THE INTERACTION OF THREE FIELDS IN ECE THEORY:

THE INVERSE FARADAY EFFECT.

by

Myron W. Evans,

Civil List Scientist,

Treasury.

(emyrone@aol.com and www.aias.us)

ABSTRACT

The simultaneous interaction of three fundamental fields is illustrated in Einstein Cartan Evans (ECE) theory with reference to the effect of gravitation on the inverse Faraday effect. The three-field interaction in this case is that of the fermionic, electromagnetic and gravitational fields. The interaction of the first two is developed in a well defined semi-classical approximation of the ECE wave equation and the effect of gravitation incorporated through the index reduced canonical energy momentum density T. The exercise is repeated using the ECE wave equations and a general rule developed for the effect of gravitation on the fermionic, electromagnetic weak and strong fields.

Keywords: Einstein Cartan Evans (ECE) field theory; interaction of three fields in ECE theory; ECE wave equation; ECE field equations; rule for gravitational interaction.

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1.INTRODUCTION

Recently a generally covariant unified field theory has been developed based on the extension of Riemann geometry with the well known Cartan torsion {1-12}. This theory is known as Einstein Cartan Evans (ECE) unified field theory and produces a self consistent framework for the investigation of field interaction in various approximations. In this paper the ECE theory is illustrated with the interaction of three fields simultaneously. In Section 2 the interaction of the fermionic and electromagnetic fields is developed in a well defined limit semi-classical limit of the ECE wave equation, and a rule introduced for the effect of gravitation on the interacting fermionic and electromagnetic fields. In Section 3 the exercise is repeated with the ECE field equations and a general method developed for the interaction of gravitation on the fermionic, electromagnetic, weak and strong fields.

2. WAVE EQUATION METHOD.

In general the interaction of n fundamental fields can be developed with the ECE wave equation:

$$(\square + kT)\, q^a_{\ \mu} = 0 \quad -(1)$$

where the Cartan tetrad {14} $q^a_{\ \mu}$ is the eigenfunction, and where the eigenvalues are defined by:

$$R = -kT \quad -(2)$$

where R is a well defined scalar curvature {1-12}, k is Einstein's constant and where T is the index reduced or scalar canonical energy momentum density. In general the interaction of fields is described by adding terms in T as first inferred by Einstein {13}. This procedure needs a numerical solution in general, here the method is simplified and illustrated with respect to the inverse Faraday effect for an ensemble of electrons {15}. A semi classical approximation is used based on the relativistic Hamilton Jacobi equation, which may be solved analytically {1-12} for the inverse Faraday effect in an electron or ensemble of N non-interacting electrons.

When there is no gravitational effect, Eq. (1) reduces to the Dirac equation for a fermion {1-12} as follows:

$$kT \rightarrow \left(\frac{mc}{\hbar}\right)^2 = \frac{1}{\lambda_c^2}. \quad -(3)$$

Here m is the mass of the fermion (in this case an electron), $\hbar$ is the reduced Planck constant and c the vacuum speed of light. The Compton wavelength of the electron is:

$$\lambda_c = \frac{\hbar}{mc}. \quad -(4)$$

Using the quantum equivalence rule {16}:

$$p^\mu = i\hbar\partial^\mu \quad -(5)$$

the Dirac equation becomes the Einstein equation of special relativity:

$$p^\mu p_\mu = m^2c^2. \quad -(6)$$

The interaction of the electron with the classical electromagnetic field is given by the minimal prescription {16}:

$$p^\mu \rightarrow p^\mu - eA^\mu \quad -(7)$$

and using Eq. (7), the Einstein equation (6) becomes the special relativistic Hamilton Jacobi equation:

$$(p^{\mu} - eA^{\mu})(p_{\mu} - eA_{\mu}) = m^2 c^2 . \quad -(8)$$

This has analytical solutions for one electron {1-12} or for N non-interacting electrons. These solutions are reproduced for ease of reference as follows. The circularly polarized electromagnetic field induces orbital angular momentum in the electron, this is a relativistic process. In the ultra relativistic limit the electron radiates synchrotron radiation in well defined {17} narrow beams, such as those observed from a pulsar.

The orbital linear velocity from Eq. (8) is {1-12} divided into X and Y components for a circularly polarized field applied in the Z axis. The field has a spin field:

$$\underline{B}^{(3)} = B^{(0)} \underline{k} \quad -(9)$$

and its angular frequency is ω . The radial vectors defining the electron orbit are r_X and r_Y . The analytical solution of Eq. (8) was first given by Landau and Lifshitz { 18 } and applied to he inverse Faraday effect by Evans and Vigier { 19 }. The radial vectors from Eq. (8) are:

$$r_X = -\frac{ec^2 B^{(0)}}{\gamma \omega^2} \sin \omega t, \quad r_Y = -\frac{ec^2 B^{(0)}}{\gamma \omega^2} \cos \omega t, \quad -(10)$$

and the orbital linear velocities are:

$$v_X = \frac{ecB^{(0)}}{m\omega} \cos \omega t, \quad v_Y = -\frac{ec B^{(0)}}{m\omega} \sin \omega t. \quad -(11)$$

Here γ is a special relativistic correction defined by:

$$\gamma = mc\left(1 + \left(\frac{eB^{(0)}}{m\omega}\right)^2\right)^{1/2}. \quad -(12)$$

The angular frequency imparted to the electron is defined by:

$$\Omega = \left(\omega^2 + \frac{e^2 B^{(0)2}}{m^2}\right)^{1/2}. \quad -(13)$$

The electronic angular momentum is:

$$\underline{J} = \frac{e^2 c^3 B^{(0)2}}{\gamma \omega^2} \underline{k} \quad -(14)$$

and the electronic orbital displacement in radians is:

$$\theta = \tau \Omega \quad -(15)$$

where τ is a well defined interval of time. In well defined approximations the graph of θ against r is a hyperbolic spiral or a combination of spirals {1-12}.

In the low frequency (radio freqeuncy) limit:

$$\omega << \frac{eB^{(0)}}{m} \quad -(16)$$

the angular momentum imparted to the electron by the electromagnetic field is approxiamted by:

$$\underline{J}^{(3)} \rightarrow \frac{ec^2}{\omega^2} \underline{B}^{(3)} \quad -(17)$$

and the angular displacement is approximated by:

$$\theta \to \frac{\tau e B^{(0)}}{m} . \quad -(18)$$

In the high frequency (laser) limit:

$$\omega \gg \frac{e B^{(0)}}{m} \quad -(19)$$

the angular momentum is approximated by:

$$\underline{J}^{(3)} \to \frac{e^2 c^2}{m \omega^3} B^{(0)} \underline{B}^{(3)} \quad -(20)$$

and the angular displacement by:

$$\theta \to \tau \omega . \quad -(21)$$

The angular frequency of the electromagnetic field is by definition:

$$\omega := \frac{v^{(0)}}{r} \quad -(22)$$

so it is seen that the graph of displacement against r in the laser limit is a hyperbolic spiral:

$$\theta = \frac{\tau v^{(0)}}{r} . \quad -(23)$$

In the low frequency limit (16) the angular displacement depends on the mass m, but in the high frequency limit (19) it does not.

The effect of gravitation on these results is now introduced by the rule:

$$m^2 c^2 \to \hbar^2 k T \quad -(24)$$

so the mass m is replaced wherever it occurs by:

$$m = \left(\frac{\hbar k^{1/2}}{c} \right) T^{1/2} . \quad -(25)$$

It can be seen that this is a rule of general relativity unified with wave (or quantum) mechanics because $\hbar$ appears as well as T. In the standard model this is not possible because its gravitational sector is generally covariant but its electromagnetic sector is only Lorentz covariant. In ECE theory {1-12} all sectors are generally covariant. The standard model is internally inconsistent, whereas ECE is internally consistent. In using this rule as follows T is left as a parameter to be determined by comparison with data, however it can be calculated from first principles and Eq. (1) used directly. However in this case Eq. (1) is analytically insoluble.

The rule (25) is illustrated by reference to the resonance frequency of the inverse Faraday effect {1-12}, i.e. the radiatively induced fermion resonance (RFR) frequency:

$$\omega_{res} = \frac{e^2 c^2}{2 m \omega^2} B^{(0)2} \quad -(26)$$

From Eq. (26) the RFR frequency in hertz in the low frequency approximation (16) is:

$$f_{res} = \left(\frac{e^2 \mu_0 c}{2\pi \hbar m} \right) \frac{I}{\omega^2} \quad -(27)$$

where I is the power density (watts per square meter) of the electromagnetic field and where μ_0 is the S.I. vacuum permeability. The effect of gravitation on RFR in the approximations used in this section is therefore to shift the RFR frequency to:

$$f_{res} \rightarrow \left(\frac{e^2 \mu_0 c^2}{2\pi \hbar^2 k^{1/2} T^{1/2}} \right) \frac{I}{\omega^2} . \quad -(28)$$

This frequency can be measured to high accuracy using contemporary instrumentation and so it may be possible to measure the effect of gravitation on it. It may be concluded that synchrotron and pulsar radiation are effected by gravitation. Pulsar radiation comes from an object of very high gravity, so its characteristics are modified by gravity. Pulsar radiation is synchrotron radiation, and the latter is calculated in the ultra relativistic limit when the angular frequency Ω is very large. In the case of the inverse Faraday effect the electron is spun by a circularly polarized electromagnetic field. In a conventional synchrotron it is spun by other electromagnetic devices. In both cases however an ultra relativistic electron will radiate and this radiation is affected by gravity. This illustrates the predictive abilities of ECE theory with a well defined semi-classical model (8).

3. EFFECT OF GRAVITATION ON RFR, FIELD EQUATIONS.

In general the field equations must be set up in SU(2) representation space to describe the interaction of a fermion with an electromagnetic field. The effect of gravitation must then be incorporated through a non-zero homogeneous current {1-12}. The RFR term is obtained straightforwardly from the definition of the magnetic field in ECE theory {1-12}:

$$\underline{B}^a = \underline{\nabla} \times \underline{A}^a - \underline{\omega}^a{}_b \times \underline{A}^b \quad -(29)$$

When there is no gravitation:

$$\underline{B}^{(1)*} = \underline{\nabla} \times \underline{A}^{(1)*} - ig\, \underline{A}^{(2)} \times \underline{A}^{(3)} \quad -(30)$$

et cyclicum.

The $\underline{B}^{(3)}$ component is:

$$\underline{B}^{(3)*} = -ig\,\underline{A}^{(1)} \times \underline{A}^{(2)} \qquad -(31)$$

In the SU(2) basis:

$$\underline{\sigma} \cdot \underline{B}^{(3)*} = -ig\,\underline{\sigma} \cdot \underline{A}^{(1)} \times \underline{A}^{(2)} \qquad -(32)$$

where $\underline{\sigma}$ is the Pauli vector. The basis set in SU(2) is made up of the three Pauli matrices. The right hand side in Eq. (32) is proportional to the RFR term {1-12}. The effect of the gravitational field in this context is to change the conjugate product so that the RFR term becomes:

$$\underline{\sigma} \cdot \underline{B}^{a} = \underline{\sigma} \cdot \left(\underline{\nabla} \times \underline{A}^{a} - \underline{\omega}^{a}{}_{b} \times \underline{A}^{b} \right). \qquad -(33)$$

In this case the spin connection $\underline{\omega}^{a}{}_{b}$ is defined by both spinning and curving space-time and is no longer dual to the potential. So it may be concluded qualitatively that there is an effect of gravitation on RFR. This is the same overall conclusion as that from the wave equation.

The SU(2) basis is defined { 16 } by the commutation of Pauli matrices:

$$\left[\frac{\sigma_x}{2}, \frac{\sigma_y}{2} \right] = i\,\epsilon_{xyz} \frac{\sigma_z}{2} \qquad -(34)$$

and the SU(3) basis by:

$$\left[\frac{\lambda_a}{2}, \frac{\lambda_b}{2} \right] = i\,f_{abc} \frac{\lambda_c}{2} \qquad -(35)$$

where c sums from 1 to 8 and where the structure factor has elements { 16 }:

$$f_{123} = 1,$$

$$f_{147} = -f_{156} = f_{246} = f_{257} = f_{345} = -f_{367} = \frac{1}{2},$$

$$f_{458} = f_{678} = \frac{\sqrt{3}}{2}. \qquad -(36)$$

In SU(3) the complete potential field is defined by its components A^a_μ and by its basis elements $\lambda_a/2$ and is:

$$A_\mu = A^a_\mu \frac{\lambda_a}{2} = \frac{1}{2}\begin{bmatrix} A^3_\mu + \frac{1}{\sqrt{3}}A^8_\mu & A^1_\mu - iA^2_\mu & A^4_\mu - iA^5_\mu \\ A^1_\mu + iA^2_\mu & -A^3_\mu + \frac{1}{\sqrt{3}}A^8_\mu & A^6_\mu - iA^7_\mu \\ A^4_\mu + iA^5_\mu & A^6_\mu + iA^7_\mu & -\frac{2}{\sqrt{3}}A^8_\mu \end{bmatrix}. \qquad -(37)$$

The strong field is then defined by the Cartan torsion as follows:

$$G^1_{\mu\nu} = G^{(0)}T^1_{\mu\nu} = \partial_\mu A^1_\nu - \partial_\nu A^1_\mu + g f_{1bc} A^b_\mu A^c_\nu$$

$$= \partial_\mu A^1_\nu - \partial_\nu A^1_\mu + g\Big(A^2_\mu A^3_\nu - A^3_\mu A^2_\nu + \frac{1}{2}\left(A^4_\mu A^7_\nu - A^7_\mu A^4_\nu\right) - \frac{1}{2}\left(A^5_\mu A^6_\nu - A^6_\mu A^5_\nu\right)\Big) \qquad -(38)$$

where $G^{(0)}$ is a proportionality. So the strong field in Eq. (38) is defined by a particular type of spin connection producing three commutators. Thus Eq. (38) is an example of the general definition:

$$G^a_{\mu\nu} = (d \wedge A^a)_{\mu\nu} + \omega^a_{\mu b} \wedge A^b_\nu \qquad -(39)$$

in which the spin connection involves the structure factor f_{1bc}. Eq. (37) is an example of the tetrad definition { 1- 12 }:

$$A_\mu = A_\mu^a \lambda_a / 2 \quad -(40)$$

where

$$A_\mu^a = A^{(0)} q_\mu^a . \quad -(41)$$

Similarly, in the SU(2) basis, the complete potential field is made up of its components A_μ^a and its basis elements $\frac{\sigma_a}{2}$:

$$A_\mu = A_\mu^a \frac{\sigma_a}{2} \quad -(42)$$

and the field is:

$$G_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g \epsilon_{abc} A_\mu^b A_\nu^c . \quad -(43)$$

This may be a fermionic field or a weak field symmetry. In the ECE theory these structures are those of general relativity, and not gauge theory.

The basis set in 3-D is defined by the Pauli matrices { 16 }:

$$\sigma_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \sigma_2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \quad \sigma_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad -(44)$$

so

$$\underline{\sigma} \cdot \underline{r} = \begin{bmatrix} Z & X - iY \\ X + iY & -Z \end{bmatrix} . \quad -(45)$$

Both the SU(2) and SU(3) fields are particular cases of:

$$F^a = d \wedge A^a + \omega^a{}_b \wedge A^b \quad -(46)$$

When there is no effect of gravitation the spin connection is dual to the tetrad as follows:

$$\omega^a{}_b = -\frac{1}{2}\kappa\,\epsilon^a{}_{bc}\,q^c \qquad -(47)$$

and the homogeneous current vanishes:

$$j^a = \frac{A^{(0)}}{\mu_0}\left(R^a{}_b \wedge q^b - \omega^a{}_b \wedge T^b\right) = 0. \qquad -(48)$$

So in the absence of gravitation:

$$d \wedge F^a = 0, \qquad -(49)$$

$$d \wedge \tilde{F}^a = \mu_0 J^a. \qquad -(50)$$

In the complex circular basis:

$$\left.\begin{aligned} \underline{B}^{(1)*} &= \underline{\nabla} \times \underline{A}^{(1)*} - ig\,\underline{A}^{(2)} \times \underline{A}^{(3)} \\ \underline{B}^{(2)*} &= \underline{\nabla} \times \underline{A}^{(2)*} - ig\,\underline{A}^{(3)} \times \underline{A}^{(1)} \\ \underline{B}^{(3)*} &= \underline{\nabla} \times \underline{A}^{(3)*} - ig\,\underline{A}^{(1)} \times \underline{A}^{(2)}. \end{aligned}\right\} -(51)$$

If $\underline{A}^{(3)}$ is Z directed then:

$$\underline{B}^{(3)*} = \underline{B}^{(3)} = -ig\,\underline{A}^{(1)} \times \underline{A}^{(2)} \qquad -(52)$$

which is the fundamental spin field {1-12}. Using the SU(2) basis gives the extra information:

$$\underline{\sigma} \cdot \underline{B}^{(3)} = \begin{bmatrix} B_Z & 0 \\ 0 & -B_Z \end{bmatrix} = -ig\,\underline{\sigma} \cdot \underline{A}^{(1)} \times \underline{A}^{(2)} \qquad -(53)$$

which produces radiatively induced fermion resonance (RFR). Gravitation changes this result by changing the conjugate product $\underline{A}^{(1)} \times \underline{A}^{(2)}$.

Electrodynamics may be developed in any representation space symmetry group SU(n), where n = 2, 3, 4, ... {1-12} using the same Cartan geometry as O(3). However, extra information can be found at the end of the calculation by using the Pauli matrices as for example in Eq. (53). This rule can also be used for SU(n), n $\geq$ 2. The SU(3) group for example is the one that governs gluons in the quark / gluon model - the complete electromagnetic potential vector field in SU(3) is given by Eq. (37) and the electromagnetic field can be represented by Eq. (38). In Eq. (42) the Cartan tangent space labeled a is a SU(2) symmetry representation space and in Eq. (37) is a SU(3) representation space (Cartan 1913). Eq. (38) is the special case:

$$\omega^{a}_{\mu b} \wedge q^{b}_{\nu} = g f_{abc} A^{b}_{\mu} A^{c}_{\nu} \quad -(54)$$

and Eq. (43) is the special case:

$$\omega^{a}_{\mu b} \wedge q^{b}_{\nu} = g \epsilon_{abc} A^{b}_{\mu} A^{c}_{\nu}. \quad -(55)$$

This process can be continued in any SU(n) symmetry, n = 2, 3, 4, ... and in general:

$$\omega^{a}_{\mu b} \wedge q^{b}_{\nu} = g \zeta_{abc} A^{b}_{\mu} A^{c}_{\nu} \quad -(56)$$

where ζ_{abc} is the general structure factor of the group SU(n).

The effect of gravitation in general is therefore to break the SU(n) symmetry. In the absence of gravitation the general symmetry is:

$$\omega^{a}{}_{b} = -\frac{1}{2} \kappa \zeta^{a}{}_{bc} q^{c} \quad -(57)$$

but this is no longer true in the presence of gravitation.

In the standard model consideration is restricted to the U(1) = O(2) symmetry group so the field is linearized to:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu . \quad -(58)$$

In differential form notation this equation is {1-12}:

$$F = d \wedge A \quad -(59)$$

so in the standard model there is no self-consistent method of describing the observable inverse Faraday effect and no self consistent method of describing RFR. In ECE and for any representation space, the IFE and RFR are both results of the spin connection term in Eq. (46), i.e. we experimentally observe spinning space-time in the IFE and RFR. Extra information about classical electrodynamics may be found by the method of Eq. (53) in all SU(n). In SU(3) for example we multiply $\underline{B}^{(3)}$ by the SU(3) 3 x 3 matrices { 16 } to describe the gluon / electromagnetic interaction.

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