DESCRIPTION OF THE FARADAY EFFECT AND INVERSE FARADAY EFFECT IN TERMS OF THE ECE SPIN FIELD.

by

Myron W. Evans,

Civil List Scientist,

(emyrone@aol.com and www.aias.us).

ABSTRACT

The inverse Faraday effect for one electron in a circularly polarized electromagnetic field is deduced directly from the Einstein equation of special relativity, a limit of the ECE wave equation. It is shown that the effect is due to the fundamental ECE spin field. An additional static magnetic field produces the Faraday effect using the same method. Therefore the inverse Faraday effect and Faraday effect are both described using the Einstein equation. The non-relativistic and hyper-relativistic limit of the inverse Faraday effect are described, and self-consistent results obtained.

Keywords: ECE spin field, ECE wave equation, inverse Faraday effect, Faraday effect, Einstein equation.

Paper 81 of the ECE Series

1. INTRODUCTION

Recently a self-consistent and fully quantized unified field theory has been developed {1-14} from standard Cartan geometry . This is a generally covariant unified field theory based on the Einstein / de Broglie philosophy, i.e. a causal and objective physics. General relativity and wave mechanics have been unified with geometry and this is known as Einstein Cartan Evans (ECE) unified field theory. It has many advantages over the standard model (see www.aias.us) and has been experimentally tested in several ways. In this paper the ECE theory is used to show that the inverse Faraday effect (IFE) and Faraday effect (FE) for one electron originate in the ECE spin field, a fundamental property of general relativity applied to electromagnetism. The inverse Faraday effect is the magnetization of matter by a circularly polarized electromagnetic field, and the Faraday effect is the rotation of the plane of polarization of electromagnetic radiation by a static magnetic field.

In Section 2 the Einstein equation is deduced as a limit of the ECE wave equation {1-14}. It is applied to one electron and a circularly polarized electromagnetic field introduced with the minimal prescription. The equation is solved directly to give the relativistic angular momentum and kinetic energy of the electron. This method gives the angular frequency of the electron (Ω) and allows the angular frequency of the applied electromagnetic field (ω) to be expressed in terms of Ω. The FE is described in Section 3 through the additional influence of a static magnetic field on the angular velocity of the electron. The static magnetic field is shown to change the electromagnetic phase by:

$$\Delta \phi = \frac{2eB}{m}\left(t - \frac{Z}{c}\right) \quad -(1)$$

and this the one electron Faraday effect. The latter is a relativistic phenomenon in general. Here $\underline{B}$ is the applied magnetic flux density in tesla and e and m are the electronic charge magnitude and mass respectively. Finally it is shown in Section 4 that the non-relativistic limit and ultra-relativistic limit of the IFE obtained by this method are both the same as obtained from the relativistic Hamilton Jacobi equation in previous work.

2. THE INVERSE FARADAY EFFECT

The starting point of the calculation is the ECE wave equation {1-14}:

$$(\square + kT)\, q^a_\mu = 0 \qquad -(2)$$

where q^a_μ is the tetrad wave-function of the electron, k is Einstein's constant, and T is the scalar canonical energy - momentum density of the electron. When the fermion field becomes independent of other fields Eq. (2) reduces to:

$$kT = k\frac{m}{V} = \frac{m^2c^2}{\hbar^2} \qquad -(3)$$

where $\hbar$ is the reduced Planck constant and where c is the speed of light in vacuo. Here V is a finite volume which the electron always occupies and which his defined by:

$$V = \frac{k\hbar^2}{mc^2}. \qquad -(4)$$

Therefore there are no singularities in ECE theory and the need for elaborate renormalization is by-passed. In this limit the ECE wave equation of the electron becomes the Dirac equation of the free electron:

$$\left(\square + \frac{m^2c^2}{\hbar^2}\right) q^a_\mu = 0. \qquad -(5)$$

The index a of the tetrad denotes the well known spin of the fermion and the index μ is that of the Pauli spinor. So the tetrad becomes the well known Dirac spinor.

The classical equivalent of the Dirac equation is well known to be the Einstein equation:

$$p^\mu p_\mu = m^2 c^2 \qquad -(6)$$

where the rest energy of the electron is:

$$E_0 = mc^2. \qquad -(7)$$

Here:

$$p^\mu = \left(\frac{E}{c}, \underline{p}\right), \; p_\mu = \left(\frac{E}{c}, -\underline{p}\right), \qquad -(8)$$

in contravariant-covariant notation, where E is the relativistic energy:

$$E = \gamma mc^2 \qquad -(9)$$

and $\underline{p}$ is the relativistic momentum:

$$\underline{p} = \gamma m \underline{v}. \qquad -(10)$$

Eq. (10) is equivalent to the well known Einstein equation:

$$E^2 = c^2 p^2 + E_0^2. \qquad -(11)$$

The relativistic kinetic energy is defined to be:

$$T = E - E_0 = \frac{c^2 p^2}{E + E_0} \qquad -(12)$$

Here:

$$\gamma = \left(1 - \frac{u^2}{c^2}\right)^{-1/2} \qquad -(13)$$

where $\underline{u}$ is the velocity of one frame with respect to another in a Lorentz boost, and $\underline{v}$ is the electron velocity. The influence of the electromagnetic field is introduced through the minimal prescription:

$$p^\mu \rightarrow p^\mu - eA^\mu \quad -(14)$$

where the four-potential is:

$$A^\mu = \left(\frac{\phi}{c}, \underline{A}\right). \quad -(15)$$

Here ϕ is the scalar potential and $\underline{A}$ is the vector potential. For a circularly polarized electromagnetic field:

$$\underline{A} = A^{(0)}\left(\underline{i}\cos\phi + \underline{j}\sin\phi\right) \quad -(16)$$

where the electromagnetic phase is defined as:

$$\phi = \omega t - \kappa Z. \quad -(17)$$

Here ω is the angular frequency at instant t and κ is the vector potential at coordinate Z. The phase can be rewritten in terms of ω as:

$$\phi = \omega\left(t - \frac{Z}{c}\right). \quad -(18)$$

The method developed here to describe the IFE is to integrate $\underline{p}$ directly to give the position vector $\underline{r}$ and to calculate the relativistic angular momentum directly:

$$\underline{J} = \underline{r} \times \underline{p} \quad -(19)$$

it is shown that the relativistic angular momentum is proportional to the ECE spin field. The angular frequency Ω of the electron in the electromagnetic field is then calculated from a comparison of the relativistic angular momentum $\underline{J}$ and kinetic energy T. Finally the field angular frequency and phase are expressed in terms of Ω. The additional effect of a static magnetic field (Section 3) is to change Ω, and therefore the phase. This is the one electron Faraday effect, which is due to a combination of the ECE spin field and a static magnetic field.

The relativistic momentum is:

$$\underline{p} = \gamma m \frac{d\underline{r}}{d\tau} = \gamma m \frac{d\underline{r}}{dt}\frac{dt}{d\tau} \quad -(20)$$

where:

$$\frac{dt}{d\tau} = \gamma = \left(1 - \frac{u^2}{c^2}\right)^{-1/2}. \quad -(21)$$

Here τ is the proper time. Therefore the position vector is given by:

$$\underline{r} = \frac{1}{\gamma}\int \underline{v}\, dt. \quad -(22)$$

The electronic momentum in the presence of the electromagnetic field is:

$$\underline{p} = \gamma m \underline{v} + e\underline{A}. \quad -(23)$$

Therefore the relativistic angular momentum is:

$$\underline{J} = \underline{r} \times \underline{p} = \left(\int \underline{v}\, dt + \frac{1}{\gamma m}\int e\underline{A}\, dt\right) \times \left(\gamma m \underline{v} + e\underline{A}\right) \quad -(24)$$

If we define:

$$r_0 = \int \underline{v}\, dt \quad -(25)$$

then:

$$\underline{J} = \gamma \underline{r_0} \times \underline{p} + e \underline{r_0} \times \underline{A} + e \int \underline{A} dt \times \underline{v} + \frac{e^2}{m\gamma} \int \underline{A}\, dt \times \underline{A} . \quad -(26)$$

The second order interaction term in Eq. (26) is:

$$\underline{J} = \frac{e^2 A^{(0)2}}{m \gamma \omega} \underline{k} \quad -(27)$$

which is expressed in terms of the ECE spin field $\underline{B}^{(3)}$ as:

$$\underline{J}^{(3)} = \frac{e^2 c^2 B^{(0)}}{m \gamma \omega^3} \underline{B}^{(3)} . \quad -(28)$$

In the non-relativistic limit:

$$\gamma \rightarrow 1 \quad -(29)$$

and the induced magnetic dipole moment is given through the gyromagnetic ratio by:

$$\underline{m}^{(3)} = \frac{e^3 c^2 B^{(0)}}{2 m^2 \omega^3} \underline{B}^{(3)} . \quad -(30)$$

The non-relativistic kinetic energy of interaction is:

$$T = \frac{1}{2} \Omega J . \quad -(31)$$

Eqs. (28) and (31) are the same as results obtained previously {1-14} from the relativistic Hamilton Jacobi equation. Therefore a check for self consistency has been obtained.

The magnitude of the momentum in Eq. (23) is:

$$p = \gamma m v + eA. \quad -(32)$$

The relativistic position vector of the electron in the electromagnetic field is:

$$\underline{r} = \underline{r}_0 + \frac{e}{m\gamma}\int \underline{A}\, dt \quad -(33)$$

and its magnitude for the plane wave (16) is:

$$r = r_0 + \frac{eA^{(0)}}{\gamma m \omega}. \quad -(34)$$

The magnitude of the relativistic angular momentum is then:

$$J = rp = \left(r_0 + \frac{eA^{(0)}}{\gamma m \omega}\right)\left(\gamma m v + eA^{(0)}\right). \quad -(35)$$

The angular momentum magnitude is:

$$J = \frac{T}{\Omega}. \quad -(36)$$

From previous work {1-14} the kinetic energy of the electron in the electromagnetic field is:

$$T = \frac{\left(\gamma m v + eA^{(0)}\right)^2 c^2}{mc^2(1+\gamma) + e\phi}. \quad -(37)$$

Thus:

$$\Omega = \frac{T}{J} \quad -(38)$$

and the angular velocity of the electron in the electromagnetic field is:

$$\Omega = \frac{(\gamma m v + eA^{(0)})c^2}{\left(r_0 + \frac{eA^{(0)}}{\gamma m \omega}\right)\left(mc^2(1+\gamma) + e\phi\right)} . \quad -(39)$$

In the non-relativistic limit for a weak applied electromagnetic field:

$$eA \ll \gamma m v, \quad eA \ll \gamma m \omega, \quad -(40)$$

$$e\phi \ll mc^2(1+\gamma), \quad -(41)$$

$$\gamma \rightarrow 1, \quad -(42)$$

and the classical non-relativistic result is recovered for the kinetic energy of the free electron:

$$T = \frac{1}{2}\Omega J . \quad -(43)$$

3. THE FARADAY EFFECT

From Eq. (39) it is seen that the angular velocity of the electromagnetic field may be expressed in terms of that of the electron:

$$\omega = \frac{eA\Omega}{\gamma m (x - \Omega r_0)} , \quad x = \frac{(\gamma m v + eA)c^2}{mc^2(1+\gamma) + e\phi} . \quad -(44)$$

The electromagnetic phase from Eq. (18) is therefore:

$$\phi = \left(t - \frac{Z}{c}\right)\left(\frac{eA\Omega}{\gamma m (x - \Omega r_0)}\right) . \quad -(45)$$

A static magnetic field is now added to this system in order to describe the one electron Faraday effect. The effect of the static magnetic field on the angular velocity of the electron is well known from electron spin resonance theory to be:

$$\Omega = \frac{eB}{m} . \quad -(46)$$

For an initially stationary electron:

$$v = 0 \quad -(47)$$

considered to be initially at

$$r = 0 \quad -(48)$$

the expression (45) simplifies. We further consider the non-relativistic limit:

$$mc^2(1+\gamma) \gg e\phi, \quad \gamma \rightarrow 1 \quad -(49)$$

to obtain the simple result:

$$\omega \rightarrow 2\Omega . \quad -(50)$$

Therefore the electromagnetic phase is changed in this approximation to:

$$\phi = \underline{2}\left(t - \frac{Z}{c}\right)\left(\Omega + \frac{eB}{m}\right) - (51)$$

i.e. there is a phase change:

$$\Delta\phi = 2\frac{eB}{m}\left(t - \frac{Z}{c}\right) - (52)$$

which is the Faraday effect. The effect of this change of phase on a plane wave such as (16)

is to change it to:

$$\underline{A}' = A^{(0)}\left(\underline{i}\cos\phi' + \underline{j}\sin\phi'\right) \quad -(53)$$

where:

$$\cos\phi' = a\cos\phi, \quad \sin\phi' = b\sin\phi. \quad -(54)$$

For example if:

$$\phi = 45^\circ, \quad \phi' = 60^\circ \quad -(55)$$

then:

$$a = 1.414, \quad b = 0.816. \quad -(56)$$

So the circular polarization of Eq. (16) is changed to the elliptical polarization of Eq. (53) and this is equivalent to rotating the plane of polarization. It is seen that both IFE and FE are due fundamentally to the ECE spin field via Eq. (30).

4. HYPER-RELATIVISTIC LIMIT.

.In this limit synchrotron radiation is emitted by the circling electron, as is well known. In the hyper-relativistic limit:

$$v \rightarrow c. \quad -(57)$$

In consequence:

$$u \rightarrow 0 \quad -(58)$$

because the speed of the electron cannot be increased from the limit c. So a Lorentz boost of the electron traveling at c will result again in the electron traveling at c. Eq. (58) follows from this law of relativity introduced by Einstein in 1905. Therefore the kinetic energy from Eq. (39) approaches:

$$T \rightarrow \left(\frac{mc^2}{e\phi} \right) \omega J. \qquad -(59)$$

In this limit the rest energy mc^2 of the electron becomes photon energy of a given frequency ω:

$$\hbar\omega = mc^2 = e\phi \qquad -(60)$$

where the quantum of energy of the electromagnetic field is defined by:

$$T = \hbar\omega. \qquad -(61)$$

The angular momentum J of the electron becomes the quantum of angular momentum of the electromagnetic field:

$$J = \hbar \qquad -(62)$$

so we obtain:

$$T \rightarrow \hbar\omega. \qquad -(63)$$

In this case the electromagnetic field (one photon) is of such intensity as to transfer all its energy ($\hbar\omega$), angular momentum ($\hbar$) and angular velocity (ω) to the electron in an inelastic collision. In the opposite non-relativistic limit (43) the kinetic energy of the electron is essentially unaffected by the weak electromagnetic field in an elastic interaction of field and electron. These results were again obtained from the relativistic Hamilton Jacobi equation of previous work {1-14}. So both methods give self-consistent information on the IFE and FE. It has been shown that both effects are due to the ECE spin field.

ACKNOWLEDGMENTS

The British Government is thanked for a Civil List pension in recognition of distinguished services to Britain and the Commonwealth in science. The staff of AIAS and many others are thanked for interesting discussions.

REFERENCES

{1} M. W. Evans, "Generally Covariant Unified Field Theory: the Geometrization of Physics" (Abramis Academic, Suffolk, 2005), vol. 1.

{2} ibid., volumes 2 and 3 (2006).

{3} ibid., vol. 4 in prep. (papers 55 to 70 on www.aias.us).

{4} ibid., vol. 5 in prep. (Papers 71 onwards on www.aias.us).

{5} L. Felker, "The Evans Equations of Unified Field Theory" (Abramis in press, also on www.aias.us).

{6}H. Eckardt, L. Felker, S. Crothers and D. Indranu, papers and notes on www.aias.us.

{7} M. W. Evans (ed.), "Modern Non-Linear Optics", a special topical issue in three parts of I. Prigogine and S. A. Rice (series eds.), "Advances in Chemical Physics" (Wiley-Interscience, New York, 2001, 2nd ed.), vols. 119(1) to 119(3).

{8} M. W. Evans and L. B. Crowell, "Classical and Quantum Electrodynamics and the B(3) Field" (World Scientific, Singapore, 2001).

{9} M. W. Evans and J.-P. Vigier, "The Enigmatic Photon" (Kluwer, Dordrecht, 1994 to 2002, hardback and softback), in five volumes.

{10} M. W. Evans and A. A. Hasanein, "The Photomagneton in Quantum Field Theory"

(World Scientific, Singapore, 1994).

{11} M. W. Evans, "The Photon's Magnetic Field" (World Scientific, 1992).

{12} M. W. Evans and S. Kielch (eds.), first edition of ref. (7), (Wiley Interscience, New York, 1992, 1993 and 1997, hardback and softback), vols. 85(1) to 85(3).

{13} S. Carroll, "Space-time and Geometry: an Introduction to General Relativity" (Addison-Wesley, New York, 2004).

{14} M. W. Evans, papers and letters in "Foundations of Physics" and "Foundations of Physics Letters" and Physica B, 182, 227, 237 (1992).