Chapter 1

Evans Field Theory Of The Sagnac Effect

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Abstract

The Sagnac effect is described straightforwardly in Evans unified field theory as an example of rotational relativity. The circulating light in a Sagnac interferometer at rest is a rotation of spacetime described by a tetrad field. This is multiplied by the scalar valued vector potential magnitude $A^{(0)}$ to produce vector potentials rotating at an angular frequency

$$\omega_1 = \frac{c}{r}$$

where r is the radius of the circular platform. The additional mechanical spinning of the platform results in a time delay which is the Sagnac effect. The time delay is that between the light circulating with the spinning platform and the light circulating against the spinning platform. This is observed as a frame independent phase shift. Thus the Sagnac effect is an example of general or rotational relativity in optics and electrodynamics.

Keywords: Evans field theory; general and rotational relativity in optics and electrodynamics; the Sagnac effect.

1.1 Introduction

There have been many attempts to explain the well known Sagnac effect using special relativity and gauge theory [1]– [3]. The effect is observed as a phase shift in a mechanically rotated Sagnac interferometer and has been developed into a high accuracy ring laser gyro. The rotational motion implies the use of general relativity to explain the effect theoretically. Thus the many attempts over more than ninety years based on special relativity are not valid because the latter theory does not deal with the accelerations automatically introduced by rotation. Barrett [4] has offered an explanation based on gauge theory and non-simply connected topology. This is in the spirit of general relativity, but is transitional towards the fully developed Evans field theory [5]– [30], which is a straightforward extension of the Einstein field theory of gravitation to the unified field.

In section 1.2 the effect is understood straightforwardly as one of general or rotational relativity [31]– [38] in electrodynamics and optics. The rotating light beam of the static Sagnac interferometer sets up a rotating tetrad field and a rotating potential. The angular frequency of rotation (radians per second) is:

$$\omega_1 = \frac{c}{r} \tag{1.1}$$

where c is the vacuum speed of light and r the radius of the circular platform, of area πr^2. The vacuum speed of light is a universal constant of general relativity [39]. The radius r can be thought of as a Thomson or photon radius. Its inverse is a wavenumber:

$$\kappa = \frac{1}{r}. \tag{1.2}$$

The mechanical rotation of the platform at an angular frequency ω_1 produces phase shifts in the circulating tetrad fields of the Evans field theory, and from these shifts a time delay can be calculated and compared with the experimental result. The time delay is:

$$\Delta t = 2\pi \left(\frac{1}{\omega_1 - \Omega} - \frac{1}{\omega_1 + \Omega} \right) = \frac{4\pi\Omega}{\omega_1^2 - \Omega^2}, \tag{1.3}$$

and is the delay between a beam rotating with the spinning platform and a beam rotating against the spinning platform. This is the Sagnac effect and is a clear experimental proof to very high precision of the fact that the electromagnetic field in general relativity is spinning spacetime [5]– [30]. These concepts do not exist in the standard model, which is based on special relativity, notably the Lorentz covariant Maxwell Heaviside equations. These are T invariant, where T is the motion reversal operator, and so cannot describe the Sagnac effect, or any type of rotational relativity such as the Faraday disc effect [31]– [38].

1.2 The Rotating Tetrad Fields

Consider the rotation of a beam of light of any polarization around a circle of area πr^2 in the XY plane at an angular frequency ω_1 to be determined. The rotation is a rotation of spacetime described by the rotating tetrad field [5]– [30]:

$$\mathbf{q}^{(1)} = \frac{1}{\sqrt{2}} (\mathbf{i} - i\mathbf{j})\, e^{i\omega_1 t} \tag{1.4}$$

i.e. rotation around the rim of the circular platform of the static Sagnac interferometer with the beam of light. The Evans Ansatz [5]– [30] converts the geometry into physics as follows:

$$\mathbf{A}^{(1)} = A^{(0)}\mathbf{q}^{(1)}. \tag{1.5}$$

The geometry is Cartan geometry, or Riemann geometry with torsion. Thus Eq.1.5 describes a vector potential field rotating around the rim of the circular Sagnac platform at rest. Rotation to the left is described by:

$$\mathbf{A}_L^{(1)} = \frac{A^{(0)}}{\sqrt{2}}\left(\mathbf{i} - i\mathbf{j}\right)e^{i\omega_1 t} \tag{1.6}$$

and rotation to the right by:

$$\mathbf{A}_R^{(1)} = \frac{A^{(0)}}{\sqrt{2}}\left(\mathbf{i} + i\mathbf{j}\right)e^{i\omega_1 t}. \tag{1.7}$$

When the platform is at rest a beam going around left-wise takes the same time to reach its starting point on the circle as a beam going around right-wise. The time delay between the two beams is:

$$\Delta t = 2\pi\left(\frac{1}{\omega_1} - \frac{1}{\omega_1}\right) = 0. \tag{1.8}$$

Note carefully that Eqs.1.6 and 1.7 do not exist in special relativity because electromagnetism is thought of as an entity superimposed on a passive or static frame which never rotates.

Now consider the beam 1.6 rotating left-wise and spin the platform left-wise at an angular frequency ω. The result is an increase in the angular frequency of the rotating tetrad, (because the spacetime is spinning more quickly):

$$\omega_1 \to \omega_1 + \Omega. \tag{1.9}$$

Similarly consider the beam 1.6 rotating left-wise and spin the platform right-wise at the same angular frequency Ω. The result is a decrease in the angular frequency of the rotating tetrad, (because the spacetime is spinning more slowly):

$$\omega_1 \to \omega_1 - \Omega. \tag{1.10}$$

The time delay between a beam circling left-wise with the platform and a beam circling left wise against the platform is:

$$\Delta t = 2\pi\left(\frac{1}{\omega_1 - \Omega} - \frac{1}{\omega_1 + \Omega}\right) \tag{1.11}$$

and this is the Sagnac effect.

In order to calculate the angular frequency ω_1 we use the well known experimental result [1]– [4], so:

$$\Delta t = \frac{4\Omega Ar}{c^2} = \frac{4\pi\Omega}{\omega_1^2 - \Omega^2} \tag{1.12}$$

where:

$$Ar = \pi r^2 \tag{1.13}$$

for a circular platform. If:

$$\Omega \ll \omega_1 \tag{1.14}$$

it is found that

$$\omega_1 = \frac{c}{r} = c\kappa \tag{1.15}$$

Q.E.D. This is the angular frequency of the rotating tetrad, or rotating spacetime.

1.3 Discussion

The well known features of the Sagnac effect are all described by this analysis of general relativity. The effect is observed as a phase shift which is frame independent, so is the same to an observer on or off the platform. The time delay itself is frame dependent but the phase is frame independent, being a scalar. The time delay is not observed directly. The effect is independent of the optical properties of the fiber that carries the beam of light around the circle, and is the same if the beam of light is guided by mirrors instead of a fiber. The reason is that the effect is due to mechanical rotation of spacetime itself, i.e of the frame of reference itself. Analogously gravitation is the bending of spacetime itself. The Sagnac effect is therefore similar [3] to the well known Tomita Chiao effect - a phase shift observed in a light beam traversing a helical optical fiber. The Sagnac effect can be thought of as a Tomita Chiao effect using a circle rather than a helix. This is usually referred to as a topological phase shift similar to the class of Berry phases. These have been shown to originate in the Evans phase of the unified field theory [5]– [30], which also gives the result 1.12 [3]. These phase shifts all originate therefore in general or rotational relativity and not in special relativity.

The Sagnac effect can be influenced [1]– [4] by gravitational or Coriolis type forces or centripetal type forces in dynamics. In the Evans field theory this type of influence is due to the fact that gravitation affects electromagnetism through the homogeneous current governing the homogeneous field equation [5]– [30]. In the absence of gravitation the current is zero, in the presence of gravitation it may be non-zero for the general spin connection. Thus solutions to the homogeneous field equation are changed by gravitation, and in consequence solutions to the rotating potential fields are changed, giving a shift in the Sagnac effect due to the influence of central gravitational forces or non-central Coriolis and centripetal forces on electromagnetism. A calculation of these effects in general must be numerical.

Closely related to the Sagnac effect is the class of geometrical phases such as that first observed by Tomita and Chiao [40]. The root cause of all geometrical phases is parallel transport [41], a basic property of general relativity. In the Evans field theory geometrical phases are due to tetrad fields. In the Tomita Chiao effect the geometrical phase manifests itself by the passage of light through a fiber wound into a helix. Nothing else is required to produce the phase, which is evidently therefore a property of spacetime itself - a property of general relativity. In special relativity (Maxwell Heaviside field theory) no such effect is predicted, contrary to the experimental data. The reason is that in special relativity the electromagnetic field is an entity which is superimposed on a passive frame of reference in flat or Minkowski spacetime. Therefore in special relativity there is no tetrad field because the tetrad is by definition the matrix which links two frames of reference [39]. In gravitational theory the upper index of the tetrad (the a index) denotes the Minkowski tangent spacetime at a point P - tangent to the base manifold indexed μ. In the Evans unified field theory the concept is extended to optics and electromagnetism. The upper index is that of the complex circular basis:

$$a = (1), (2), (3) \tag{1.16}$$

and the lower index is that of the cartesian basis:

$$\mu = X, Y, Z. \tag{1.17}$$

The existence of spin in electromagnetic theory is therefore defined by $((1), (2), (3))$ superimposed on (X, Y, Z). More generally this is the way that spin is treated [5]– [30] within the Evans field theory for any radiated or matter field. This concept can be applied to give a straightforward explanation of the Tomita Chiao effect (and all geometrical phases [1]– [4]) as follows.

The geometrical phase of Tomita and Chiao is usually expressed [40] as:

$$\phi = e^{\left(2\pi i\left(1-\frac{p}{s}\right)\right)} \tag{1.18}$$

where p is the pitch of the helix and s is its length. A helix turns 2π radians in a pitch p, so p is the wavelength λ. Align s in Z. Therefore:

$$\phi = e^{\left(2\pi i\left(1-\frac{\lambda}{Z}\right)\right)} = e^{(2\pi i)} e^{\left(-2\pi i\frac{\lambda}{Z}\right)} \tag{1.19}$$

$$= e^{\left(-2\pi i\frac{\lambda}{Z}\right)} \tag{1.20}$$

using

$$e^{2\pi i} = \cos 2\pi + i \sin 2\pi = 1. \tag{1.21}$$

In general relativity (Evans field theory) the vector potential of the light traversing the helical path is a rotating and translating tetrad field $\mathbf{q}^{(1)}$ multiplied by $A^{(0)}$:

$$\mathbf{A}^{(1)} = A^{(0)}\mathbf{q}^{(1)} \tag{1.22}$$

where:

$$\mathbf{q}^{(1)} = \frac{1}{\sqrt{2}}\left(\mathbf{i} - i\mathbf{j}\right) e^{(i(\omega t - \kappa Z))}. \tag{1.23}$$

In general:

$$\omega = \frac{v}{r}, \qquad \kappa = \frac{1}{r}, \qquad \omega = \kappa v. \tag{1.24}$$

Here ω is an angular frequency in radians per second, r is a distance in meters, κ is a wave-number in inverse meters, and v is a velocity in meters per second. Thus Eq.1.23 denotes in general a propagating and circularly polarized wave of space-time, and can be expressed as:

$$\mathbf{q}^{(1)} = \frac{1}{\sqrt{2}}\left(\mathbf{i} - i\mathbf{j}\right) e^{\left(-i\frac{\widehat{Z}}{r}\right)} \tag{1.25}$$

where:

$$\widehat{Z} = Z - vt. \tag{1.26}$$

This is a wave of spacetime which propagates in the same way in the absence of matter. Therefore by general relativity theory it propagates at the speed of light c. Thus:

$$v = c. \tag{1.27}$$

This inference is supported experimentally by the fact that the geometrical phase is independent of the type of glass or dielectric making up the fiber. If the optical fiber were dispensed with completely and the beam of light were guided in a helical path by a system of mirrors in a high vacuum, the geometrical phase would be the same, Eq.1.19. Therefore the geometrical phase must be a property of spacetime itself, i.e. must be the tetrad field [24] of the Evans field theory. As in the Sagnac effect, this is clear experimental proof of the fact that optics and electrodynamics are governed by general relativity and not by special relativity.

Comparing Eqs1.19 and 1.25:

$$r = \frac{Z\widehat{Z}}{2\pi\lambda}. \tag{1.28}$$

In the special case:

$$Z = \widehat{Z} = \lambda \tag{1.29}$$

then:

$$r = \frac{\lambda}{2\pi} = \frac{1}{\kappa}. \tag{1.30}$$

In special relativity (Maxwell Heaviside field theory) the electromagnetic field is a nineteenth century entity separate from the frame of reference. Therefore there is no propagating wave of space-time since space-time in special relativity is the flat Minkowski space-time [39]. In the Maxwell Heaviside field theory a light beam traveling in a helix is predicted to have the same (dynamical) phase as a light beam traveling in a straight line, contrary to the experimental observation of the geometric phase. The latter is due to parallel transport of space-time and there is no parallel transport of space-time in special relativity. The parallel transport methods of gauge theory [39] use an abstract gauge space superimposed on Minkowski space-time. The abstract gauge space is purely mathematical in nature and so is extraneous to general relativity. In Evans field theory this procedure, introduced by Yang and Mills, is replaced entirely by Cartan geometry, which is general relativity itself. Cartan geometry is rigorously equivalent to the most general type of Riemann geometry. Thus, by Okham's Razor, Evans field theory is preferred to Yang Mills field theory. The Evans field theory gives all the results [5]– [30] of the Yang-Mills field theory of the weak and strong forces, but using geometry alone, as demanded by general relativity. The conventional Yang Mills field theory is a theory of special relativity.

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