

GENERALLY COVARIANT QUANTUM MECHANICS IN m SPACE

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ABSTRACT

Schroedinger quantization is developed in m space, producing a new and generally covariant quantum mechanics, a unification of general relativity and quantum mechanics using the principles of m theory. The results are applied to the hydrogen atom, and it is shown that the generally covariant H atom contains shifts and splittings that are not present in the Dirac H atom of special relativity and its non relativistic limit, the Schroedinger H atom. The shifts produced by the unification of quantum mechanics and general relativity are identified with the Lamb shifts.

Keywords, ECE2 theory, m theory, unification of quantum mechanics and general relativity.

4FT 434

1. INTRODUCTION

In recent papers of this series {1 - 41} the theory of physics has been developed in the most general spherically symmetric space ("m space") in UFT415 to UFT433 to date, producing many original results. In Section 2 the m theory is used to unify quantum mechanics and general relativity. The fundamental Schroedinger quantization is developed in m space, producing generally covariant de Broglie / Einstein wave particle equations. The m theory is applied to the time dependent Schroedinger equation and to the H atom, thereby producing new shifts and splittings of energy levels which are identified with the Lamb shifts. The generally covariant H atom of m theory therefore describes the Lamb shifts as being due to the geometry of space itself. The Dirac H atom of special relativity does not produce the Lamb shift as is well known, and the Schroedinger H atom is the non relativistic limit of the Dirac H atom.

This paper is a brief synopsis of detailed calculations contained in the notes accompanying UFT434 on www.aias.us. Note 434(1) summarizes the equations of m theory. Note 434(2) develops the quantization of linear momentum in m space. Note 434(3) develops the quantization of energy in m space. Note 434(4) summarizes the unification of general relativity and quantum mechanics in m space. Finally Note 434(5) develops the time dependent Schroedinger equation in m space, and develops the generally covariant H atom, in which there are energy levels which are not present in the Dirac H atom of special relativity, and not present in the non relativistic limit of the Dirac H atom, the Schroedinger H atom. These new energy levels and shifts are interpreted as the Lamb shifts.

2. THEORETICAL FUNDAMENTALS

As in Notes 434(1) to 434(3), and using the principles developed in UFT415

ff., the fundamental Schroedinger quantization of energy in m space is:

$$E_1 \psi = i \hbar \frac{\partial \psi}{\partial t_1} \quad - (1)$$

where

$$\frac{\partial \psi}{\partial t_1} = \frac{1}{m(r)^{1/2}} \frac{\partial \psi}{\partial t} \quad - (2)$$

The $m(r)$ function is defined by the infinitesimal line element of m space, the most general spherically symmetric space:

$$ds^2 = c^2 d\tau^2 = m(r) c^2 dt^2 - \frac{dr^2}{m(r)} - r^2 d\phi^2 \quad - (3)$$

where τ is the proper time. Eq. (3) has been written in the plane polar coordinate system, (r, ϕ) , but can be written in any coordinate system. Eq. (2) follows from the fact that:

$$dt_1 = m(r)^{1/2} dt \quad - (4)$$

The expectation values of energy in m space are therefore:

$$\langle E_1 \rangle = i \hbar \int \psi^* \frac{1}{m(r)^{1/2}} \frac{\partial \psi}{\partial t} d\tau \quad - (5)$$

in which the presence of $m(r)^{1/2}$ produces shifts and splittings which are not present in the limit of special relativity:

$$m(r) = 1 \quad - (6)$$

when:

$$\langle E \rangle = i \hbar \int \psi^* \frac{\partial \psi}{\partial t} d\tau \quad - (7)$$

Using the wave function:

$$\psi = e^{-iat} \quad - (8)$$

produces the Planck photon:

$$\langle E \rangle = \hbar \omega \quad - (9)$$

Q. E. D. More photons are produced in m space, so the synthesis of the early universe was due to the nature of space itself, ^{not} due to the obsolete idea of "Big Bang". The universe has no beginning and no end.

Schroedinger quantization of momentum in m space is defined by:

$$\underline{p} \psi = -i\hbar \underline{\nabla} \psi \quad - (10)$$

where the del operator is defined in m space. As shown in Note 434(2) the radial part of the quantization is defined by:

$$p_1 \psi = -i\hbar \frac{d\psi}{dr} = -i\hbar \left(\frac{2m(r)^{3/2}}{2m(r) - r \frac{dm(r)}{dr}} \right) \frac{d\psi}{dr} \quad - (11)$$

Using the wavefunction:

$$\psi = e^{ikr} \quad - (12)$$

it follows that:

$$p_1 = \langle p_1 \rangle = \hbar k \int e^{-ikr} \left(\frac{2m(r)^{3/2}}{2m(r) - r \frac{dm(r)}{dr}} \right) e^{ikr} dr \quad - (13)$$

and in the limit of special relativity:

$$m(r) = 1 \quad - (14)$$

the de Broglie equation is obtained:

$$p = \hbar k \quad - (15)$$

Q. E. D. In m theory, generally covariant momentum quantization produces more energy levels than the de Broglie quantization (15), so new quantized photon momenta are produced. This again provides an explanation for the synthesis of the early universe because these equations apply to any wave particle.

The de Broglie / Einstein wave particle equations of special relativity are:

$$E = \gamma mc^2 = \hbar \omega \quad - (16)$$

$$\underline{p} = \gamma m \underline{v} = \hbar \underline{k} \quad - (17)$$

where E is the total relativistic energy, γ is the Lorentz factor of special relativity, \underline{p} is the relativistic momentum and \underline{v} the Newtonian linear velocity. In general relativity and m theory these equations become:

$$E_1 = \gamma_1 m(r_1) mc^2 = i \hbar \int \psi^* \frac{\partial \psi}{\partial t_1} d\tau \quad - (18)$$

$$\underline{p}_1 = \gamma_1 m \underline{v}_1 = - i \hbar \int \psi^* \underline{\nabla}_1 \psi d\tau \quad - (19)$$

and more energy levels and splittings appear. From UFT415 ff, the total relativistic energy in m space is:

$$E_1 = \gamma_1 m(r_1) mc^2 \quad - (20)$$

the relativistic momentum in m space is:

$$\underline{p}_1 = \gamma_1 m \underline{v}_1 \quad - (21)$$

where:

$$\underline{v}_1 = \frac{v}{m(r_1)^{1/2}} \quad - (22)$$

and the generalized Lorentz factor is:

$$\gamma_1 = \left(m(r_1) - \frac{v_1^2}{m(r_1)c^2} \right)^{-1/2} \quad - (23)$$

It follows that:

$$E_1^2 = m(r_1) (p_1^2 c^2 + m^2 c^4) \quad - (24)$$

which is the Einstein energy equation in m space.

Using the usual assumption of quantum mechanics:

$$\psi(r, t) = e^{-i\omega t} \psi(r) \quad - (25)$$

it follows that energy levels in m space are defined by:

$$E_1 = \langle E_1 \rangle = \frac{\int \psi^* \frac{1}{m(r)^{1/2}} \psi d\tau}{\int \psi^* \psi d\tau} \quad - (26)$$

so the original Planck photon (9), is augmented by other photons. energy levels of Eq.

(26). The momentum levels in m space can be expressed as:

$$P_1 = \langle P_1 \rangle = -i\hbar \int \psi^* \left(\frac{2m(r)^{3/2}}{2m(r) - r \frac{dm(r)}{dr}} \right) \frac{d\psi}{dr} d\tau \quad - (27)$$

All observed particles can be described in this way. each particle has energy levels (26)

and momentum levels (27).

Finally consider the conventional time dependent Schroedinger equation, it follows

that:

$$i\hbar \frac{d\psi}{dt} = E\psi = H\psi, \quad - (28)$$

$$i\hbar \frac{\partial \psi}{\partial t} = H\psi \quad - (29)$$

In m theory and general relativity this becomes:

$$i\hbar \frac{\partial \psi}{\partial t_1} = H_1 \psi \quad - (30)$$

i. e.

$$\frac{i\hbar}{m(r)^{1/2}} \frac{\partial \psi}{\partial t} = H_1 \psi \quad - (31)$$

where the hamiltonian in m space is:

$$H_1 = \gamma_1 m(r) mc^2 + \bar{U}_1 \quad - (32)$$

in which γ_1 is defined by Eq. (23), and in which \bar{U}_1 is the potential energy in m space. The Coulomb potential for example is:

$$\bar{U}_1 = - \frac{e^2 m(r)^{1/2}}{4\pi \epsilon_0 r} \quad - (33)$$

Using the assumption:

$$\psi(r, t) = e^{-i\omega t} \psi(r) \quad - (34)$$

in Eq. (31) it follows that:

$$\langle H_1 \rangle = \int \psi^* H_1 \psi d\tau = \hbar \omega \int \psi^*(r) \frac{1}{m(r)^{1/2}} \psi(r) d\tau$$

In spherical polar coordinates:

$$\psi(r, \theta, \phi, t) = e^{-i\omega t} \psi(r, \theta, \phi) \quad - (36)$$

In the non relativistic limit, Eq. (14), Eq. (29) is the time dependent Schroedinger

equation of the H atom, with:

$$\psi(r, \theta, \phi, t) = e^{-i\omega t} \psi_H(r, \theta, \phi) \quad - (37)$$

where ψ_H are the well known hydrogenic wavefunctions. In this case:

$$|E| = \left| \int \psi^* H \psi d\tau \right| = \left(\frac{\mu e^4}{32\pi^2 \epsilon_0^2 \hbar^2} \right) \frac{1}{n^2} = \hbar \omega_n \quad - (38)$$

The well known energy levels of the Schroedinger H atom are negative:

$$E = - \frac{\mu e^4}{32\pi^2 \epsilon_0^2 \hbar^2} \cdot \frac{1}{n^2} \quad - (39)$$

where n is the principal quantum number so the moduli of the energy levels are used in Eq.

(38) because $\hbar \omega_n$ must be positive.

In the m theory of Eq. (35) more energy levels of the H atom appear, and these are detected experimentally as Lamb shifts. Therefore in m theory, the Lamb shift is the effect of the m space on the H atom. In the obsolete standard model, the Lamb shift is the effect of the vacuum on the H atom. Therefore "the vacuum" is m space, Q. E. D.

Generally covariant quantum mechanics in m space

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3 Energy levels of m theory for Hydrogen

According to Eq. (35) the expectation value of energy in m space is the de Broglie frequency $\hbar\omega$ multiplied by a correction factor F :

$$\langle H_1 \rangle = \hbar\omega F \quad (40)$$

with

$$F = \int \psi^*(r) \frac{1}{m(r)^{1/2}} \psi(r) d\tau. \quad (41)$$

In Schrödinger theory where we have $m(r)=1$, F is the normalization integral and is unity for normalized wave functions ψ . In this section we compute F for several approaches of $m(r)$ in m space of general relativity. We use four approaches:

$$m_1(r) = 2 - \exp\left(\log(2) \exp\left(\frac{-r}{R}\right)\right) \quad (42)$$

$$m_2(r) = 1 - \exp\left(\frac{-r}{R}\right) \quad (43)$$

$$m_3(r) = \frac{r^2}{r^2 + 4Rr + 4R^2} \quad (44)$$

$$m_4(r) = \frac{r^2}{r^2 + Rr + \frac{R^2}{4}} \quad (45)$$

The first is the m function used earlier, the second is a simplified version thereof. The third and fourth $m(r)$ are the rational form found in UFT 432, with halved and doubled constant R . The radius R was used as a parameter to evaluate the

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integral (41). We used the wave functions of Hydrogen and put R in the range being found relevant when computing the Lamb shift, see UFT 429. The units are atomic units where the proton radius is $1.6e-5 a_0$.

We started with computing the expectation value of $m_1(r)$. The results are shown in Fig. 1 in dependence of R for the angular momentum eigen functions of the first three principal quantum numbers. The deviations from unity are quite small because $m(r)$ varies only near to the nucleus. The splitting is highest for the $1s$ state. There is a splitting between $2s$ and $2p$ which is not present in the non-relativistic Schrödinger energies of Hydrogen.

The values of the energy factor F are presented in Figs. 2-5 for the m functions m_1 to m_4 . When the radius parameter R is enlarged, the integral takes larger values as expected. Again the effect is highest for the $1s$ state, however the effects deviate only by 10^{-9} from unity for the exponential m functions. The effects are larger for the rational functions because they have a much wider range.

There is an interesting difference between the exponential and rational m functions. The exponential m functions (Figs. 2 and 3) show only shifts of the s orbitals. p and d orbitals are unaffected. For the rational functions (Figs. 4 and 5), the p and d levels are affected, but the shift is the same as for the s orbitals, that means it depends on the principal quantum number only. There seems to be a rich structure to appear when several kinds of m functions are considered.

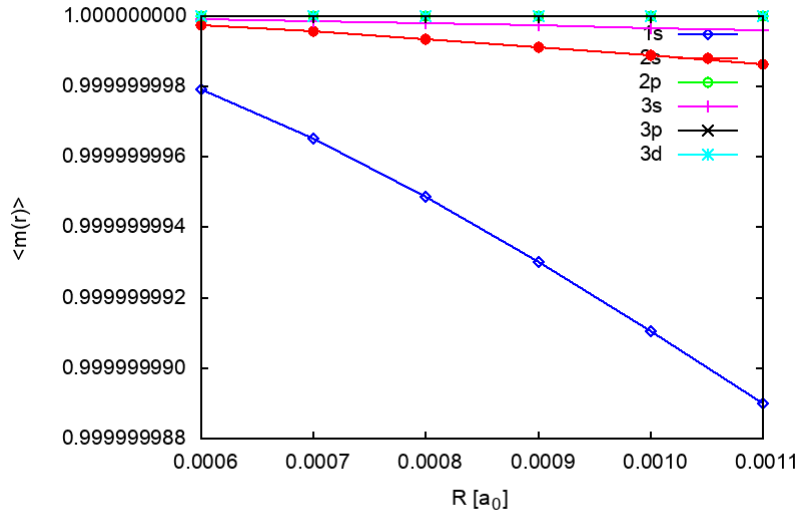


Figure 1: Expectation values of $m_1(r)$.

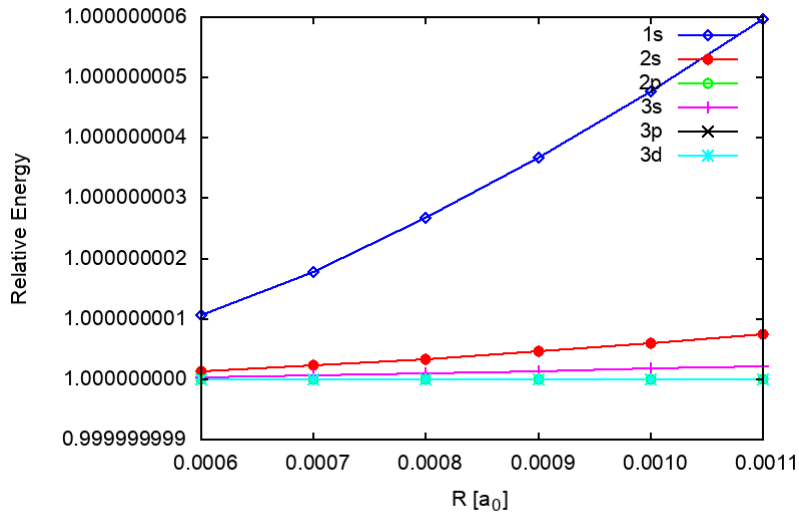


Figure 2: Energy shift factors of $m_1(r)$.

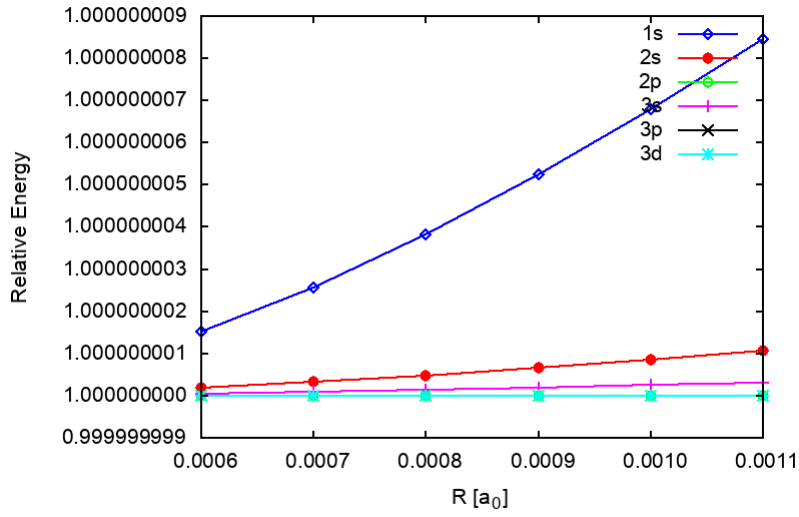


Figure 3: Energy shift factors of $m_2(r)$.

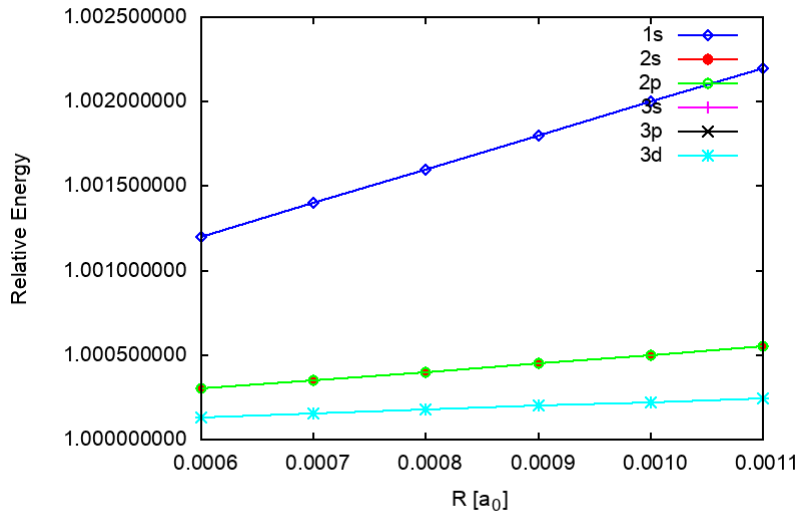


Figure 4: Energy shift factors of $m_3(r)$.

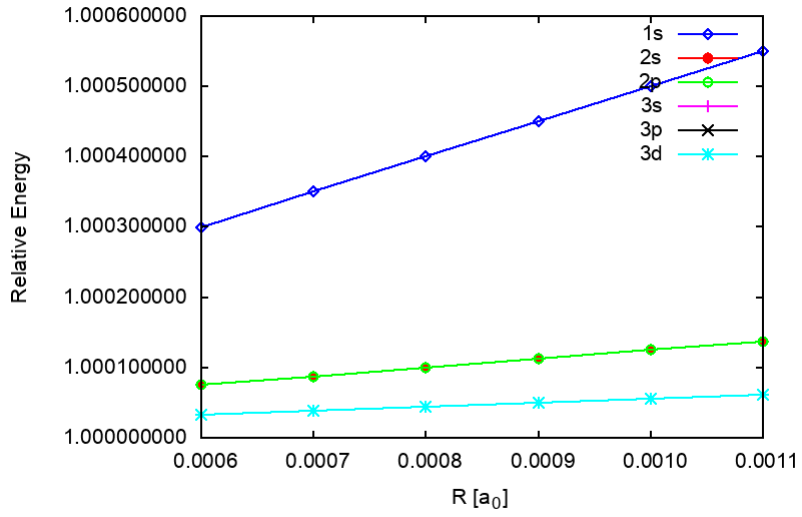


Figure 5: Energy shift factors of $m_4(r)$.

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