CALCULATION OF THE LARMOR RADIUS OF THE INVERSE FARADAY EFFECT IN AN ELECTRON ENSEMBLE FROM THE EINSTEIN CARTAN EVANS (ECE) UNIFIED FIELD THEORY.

by

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ABSTRACT

The Larmor radius is calculated for an electron ensemble in a circularly polarized electromagnetic field. This radius is part of the inverse Faraday effect and is calculated from a well defined limit of the Einstein Cartan Evans (ECE) wave equation when it reduces to the relativistic Hamilton Jacobi equation. The radius is calculated under the experimental conditions proposed to detect the inverse Faraday effect in an electron ensemble.

Keywords: Einstein Cartan Evans unified field theory, inverse Faraday effect in an electron ensemble, Larmor radius.

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1. INTRODUCTION

Recently, a unified theory of fields has been proposed based on Cartan geometry {1-12} and is known as Einstein Cartan Evans (ECE) field theory. This theory unifies general relativity and wave mechanics by deriving a wave equation from the fundamental tetrad postulate {13} of Cartan geometry, the fundamental requirement that a complete vector field be independent of its components and basis elements. The tetrad postulate is true for any application in physics, and so therefore is the ECE wave equation. In well defined limits the latter reduces to the well known wave equations of physics, notably the Dirac and Proca equations for the electron and photon respectively. In Section 2 the Dirac equation is derived from the ECE wave equation and expressed as the well known Einstein equation of special relativity using the fundamental operator rules of wave mechanics. The Einstein equation governs the special relativistic classical dynamics of an electron. The interaction of this electron with a circularly polarized electromagnetic field is then developed form the minimal prescription and the relativistic Hamilton Jacobi equation {1-12, 14}. The results are used to calculate the Larmor radius of an electron in a circularly polarized electromagnetic field, part of the inverse Faraday effect. The latter is due to the ECE spin field $\underline{B}^{(3)}$, and this produces a different motion of the electron from that produced by a static magnetic field of the standard model.

2. RELATIVISTIC CALCULATION OF THE LARMOR RADIUS.

The starting point is the ECE wave equation:

$$(\square + kT)\, q^a_\mu = 0 \qquad -(1)$$

where q^a_μ is the Cartan tetrad, k is Einstein's constant and T is the index reduced canonical energy momentum density. In the limit of a free electron this equation reduces to the Dirac equation using

$$kT = \frac{\hbar m}{V} = \frac{m^2 c^2}{\hbar^2} \qquad -(2)$$

where m is the mass of the electron occupying a finite volume V, and where the Compton wavelength of the electron is:

$$\lambda_c = \frac{\hbar}{mc} . \qquad -(3)$$

Here $\hbar$ is the reduced Planck constant and c is the speed of light in vacuo. Using the fundamental operator definition of wave mechanics:

$$p^\mu = i\hbar \partial^\mu \qquad -(4)$$

the Dirac equation becomes the Einstein equation of special relativity:

$$p^\mu p_\mu = m^2 c^2 \qquad -(5)$$

where:

$$p^\mu = \left(\frac{E}{c} , \underline{p} \right) \qquad -(6)$$

is the four-momentum of the electron. Here E is the relativistic energy and $\underline{p}$ the relativistic momentum. Eq. (5) describes the special relativistic motion of the electron in classical physics. The interaction of the electron with an electromagnetic field is given by the minimal prescription:

$$p^\mu \rightarrow p^\mu - eA^\mu , \qquad -(7)$$

$$(p^\mu - eA^\mu)(p_\mu - eA_\mu) = m^2 c^2 , \qquad -(7a)$$

where A^μ is the four-potential of the electromagnetic field:

$$A^{\mu} = \left(\frac{\phi}{c}, \underline{A} \right) \quad -(8)$$

where ϕ is the scalar potential and $\underline{A}$ the vector potential. Eq. (7a) is the relativistic Hamilton Jacobi equation of motion of the electron in a classical electromagnetic field.

In previous work {1-12} this equation has been solved for the dynamics of the system adopting a method originally given by Landau and Lifshitz {14}. The Larmor radius of the electron in a circularly polarized electromagnetic field is defined by:

$$r_L = \frac{v}{\Omega} \quad -(9)$$

where v is the orbital linear velocity of the electron and where Ω is its angular frequency. These quantities are given from Eq. (7a) as follows:

$$v = \frac{ecB^{(0)}}{m\omega}, \quad -(10)$$

$$\Omega = \left(\omega^2 + \left(\frac{eB^{(0)}}{m} \right)^2 \right)^{1/2}. \quad -(11)$$

Here ω is the angular frequency of the applied electromagnetic field and $B^{(0)}$ is the magnitude of its flux density in tesla. . The power density in watts per square meter of the applied electromagnetic field is {1-12}:

$$\underline{I} = \frac{c}{\mu_0} B^{(0)2} \quad -(12)$$

where μ_0 is the S.I. vacuum permeability. So the Larmor radius may be calculated in terms of ω and $\underline{I}$ and may be detected experimentally using a detector of given radius, such as a Faraday cup. Experimental conditions may be adjusted such that the Larmor radius is less than that the radius of the Faraday cup detector, when a large signal should become detectible. The experimental conditions are such that:

$$\omega \gg \frac{eB^{(0)}}{m} \quad - (13)$$

in which case the Larmor radius is defined by the limit:

$$r_L \rightarrow \left(\frac{e c^{1/2} \mu_0^{1/2}}{m} \right) \frac{I^{1/2}}{\omega^2}. \quad - (14)$$

At an applied frequency of 2.5 G Hz:

$$r_L = 1.39 \times 10^{-6} I^{1/2} \text{ cm}. \quad - (15)$$

So the power density I must be adjusted using this equation in order that the Larmor radius is less than that of the Faraday cup detector.

It is emphasized that the dynamics of the electron in a circularly polarized electromagnetic field are different entirely from the dynamics of an electron in a static magnetic field in the standard model. The latter are well known to be described by the interaction hamiltonian {1-12}:

$$H = \frac{1}{2m} (\underline{p} + e\underline{A}) \cdot (\underline{p} + e\underline{A}). \quad - (16)$$

The standard model static magnetic field is described by:

$$\underline{B} = \underline{\nabla} \times \underline{A} \quad - (17)$$

and by:

$$\underline{A} = \frac{1}{2} \underline{B} \times \underline{r} \quad - (18)$$

where $\underline{r}$ is a radius vector defining the angular momentum:

$$\underline{L} = \underline{r} \times \underline{p}. \quad - (19)$$

The relevant interaction term is:

$$H = \frac{e}{m}\underline{p}\cdot\underline{A} \qquad -(20)$$

using the vector property:

$$= \frac{e}{2m}\underline{p}\cdot(\underline{B}\times\underline{r})$$

$$\underline{a}\cdot(\underline{b}\times\underline{c}) = \underline{b}\cdot(\underline{c}\times\underline{a}) = \underline{c}\times(\underline{a}\times\underline{b}). \qquad -(21)$$

The hamiltonian is usually expressed as:

$$H = \underline{m}\cdot\underline{B} \qquad -(22)$$

where $\underline{m}$ is the magnetic dipole moment:

$$\underline{m} = \frac{e}{2m}\underline{L} \qquad -(23)$$

and where:

$$\gamma_r = \frac{e}{2m} \qquad -(24)$$

is the gyromagnetic ratio. In this non-relativistic calculation the kinetic energy is the Newtonian kinetic energy:

$$T = \frac{p^2}{2m} = \frac{1}{2}\omega L \qquad -(25)$$

and the angular frequency is:

$$\omega = \frac{d\theta}{dt} = \frac{v}{r}. \qquad -(26)$$

The angular momentum is therefore:

$$L = rp = \frac{p^2}{m\omega} \qquad -(27)$$

and the Larmor radius is:

$$r_L = \frac{p}{m\omega}. \quad -(28)$$

This result is evidently different from Eq. (9). There are several reasons for this difference, primarily, $\underline{B}^{(3)}$ is a radiated field due to circularly polarized electromagnetic radiation, and $\underline{B}$ is a static field of a magnet. We emphasize this difference as follows, in order to avoid confusion between the concepts of $\underline{B}^{(3)}$ and $\underline{B}$.

In classical special relativity:

$$p^\mu p_\mu = E^2/c^2 - p^2, \quad A^\mu A_\mu = \phi^2/c^2 - A^2 \quad -(29)$$

so the special relativistic Hamilton Jacobi equation (7a) is:

$$E^2 = c^2 p^2 + m^2 c^4 - e^2 \phi^2 + e^2 c^2 A^2. \quad -(30)$$

This equation reduces in the absence of an electromagnetic field to:

$$E^2 = c^2 p^2 + E_0^2 \quad -(31)$$

where

$$E_0 = mc^2 \quad -(32)$$

is the rest energy. In these equations $\underline{p}$ is the relativistic momentum:

$$\underline{p} = \gamma m \underline{v} \quad -(33)$$

where:

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}. \quad -(34)$$

This equation is proven to be the same as Eq. (31) as follows. From Eq. (33):

$$p^2 c^2 = \gamma^2 m^2 c^4 \left(\frac{v^2}{c^2} \right). \quad -(35)$$

Now use:

$$\frac{v^2}{c^2} = 1 - \frac{1}{\gamma^2} \quad -(36)$$

so

$$p^2 c^2 = \gamma^2 m^2 c^4 \left(1 - \frac{1}{\gamma^2} \right) = E^2 - E_0^2 \quad -(37)$$

Q.E.D.

Here, the relativistic energy is:

$$E = \gamma m c^2 \quad -(38)$$

and the relativistic kinetic energy is:

$$T = mc^2 (\gamma - 1) \quad -(39)$$
$$= E - E_0 .$$

Under the condition:

$$v << c \quad -(40)$$

we regain the Newtonian kinetic energy as follows:

$$T = mc^2 \left(1 - \frac{v^2}{c^2} \right)^{-1/2} - mc^2 \sim mc^2 \left(1 + \frac{1}{2} \frac{v^2}{c^2} + \dots \right) - mc^2 \quad -(41)$$

and we regain the non-relativistic:

$$T = \frac{1}{2} m v^2 \rightarrow \frac{p^2}{2m} + \frac{2peA}{2m} + \frac{e^2 A^2}{2m} . \quad -(42)$$

This result is sufficient to describe the inverse Faraday effect when the electronic v << c. If we average over many cycles of the applied electromagnetic field:

$$\langle A \rangle = 0 \qquad -(43)$$

so the extra effect of the electromagnetic field is, on average:

$$T_{em} = \frac{e^2 A^2}{2m}. \qquad -(44)$$

Now use the fundamental properties of the electromagnetic field in free space {1-12}:

$$B^{(0)} = \kappa A^{(0)} = E^{(0)}/c, \quad \kappa = \frac{\omega}{c}, \qquad -(45)$$

to obtain:

$$T_{em} = \left(\frac{e^2 c^2}{2m\omega^2}\right) B^{(0)2}. \qquad -(46)$$

This is the induced rotational energy in the limit v << c:

$$T_{em} = \frac{1}{2}\omega J \qquad -(47)$$

where:

$$J = rp \qquad -(48)$$

where J is the induced orbital angular momentum of the electron:

$$J = \left(\frac{e^2 c^2}{m\omega^3}\right) B^{(0)2}. \qquad -(49)$$

The induced magnetic dipole moment is

$$\underline{m}^{(3)} = \left(\frac{e^3 c^2}{2m^2\omega^2}\right) B^{(0)} \underline{B}^{(3)} \qquad -(50)$$

where:

$$\underline{B}^{(3)} = B^{(0)}\underline{k} \quad -(51)$$

is the ECE spin field.

This result can be cross checked for internal self-consistency by using the angular momentum calculated from Eq. (7a) {1-12}:

$$\underline{J}^{(3)} = \frac{e^2c^2}{\omega^2}\left(\frac{B^{(0)}}{(m^2\omega^2 + e^2B^{(0)2})^{1/2}}\right)\underline{B}^{(3)}. \quad -(52)$$

When:

$$\omega \gg \frac{eB^{(0)}}{m} \quad -(53)$$

Eq. (52) becomes Eq. (49), Q.E.D. In the opposite limit:

$$\omega \ll \frac{eB^{(0)}}{m} \quad -(54)$$

Eq. (52) becomes:

$$\underline{J}^{(3)} = \left(\frac{ec^2}{\omega^2}\right)\underline{B}^{(3)} \quad -(55)$$

giving the induced magnetic dipole moment:

$$\underline{m}^{(3)} = \left(\frac{e^2c^2}{2m\omega^2}\right)\underline{B}^{(3)}. \quad -(56)$$

The one electron susceptibility is defined by:

$$\chi_e := \frac{e^2c^2}{2m\omega^2} \quad -(57)$$

and the one electron hyper-magnetizability by:

$$\beta_e := \frac{e^3 c^2}{2m^2 \omega^3} . \quad -(58)$$

(3) It is seen that there are basic differences between the motion of an electron in the $\underline{B}$ and $\underline{B}$ fields. In the presence of the electromagnetic field, the complete momentum of the electron is {1-12, 14}:

$$\underline{p} = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} m\underline{v} + e\underline{A} . \quad -(59)$$

From Eqs. (31) and (39):

$$E^2 - E_0^2 = (E + E_0)(E - E_0) = p^2 c^2 \quad -(60)$$

so the relativistic kinetic energy is:

$$T = E - E_0 = \frac{p^2 c^2}{E + E_0} . \quad -(61)$$

In the presence of an electromagnetic field, p in Eq. (61) is defined by Eq. (59), so after averaging over many cycles:

$$T_{em} = \frac{e^2 A^2 c^2}{E + E_0} . \quad -(62)$$

This is the special relativistic energy of an electron to second order in the electromagnetic field. Eq (62) bears a resemblance to Eq. (52). If it is assumed that:

$$E \sim E_0 \quad -(63)$$

then

$$T_{em} \to \left(\frac{e^2 c^2}{2m\omega^2}\right) B^{(0)2} \quad -(64)$$

which is Eq. (46). Therefore Eq. (63) means that the electronic velocity is v << c. In this limit:

$$T = \frac{p^2 c^2}{E + E_0} \to \frac{1}{2} m v^2 = \frac{1}{2} \omega J \quad -(65)$$

i.e.

$$\frac{\gamma^2 v^2 c^2 m^2}{E + E_0} \to \frac{1}{2} m v^2 \quad -(66)$$

and

$$E + E_0 \to 2\gamma^2 m c^2 \quad -(67)$$

or

$$E \to 2\gamma^2 c^2 m - mc^2$$
$$= 2\left(1 - \frac{v^2}{c^2}\right)^{-1} mc^2 - mc^2 \sim mc^2. \quad -(68)$$

if v << c. In this limit the total energy and rest energy are about equal, so T is very small and this limit corresponds to Eq. (53).

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