

# Chapter 11

## Spin connection resonance (SCR) in magneto-statics

(Paper 65)

by

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### Abstract

The existence of spin connection resonance (SCR) is demonstrated in magneto-statics by developing the Gauss Law of magnetism and the Ampère Law in the context of generally covariant unified field theory, i.e. Einstein Cartan Evans (ECE) field theory. One example is given where an analytical solution is possible. At SCR in magneto-statics the magnetic field is greatly amplified for a given driving current density. Tuning to SCR means tuning a wave-number or frequency, and constructing an equivalent circuit based on the relevant resonance equations.

Keywords: Spin connection resonance (SCR), Magneto-statics, Einstein Cartan Evans (ECE) field theory, generally covariant unified field theory.

### 11.1 Introduction

Recently, classical electrodynamics has been made generally covariant [1]– [18] following the rules of Cartan geometry. This methodology parallels the development by Einstein and others of gravitational theory using the rules of Riemann geometry. The type of Riemann geometry used by Einstein is a limiting case of Cartan geometry when the Cartan torsion [19] vanishes. Geometry is used as the foundation stone of objectivity in natural philosophy. Without objectivity science means all things to all people and there would be no science. The well known Maxwell Heaviside (MH) field theory of classical electrodynamics is a theory of special relativity, and not a theory of general relativity as required

by objectivity. The MH theory is Lorentz covariant, but not generally covariant. This means that the field equations of MH theory are covariant under the Lorentz transformation of special relativity [19] but are not covariant under any type of coordinate transformation as required by general relativity and objectivity. In consequence MH theory loses a great deal of information. The Einstein Cartan Evans (ECE) field theory [1]– [18] is a generally covariant unified field theory as required by objectivity in science. This means that all the foundational fields of physics are developed in a rigorously objective manner, and in consequence become aspects of a unified field, i.e. aspects of Cartan geometry. Thus, electromagnetism may be influenced by gravitation on the classical level, as observed in light bending by gravitation. Classical electromagnetism is recognized to be the Cartan torsion form [1]– [19] within a  $C$  negative scalar factor denoted  $A^{(0)}$ . In S.I. units the quantity  $cA^{(0)}$  has the units of volts, and in ECE theory is referred to as the primordial voltage of the universe. Voltage originates in geometry, the geometry of four dimensional space-time (known as ECE space-time to distinguish it from the torsion-less space-time used by Einstein in the theory of gravitation, and from the flat or Minkowski space-time of MH theory). In order to develop a generally covariant theory of classical electro-magnetism, it is recognized in ECE theory that the electromagnetic field is due to the spinning of space-time. Similarly it was recognized by Einstein that the gravitational field is due to the curving of space-time. Spinning space-time is characterized by the torsion form, and curving space-time by the Cartan curvature form, or Riemann form [1]– [19]. Experimental evidence for the spinning of space-time is available from a number of sources summarized in refs. ([1]– [18]), notably the magnetization of matter by an electromagnetic field, the inverse Faraday effect. The latter has been shown to be due to the ECE spin field  $\mathbf{B}^{(3)}$  [1]– [18] generated by the spin connection which defines the Cartan torsion and curvature. In MH theory there is no spin connection, no spin field, and no inverse Faraday effect, contrary to data.

The phenomenon of spin connection resonance (SCR) is due to the fundamental structure of the equations of generally covariant or objective electrodynamics as developed in ECE theory [1]– [18]. The resonance equations emerge directly from Cartan geometry. The ECE field theory of classical electrodynamics is Cartan geometry itself within the factor  $A^{(0)}$ . In consequence SCR exists in the laws of electrodynamics. It also exists in the laws of gravitation. It has been developed for the Coulomb Law to show the existence of an undamped driven oscillator equation [1]– [18]. At resonance the voltage becomes theoretically infinite. If the experimental means were found to generate SCR, a new source of electric power would become available.

In Section 11.2, SCR is demonstrated for magneto-statics by developing the Gauss Law of magnetism and the Ampère Law into simultaneous, generally covariant, field equations using ECE theory. The spin connection is first simplified by using the approximation of an electromagnetic field uninfluenced by the gravitational field. In this approximation there are equations [1]– [18] which inter-relate tetrad and spin connection elements. Before proceeding to magneto-statics the role of the spin connection is illustrated in electro-statics, and it is shown that in ECE theory, the static electric field becomes characterized by a vector boson similar to that already known from gauge theory in the weak field. After simplifying the spin connection SCR in magneto-statics is illustrated using an example to which there is an analytical solution. It is

shown from this solution that the magnetic field becomes theoretically infinite at resonance. In practice the magnetic field would be enhanced at resonance because of inevitable circuit inefficiencies. Finally in Section 11.3 this methodology is extended to electrodynamics using the Faraday Law of induction as an example.

## 11.2 Magneto-Statics

The spin connection is the most important quantity in generally covariant electrodynamics and for practical applications it is important to simplify the theory and to define the spin connection precisely. In the Coulomb law for example [1]–[18] the basic equations of ECE theory are:

$$A^a{}_{\mu} = A^{(0)}q^a{}_{\mu} \quad (11.1)$$

and

$$\mathbf{E}^a = -\nabla\phi^a - \omega^a{}_b\phi^b. \quad (11.2)$$

Here  $A^a{}_{\mu}$  is the potential one-form and  $q^a{}_{\mu}$  is the tetrad one-form, the fundamental field. In the absence of a vector potential (space-like part of  $A^a{}_{\mu}$ ) the generally covariant electric field is the vector:

$$\mathbf{E}^a = \mathbf{E}^a \quad (a = 1, 2, 3) \quad (11.3)$$

where  $\phi^a$  denotes the scalar potential and  $\omega^a{}_b$  denotes the spin connection vector, the space-like part of the complete spin connection [1]–[18]. In electrostatics only the scalar potential is considered, so Eq.(11.1) becomes:

$$\phi^a{}_0 = \phi^{(0)}q^a{}_0. \quad (11.4)$$

Therefore only the time-like part of the tetrad,  $q^a{}_0$ , is considered in electrostatics. When the electromagnetic and gravitational fields are independent in ECE theory there exists the following relation [1]–[18] between the tetrad and spin connection:

$$\omega^a{}_{\mu b} = -\frac{\kappa}{2}\epsilon^a{}_{bc}q^c{}_{\mu} \quad (11.5)$$

where  $\kappa$  has the units of inverse meters. The anti-symmetric tensor  $\epsilon^a{}_{bc}$  is defined by:

$$\epsilon^a{}_{bc} = \eta^{ad}\epsilon_{dbc} \quad (11.6)$$

where  $\eta^{ad}$  is the Minkowski metric. Eq.(11.5) can be developed into the following six relations between elements [1]–[18]:

$$\omega^0{}_1 = -\frac{\kappa}{2}(q^2 + q^3) = -\omega^1{}_0, \quad (11.7)$$

$$\omega^0{}_2 = -\frac{\kappa}{2}(q^3 - q^1) = -\omega^2{}_0, \quad (11.8)$$

$$\omega^0{}_3 = -\frac{\kappa}{2}(-q^1 - q^2) = -\omega^3{}_0, \quad (11.9)$$

$$\omega^1{}_2 = \frac{\kappa}{2}(q^0 + q^3) = -\omega^2{}_1 \quad (11.10)$$

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$$\omega^1_3 = \frac{\kappa}{2} (-q^2 + q^0) = -\omega^3_1, \quad (11.11)$$

$$\omega^2_3 = \frac{\kappa}{2} (q^1 + q^0) = -\omega^3_2. \quad (11.12)$$

In each equation the index of the base manifold [19] is implied, as usual in differential geometry. Thus Eq.(11.7) for example means:

$$\omega^0_{\mu 1} = -\frac{\kappa}{2} (q^2_{\mu} + q^3_{\mu}) = -\omega^1_{\mu 0}. \quad (11.13)$$

For

$$\mu = 0 \quad (11.14)$$

Eqs.(11.7) to (11.12) simplify to:

$$\omega^1_{02} = -\omega^2_{01} = \frac{\kappa}{2} q^0_0, \quad (11.15)$$

$$\omega^1_{03} = -\omega^3_{01} = \frac{\kappa}{2} q^0_0, \quad (11.16)$$

$$\omega^2_{03} = -\omega^3_{02} = \frac{\kappa}{2} q^0_0. \quad (11.17)$$

So for electro-statics there is only one independent spin connection element.

The scalar potential  $\phi_0^a$  must be the same scalar in all frames of reference. It follows that:

$$\phi_0^a = \phi^{(0)} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad q_0^a = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (11.18)$$

The product of  $\omega^a_b$  and  $\phi^b$  must be:

$$\begin{aligned} \omega^a_{ib} \phi^b &= \omega^a_{i1} \phi^1 + \omega^a_{i2} \phi^2 + \omega^a_{i3} \phi^3 \\ &= \phi^{(0)} (\omega^a_{i1} + \omega^a_{i2} + \omega^a_{i3}) \end{aligned} \quad (11.19)$$

From Eq.(11.15)–(11.17) it is seen that:

$$a = 1, 2, 3 \quad (11.20)$$

Thus:

$$\mathbf{E}^1 = -\nabla\phi - \kappa\phi, \quad (11.21)$$

$$\mathbf{E}_1 := -\mathbf{E}^1 \quad (11.22)$$

$$\mathbf{E}^2 = -\nabla\phi \quad (11.23)$$

$$\mathbf{E}_0 := -\mathbf{E}^2 \quad (11.24)$$

$$\mathbf{E}^3 = -\nabla\phi + \kappa\phi \quad (11.25)$$

$$\mathbf{E}_{-1} := \mathbf{E}^3 \quad (11.26)$$

The electric field has the three indices,  $-1, 0, +1$  of a boson and develops the character of a vector boson.

In the Coulomb law it is almost always observed with great precision [20] that:

$$\phi = -\frac{e}{4\pi\epsilon_0 r} \quad (11.27)$$

In ECE theory this well known potential is used to identify the vector spin connection in the off resonant condition:

$$\nabla\phi = \pm\kappa\phi \quad (11.28)$$

i.e.

$$\kappa = \pm\frac{1}{r}\mathbf{e}_r \quad (11.29)$$

where  $\mathbf{e}_r$  is the radial unit vector in spherical polar coordinates. At resonance it has been shown [1]–[18] that the spin connection is still given by Eq.(11.29) but that the scalar potential with the units of volts may become infinite. This gives a new source of power if the experimental method were found of tuning to SCR.

In magneto-statics there are four basic equations:

$$A^a{}_\mu = A^{(0)}q^a{}_\mu \quad (11.30)$$

$$\nabla \cdot \mathbf{B}^a = \mu_0 j^{0a} \quad (11.31)$$

$$\nabla \times \mathbf{B}^a = \mu_0 \mathbf{J}^a{}_m \quad (11.32)$$

$$\mathbf{B}^a = \nabla \times \mathbf{A}^a - \boldsymbol{\omega}^a{}_b \times \mathbf{A}^b \quad (11.33)$$

where  $\mathbf{A}^a$  is the vector potential,  $\mu_0$  is the vacuum permeability,  $j^{0a}$  is the homogeneous charge density of ECE theory and  $\mathbf{J}^a{}_m$  is the current density including magnetization [1]–[18]. There is no scalar potential in magneto-statics so the space-like elements of  $A^a{}_\mu$  are used:

$$A^a{}_\mu = (0, -\mathbf{A}^a) \quad (11.34)$$

Therefore only the space-like part of the tetrad is used in magneto-statics:

$$q^a{}_\mu = (0, -\mathbf{q}^a) \quad (11.35)$$

When electromagnetism and gravitation are independent:

$$j^{0a} = 0 \quad (11.36)$$

It has been shown that Eq.(11.36) implies [1]–[18]:

$$\mathbf{B}^1 = \nabla \times \mathbf{A}^1 - g\mathbf{A}^2 \times \mathbf{A}^3 \quad (11.37)$$

$$\mathbf{B}^2 = \nabla \times \mathbf{A}^2 - g\mathbf{A}^3 \times \mathbf{A}^1 \quad (11.38)$$

$$\mathbf{B}^3 = \nabla \times \mathbf{A}^3 - g\mathbf{A}^1 \times \mathbf{A}^2 \quad (11.39)$$

$$\nabla \times \mathbf{A}^0 = -\nabla \times \mathbf{A}^3 \quad (11.40)$$

The basic equations of ECE magneto-statics are therefore:

$$\nabla \cdot \mathbf{B}^a = 0 \quad (11.41)$$

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$$\nabla \times \mathbf{B}^a = \mu_0 \mathbf{J}_m^a \quad (11.42)$$

and

$$\mathbf{B}^a = \nabla \times \mathbf{A}^a - g \mathbf{A}^b \times \mathbf{A}^c \quad (11.43)$$

Here:

$$g = \frac{\kappa}{A^{(0)}} \quad (11.44)$$

SCR in magneto-statics is therefore defined by the simultaneous equations:

$$\nabla \times \left( \nabla \times \mathbf{A}^a - g \mathbf{A}^b \times \mathbf{A}^c \right) = \mu_0 \mathbf{J}_m^a \quad (11.45)$$

and

$$\nabla \cdot \mathbf{A}^b \times \mathbf{A}^c = 0 \quad (11.46)$$

Eq.(11.45) can be developed with the vector identities [21]:

$$\nabla \times (\nabla \times \mathbf{A}^a) = -\nabla^2 \mathbf{A}^a + \nabla (\nabla \cdot \mathbf{A}^a) \quad (11.47)$$

and

$$\begin{aligned} \nabla \times (\mathbf{A}^b \times \mathbf{A}^c) = & \\ & \mathbf{A}^b \nabla \cdot \mathbf{A}^c - \mathbf{A}^c \nabla \cdot \mathbf{A}^b + \\ & (\mathbf{A}^c \cdot \nabla) \mathbf{A}^b - (\mathbf{A}^b \cdot \nabla) \mathbf{A}^c \end{aligned} \quad (11.48)$$

To simplify the problem for the sake of illustration only, assume that the vector potential has no divergence:

$$\nabla \cdot \mathbf{A}^a = \nabla \cdot \mathbf{A}^b = \nabla \cdot \mathbf{A}^c = 0 \quad (11.49)$$

and assume that  $\mathbf{A}^c$  is space independent so that:

$$(\mathbf{A}^b \cdot \nabla) \mathbf{A}^c = \mathbf{0} \quad (11.50)$$

Eq.(11.45) then becomes:

$$\nabla^2 \mathbf{A}^a + g (\mathbf{A}^c \cdot \nabla) \mathbf{A}^b = -\mu_0 \mathbf{J}_m^a \quad (11.51)$$

Considering the  $\mathbf{k}$  component:

$$\begin{aligned} & \left( \frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Y^2} + \frac{\partial^2}{\partial Z^2} \right) A^a_Z + \\ & g \left( A^c_X \frac{\partial}{\partial X} + A^c_Y \frac{\partial}{\partial Y} + A^c_Z \frac{\partial}{\partial Z} \right) A^b_Z \\ & \qquad \qquad \qquad = -\mu_0 J^a_{mZ} \end{aligned} \quad (11.52)$$

If:

$$A^a_Z = A^b_Z = A^c_Z \quad (11.53)$$

and:

$$\frac{\partial A^a_Z}{\partial Z} = \frac{\partial A^a_Z}{\partial Y} = 0 \quad (11.54)$$

then:

$$\frac{\partial^2 A^a_Z}{\partial X^2} + g \left( \frac{\partial A^b_Z}{\partial X} \right) A^c_Z = -\mu_0 J^a_{mZ} \quad (11.55)$$

Finally if it is assumed that:

$$g \left( \frac{\partial A_Z^b}{\partial X} \right) = \kappa_0^2 \quad (11.56)$$

Eq.(11.55) becomes the undamped driven oscillator [22]:

$$\frac{\partial^2 A_Z^a}{\partial X^2} + \kappa_0^2 A_Z^a = \mu_0 J_{mZ}^a(0) \cos(\kappa X) \quad (11.57)$$

if there is an initial oscillatory driving current density:

$$J_{mZ}^a = -J_{mZ}^a(0) \cos(\kappa X) \quad (11.58)$$

The solution of Eq.(11.57) is:

$$A_Z^a = \frac{\mu_0 J_{mZ}^a(0)}{\kappa_0^2 - \kappa^2} \cos(\kappa X) \quad (11.59)$$

Spin connection resonance occurs at:

$$\kappa_0 = \kappa \quad (11.60)$$

at which point  $A_Z^a$  goes to infinity and there is a surge or spike in the magnetic field. Eq.(11.53) is compatible with:

$$\nabla \cdot \mathbf{A}^b \times \mathbf{A}^a = 0 \quad (11.61)$$

and

$$\mathbf{A}^a = \mathbf{A}^b \quad (11.62)$$

so this is a self-consistent development.

### 11.3 Spin connection resonance in the Faraday Law of Induction

The Faraday Law of induction in ECE theory is [1]- [18]:

$$\nabla \times \mathbf{E}^a + \frac{\partial \mathbf{B}^a}{\partial t} = \mu_0 \mathbf{j}^a \quad (11.63)$$

where the electric field is defined now by both the scalar and vector potentials:

$$\mathbf{E}^a = -\frac{\partial A^a}{\partial t} - c \nabla A^{0a} - c \omega^{0a}_b \mathbf{A}^b + c \omega^a_b A^{0b} \quad (11.64)$$

and where the magnetic field is defined by Eq.(11.33). The homogeneous current density [1]- [18] is non-zero if and only if there is interaction between the electromagnetic and gravitational fields. Therefore SCR in the Faraday Law of induction can exist only when the two fields interact, as in light bending by gravity. In the standard model (special relativity) the Faraday law of induction in S.I. units is well known to be:

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = \mathbf{0} \quad (11.65)$$

where:

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} - \nabla \phi \quad (11.66)$$

and

$$\mathbf{B} = \nabla \times \mathbf{A} \quad (11.67)$$

Using the vector identity:

$$\nabla \times \nabla \phi := \mathbf{0} \quad (11.68)$$

then:

$$-\nabla \times \frac{\partial \mathbf{A}}{\partial t} + \frac{\partial}{\partial t} \nabla \times \mathbf{A} = \mathbf{0} \quad (11.69)$$

and there is no SCR in the standard model Faraday law of induction and no homogeneous current density. In consequence the effect of gravitation on electromagnetism cannot be described in special relativity, and the well known bending of light by gravity cannot be described on the classical level. Its well known prediction by Einstein rests on the assumption of photon mass, which gravitates with the mass of an object such as the sun. This is therefore a semi-classical description. The photon does not exist on the classical level by definition. ECE explains the bending of light by gravity [1]– [18] using the fact that it is a generally covariant unified field theory able to describe the interaction of electromagnetism and gravitation through the first Bianchi identity of Cartan geometry. This becomes the homogeneous electromagnetic field equation, whose Hodge transform gives the inhomogeneous field equation. The Faraday law of induction is part of the homogeneous field equation and thus part of the first Bianchi identity.

For the sake of illustration only, a simple resonance equation can be developed if it is assumed that there is no scalar potential and also assume that the time-like part of the spin connection is zero:

$$\phi^a = 0, \quad \omega^{0a}_b = 0 \quad (11.70)$$

In this approximation the electric field reduces to:

$$\mathbf{E}^a = -\frac{\partial \mathbf{A}^a}{\partial t} \quad (11.71)$$

and Eq.(11.63) becomes:

$$\frac{\partial \mathbf{A}^b}{\partial t} \times \omega^a_b + \mathbf{A}^b \times \frac{\partial \omega^a_b}{\partial t} = \mu_0 \mathbf{j}^a \quad (11.72)$$

Now assume that the homogeneous current density is oscillatory in time:

$$\mathbf{j}^a = \mathbf{j}^a(0) \sin \omega t \quad (11.73)$$

and differentiate Eq.(11.72) to obtain:

$$\frac{\partial^2 \mathbf{A}^b}{\partial t^2} \times \omega^a_b + 2 \frac{\partial \mathbf{A}^b}{\partial t} \times \frac{\partial \omega^a_b}{\partial t} + \mathbf{A}^b \times \frac{\partial^2 \omega^a_b}{\partial t^2} = \mu_0 \omega \mathbf{j}^a(0) \cos \omega t \quad (11.74)$$

This is a second order differential equation which under certain circumstances [1]– [18] can show resonance. In this case there must be interaction between gravitation and electromagnetism for resonance to occur. For example light emitted by a pulsar could be explained as pulses of electromagnetic radiation at resonance. The pulsar has a very strong gravitational field and in this situation the homogeneous current density is non-zero. A very small initial driving force from this current is greatly amplified at resonance.



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