orbit of central motion from Hamilton equations, m=1.](image)

Figure 2: Orbit of central motion from Hamilton equations, exponential m function.

Figure 3: Solutions for the action $S_r(q_1)$ of m theory.
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