PHYSICAL OPTICS, THE SAGNAC EFFECT, AND THE AHARONOV-BOHM EFFECT IN THE EVANS UNIFIED FIELD THEORY

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A generally covariant and gauge invariant description of physical optics, the Sagnac effect, and the Aharonov-Bohm (AB) effect is developed using the appropriate phase factor for electrodynamics. The latter is a generally covariant development of the Dirac-Wu-Yang phase factor based on the generally covariant Stokes theorem. The Maxwell-Heaviside (MH) field theory fails to describe physical optics, interferometry, and topological phase effects in general because the phase factor in that theory is under-determined. A random number from a U(1) gauge transformations can be added to the MH phase factor. The generally covariant phase factor of the Evans unified field theory is gauge invariant and has the correct property under parity inversion to produce observables such as reflection, interferograms, the Sagnac and AB effects, the Tomita-Chao effect and topological phase effects in general. These effects are described simply and self consistently with the generally covariant Stokes theorem in which ordinary exterior derivative is replaced by the covariant exterior derivative of differential geometry.

Key words: Evans unified theory, $\mathbf{B}^{(3)}$ field, physical optics, reflection, interferometry, Sagnac effect, AB effect, Tomita-Chao effect, topological phase effects.

1. INTRODUCTION

In generally covariant electrodynamics [1-8] the potential field is the tetrad within a C negative coefficient, and the gauge invariant electromagnetic field is a wedge product of tetrads. The covariant exterior derivative is used to describe the electromagnetic field using the first Maurer-Cartan structure relation of differential geometry [9]. The generally covariant field theory leads to $O(3)$ electrodynamics and the Evans-Vigier field $\mathbf{B}^{(3)}$. The latter is observed in several experiments, including the inverse Faraday effect [10]. The experimental evidence for the $\mathbf{B}^{(3)}$ field, and thus for generally covariant electrodynamics, is reviewed in the literature [11].

In this paper a generally covariant Dirac-Wu-Yang phase is defined within Evans unified field theory by using a generally covariant exterior derivative to develop the Stokes theorem of differential geometry [12]. The electromagnetic phase factor is described by this generally covariant Stokes theorem. This procedure produces the correct parity inversion symmetry needed to describe fundamental physical optical phenomena such as reflection and interferometry. The MH theory of electrodynamics is not generally covariant, i.e., is a theory of special relativity, and for this reason is unable to describe physical optics and interferometry, and unable to describe the Sagnac and AB effects. In Sec. 2, these shortcomings of the MH theory are summarized. In Sec. 3 the Evans unified field theory is applied to physical optics, which is correctly described with a phase factor constructed from the appropriate contour and area integrals of the generally covariant Stokes theorem. This procedure gives the correct parity inversion symmetry of the electromagnetic phase factor, and so is able to correctly describe physical optical phenomena such as reflection, interferometry, the Sagnac effect and the AB effect. In each case the generally covariant theory is simpler than the MH theory and at the same time is the first correct theory of physical optics as a theory of electrodynamics.

2. SHORTCOMINGS OF THE MAXWELL-HEAVISIDE FIELD THEORY IN PHYSICAL OPTICS

In the MH field theory the phase factor is defined by

$$\Phi_{\text{MH}} = \exp[i(\omega t - \kappa Z)], \tag{1}$$

where ω is the angular frequency of radiation at instant t and κ is the wave number of radiation at point Z in the propagation axis. Under a U(1) gauge transformation [13] the phase factor (1) becomes

$$\Phi_{\text{MH}} \rightarrow \exp(i\alpha)\Phi_{\text{MH}} = \exp[i(\omega t - \kappa Z + \alpha)], \tag{2}$$

where α is arbitrary. In consequence the phase factor is not gauge invariant, a major failure of MH field theory. In other words the phase factor (2) is undetermined theoretically, because any number α can be added to it without affecting the description of experimental data. In Michelson interferometry [14], for example, an interferogram is formed by displacing a mirror in one arm of the interferometer, thus changing Z in Eq. (1) for constant ω, t and κ. The path of a beam of light from the beam-splitter to the mirror and back to the beam-splitter is increased by $2Z$. The conventional description of Michelson interferometry incorrectly asserts that this increase $2Z$ in the path of the light beam results in a change of phase factor

$$\Delta\Phi_{\mathrm{MH}} = \exp(2i\kappa Z) \tag{3}$$

producing the observed interferogram, $\cos(2\kappa Z)$, for a monochromatic light beam in Michelson interferometry.

This basic error in the conventional theory has been reviewed in detail in Ref. [11]. The main purpose of this paper is to show that the error is corrected when the theory of electrodynamics is developed into a generally covariant unified field theory.

The origin of the error is found when parity inversion symmetry is examined. Consider the parity inversion symmetry of the product $\boldsymbol{\kappa} \cdot \mathbf{r}$ of wave vector $\boldsymbol{\kappa}$ and position vector $\mathbf{r}$:

$$\hat{P}(\boldsymbol{\kappa} \cdot \mathbf{r}) = \boldsymbol{\kappa} \cdot \mathbf{r} \tag{4}$$

because

$$\hat{P}(\boldsymbol{\kappa}) = -\boldsymbol{\kappa}, \tag{5}$$

$$\hat{P}(\mathbf{r}) = -\mathbf{r}. \tag{6}$$

Parity inversion is equivalent to reflection, and so κZ does not change under reflection in the MH phase factor (1). This means that the reflected phase factor is the same as the phase factor of the beam before reflection. In terms of the Fermat principle of least time in optics [15], this means

$$\Phi_2 = e^0 \Phi_1 = \Phi_1, \tag{7}$$

where Φ_1 is the phase factor of the wave before reflection and Φ_2 the factor after reflection.

The experimentally observed interferogram in the Michelson interferometer [14] and in reflection in general must be described, however, by

$$\Phi_2 = e^{2i\kappa Z} \Phi_1, \tag{8}$$

and so the MH field theory of electrodynamics fails at a fundamental level to describe physical optics. Various hand-waving arguments have been used over the years to address this problem, but there has never been a solution. The reason is that there cannot be a solution in special relativistic gauge field theory because, as we have seen, the phase factor is not gauge invariant; a random number α can be added to it [13]. This number is random in the same sense that χ is random in the usual gauge transform of the MH potential, i.e.,

$$\mathbf{A} \to \mathbf{A} + \boldsymbol{\nabla}\chi. \tag{9}$$

The U(1) gauge transformation is a rotation of the phase factor (2), a rotation of a gauge field brought about by multiplication by the rotation generator of U(1) gauge field theory, the exponential $e^{i\alpha}$. It is always argued in U(1) gauge transformation that χ is unphysical; so, for self-consistency, α must be unphysical. However, an unphysical factor α cannot appear in the phase factor [13] of a physical theory, i.e., a theory of physics aimed at describing data. So the MH field theory fails at a fundamental level to describe physical optics. In other words, MH gauge field theory fails to describe reflection and interferometry.

The Sagnac interferometer [13] with platform at rest produces an observable interferogram both in electromagnetic waves and matter waves (interfering electron beams [11]):

$$\gamma = \cos(2\omega^2/c^2 Ar), \tag{10}$$

where Ar is the area enclosed by the interfering beams (electromagnetic or electron beams). When the platform is rotated at an angular frequency Ω, a shift is observed in the interferogram:

$$\Delta\gamma = \cos(4\omega\Omega/c^2 Ar). \tag{11}$$

The interferograms (10) and (11) are independent of the shape of the area Ar, and are the same to an observer on and off the rotating platform. There have been many attempts to explain the Sagnac effect since it was first observed, all have their shortcomings, as reviewed by Barrett [13]. In particular the MH field theory (a U(1) gauge field theory) is wholly incapable of explaining the effect [13], either with platform at rest or in motion. In Sec. 3 it is shown that the Sagnac effect is described straightforwardly in the Evans unified field theory as a change in the Cartan tetrad [1-8]. It is well known that the tetrad plays the role of metric in differential geometry. This gets to the core of the problem with MH theory – it is a theory of flat spacetime and so is metric-invariant [13]. The phase factor (1) of the MH field theory is a number, and so is invariant under both parity and motion reversal symmetry. The phase factor (1) has no sense of parity, of handedness or of chirality (of being clockwise of anticlockwise) and does not change under frame rotation or platform rotation. The MH theory cannot describe any feature of the Sagnac effect, and again fails at a fundamental level.

There has been a forty-year controversy over the AB effect [12]. The only way to describe it with MH field theory is to shift the problem to the nature of spacetime itself. It is asserted conventionally [12] that the latter is multiply connected and that this feature (rather magically) produces an AB effect by U(1) gauge transformation into the vacuum. It is straightforward to show as follows that this assertion is incorrect because it violates the Poincaré lemma. In Sec. 3 we show that the Evans unified field theory explains the AB effect with the correct generally covariant phase factor. This is the same in mathematical structure as for the Sagnac effect and physical optics, only it interpretation is different. The Evans unified theory is therefore much simpler, as well as much more powerful, than the earlier MH field theory. To prove that the MH theory of the AB effect is incorrect, consider the Stokes theorem in the well-known [9] notation of differential geometry:

$$\exp\left(ig\oint_{\delta S} A\right) = \exp\left(ig\oint_{S} d\wedge A\right), \tag{12}$$

where A is the potential one-form and $d\wedge A$ is the exterior derivative of A, a two-form. The factor g in the AB effect is $e/\hbar$, where $-e$ is the charge on the electron and $\hbar$ is the Dirac constant $(h/2\pi)$. Under the U(1) gauge transformation

$$A \rightarrow A + d\chi \tag{13}$$

the two-form becomes

$$d\wedge A \rightarrow d\wedge A + d\wedge d\chi. \tag{14}$$

However, the Poincaré lemma asserts that, for any topology,

$$d\wedge d := 0. \tag{15}$$

Therefore the two-form $d\wedge A$ is unchanged under the U(1) gauge transformation (13). The Stokes theorem (12) then shows that

$$\oint_{\delta S} d\chi := 0. \tag{16}$$

However, the conventional description of the AB effect [12] relies on the incorrect assertion

$$\oint_{\delta S} d\chi \neq 0. \tag{17}$$

This is incorrect because it violates the Poincaré lemma (15). The lemma is true for multiply-connected as well as simply-connected regions. In ordinary vector notation the lemma states that, for any function χ,

$$\boldsymbol{\nabla}\times\boldsymbol{\nabla}\chi := 0, \tag{18}$$

and this is true for a periodic function because it is true for any function. A standard freshman textbook [16] will show that the Stokes theorem and the Green theorem are both true for multiply-connected regions as well as for simply-connected regions.

There is therefore no correct explanation of the AB effect in MH theory and special relativity. Similar arguments show that MH theory cannot be used to describe a topological phase effect such as the Tomita-Chao effect, a shift in phase brought about by rotating a beam of light around a helical optical fibre. In Sec. 3 we show that the Tomita-Chao effect is a Sagnac effect with several loops, and is a shift in the Cartan tetrad of the Evans unified field theory. Similarly, the Berry phase of matter wave theory is a shift in the tetrad of the Evans unified field theory. Many more examples could be given of the advantages of the Evans unified field theory over the MH theory.

3. GENERALLY COVARIANT PHASE FACTOR FROM THE EVANS UNIFIED FIELD THEORY

The first correct description of the phase factor in electrodynamics was given in Ref. [11]. Since then, a unified field theory has been developed [1-8] which in turn gives a further insight to the phase factor of electrodynamics and physical optics. In order to construct a phase factor that has the correct symmetry under parity inversion, which is generally covariant and valid for all topologies, and which furthermore is correctly gauge invariant, we use the generally covariant Stokes theorem in differential geometry within the exponent of the electromagnetic phase factor. The phase factor is therefore an application of Eq. (12) for the propagating electromagnetic field, which is considered to be part of the generally covariant unified field theory [1-8]. Therefore the phase factor of electrodynamics and physical optics is

$$\Phi = \exp\left(ig\oint_{DS} A\right) = \exp\left(ig\int_{S} D\wedge A\right) \tag{19}$$

under all conditions (free or radiated field and field matter interaction).

We shall show in this section that the phase factor (19) produces fundamental phenomena of physical optics such as reflection and interferometry through a difference in contour integrals of Eq. (19)

$$\Delta\Phi = \exp\left(i\left(\oint_{0}^{Z}\boldsymbol{\kappa}\cdot d\mathbf{r} - \oint_{Z}^{0}\boldsymbol{\kappa}\cdot d\mathbf{r}\right)\right). \tag{20}$$

It is also shown that the phase factor (19) can be written as

$$\Phi = \exp\left(i\oint\boldsymbol{\kappa}\cdot d\mathbf{r}\right) = \exp\left(i\int\kappa^2 dAr\right) \tag{21}$$

and automatically gives the Sagnac effect from the right-hand side of Eq. (21). Equation (21) follows from the generally covariant Stokes formula applied to the one-form κ of differential geometry, the one-form that represents the wave number. In Eq. (19), $D \wedge A$ is the covariant exterior derivative [1-9] necessary to make the theory correctly covariant in general relativity. Equation (19) is the generally covariant Stokes theorem, i.e., the Stokes theorem defined for the non-Euclidean geometry of the base manifold and therefore in general relativity [11], and it follows from differential geometry that $D \wedge A$ is a two-form. The electromagnetic phase can be written in general

$$\Phi = \exp\left(i \oint_{DS} \kappa\right) = \exp\left(i \int_S D \wedge \kappa\right); \tag{22}$$

and Eq. (19) and (22) are equivalent definitions provided that we recognise the following duality between the electromagnetic field as a potential one-form, denoted by A in differential geometry [9], and the wave number one-form κ:

$$\kappa = gA. \tag{23}$$

Here g is a constant of proportionality between the A and κ one-forms. From gauge theory [11] g is given by the wave-particle dualism represented by

$$p = \hbar\kappa = eA, \tag{24}$$

where p is a momentum one-form. The duality (24) asserts that the photon is a particle with momentum $\hbar\kappa$ and also a field with momentum eA, where $-e$ is the charge on the electron. The meaning of the duality (24) has been explained extensively in the literature on $O(3)$ electrodynamics [11]. The charge e is defined as the ratio of magnitudes:

$$e = |p|/|A|, \tag{25}$$

and so Eq. (23) means that the wave number in physical optics and electrodynamics is multiplied by the potential and divided by the magnitude of the potential to inter-relate the field as particle with momentum $\hbar\kappa$ and the field as wave, with momentum eA. This is the de Broglie wave-particle duality applied to the electromagnetic field.

The electromagnetic phase in unified field theory and general relativity is therefore

$$\Phi = \exp\left(i \oint_{DS} \kappa\right) = \exp\left(\frac{i}{\hbar} \oint_{DS} p\right) = \exp\left(ig \oint_{DS} A\right), \tag{26}$$

which is the generally covariant Dirac phase or Wilson loop for electrodynamics as part of a unified field theory [1-8]. The covariant derivative appearing in Eq. (19) is defined in the notation of differential geometry by

$$D \wedge A = d \wedge A + gA \wedge A. \tag{27}$$

The symbol A is convenient shorthand notation for the tetrad A^a_μ [1-9], so Eq. (27), for example, denotes

$$(D \wedge A^c)_{\mu\nu} = (d \wedge A^c)_{\mu\nu} + g A^a_\mu \wedge A^b_\nu. \tag{28}$$

For $O(3)$ electrodynamics with orthonormal space index

$$a = (1), (2), (3) \tag{29}$$

the complex circular basis, then:

$$D \wedge A^{(1)*} = d \wedge A^{(1)*} - igA^{(2)} \wedge A^{(3)}, \quad \text{et cyclicum.} \tag{30}$$

In Eq. (30) the indices μ and ν of the base manifold have been suppressed, as is the custom in differential geometry [9], because Eqs. (27) to (30) are equations of differential geometry valid for all geometries of the base manifold. So, the indices μ and ν are always the same on both sides and can be suppressed for convenience of notation [9]. The shorthand notation of Eq. (27) is therefore a summary of the essential features of Eqs. (28) to (30). The $B^{(3)}$ form of differential geometry

$$B^{(3)} = D \wedge A^{(3)} = -igA^{(1)} \wedge A^{(2)}, \tag{31}$$

is the expression of the fundamental Evans-Vigier field [11] in differential geometry. The $B^{(3)}$ form is a component of a torsion tensor (a vector valued two-form or antisymmetric tensor), and is the spin Casimir invariant of the Einstein group. It will be demonstrated in this section that it is responsible for and therefore observed in all physical optical effects through the generally covariant and gauge invariant phase factor of electrodynamics

$$\Phi = \exp\left(ig \oint_{DS} A^{(3)}\right) = \exp\left(ig \int_S B^{(3)}\right). \tag{32}$$

Equation (32) is the result of a generally covariant theory of electrodynamics and it is necessary to be precise about the meaning of the contour integral on the left-hand side and the area integral on the right-hand side of Eq. (32). In particular, $A^{(3)}$ is an irrotational function, whose curl vanishes. It follows that Eq. (32) cannot be the result of the ordinary Stokes theorem

$$\oint_{\delta S} A = \int_S B = \int_S d \wedge A, \tag{33}$$

in which $d\wedge$ is the ordinary exterior derivative [11] of differential geometry. In vector notation, Eq. (33) reads

$$\oint \mathbf{A} \cdot d\mathbf{r} = \int \mathbf{B} \cdot \mathbf{k} dAr = \int \boldsymbol{\nabla} \times \mathbf{A} \cdot \mathbf{k} dAr, \tag{34}$$

and for an irrotational function $\mathbf{A}$ both sides of Eq. (34) vanish. Therefore, in Eq. (32), $\oint A^{(3)}$ is not the result of an integration around a closed loop as defined in the ordinary Stokes theorem [16].

Another way of seeing this is that, if we attempt to integrate the ordinary plane wave

$$\mathbf{A} = \frac{A^{(0)}}{\sqrt{2}}(\mathbf{i} - i\mathbf{j})e^{i\phi} \tag{35}$$

around a closed loop (such as a circle) using the ordinary Stokes theorem, we obtain the result

$$\oint \mathbf{A} \cdot d\mathbf{r} = \int \boldsymbol{\nabla} \times \mathbf{A} \cdot \mathbf{k} dAr = 0. \tag{36}$$

The right-hand side is true because $\mathbf{k}$ is perpendicular to $\mathbf{i}$ and $\mathbf{j}$, and the left-hand side is proven by parameterizing the circle as

$$dx = -x_0 \sin\theta d\theta, \quad dy = y_0 \cos\theta d\theta \tag{37}$$

and using

$$\int_0^{2\pi} \sin\theta d\theta = \int_0^{2\pi} \cos\theta d\theta = 0. \tag{38}$$

This means that the ordinary Stokes theorem of special relativity cannot be used to represent the phase factor of electrodynamics. The essential reason for this is that electrodynamics must be a theory of general relativity in which the exterior derivative is replaced by the covariant exterior derivative [1-9] under all conditions (free field and field-matter interaction). Only in this way can a unification of gravitation and electromagnetism be achieved [1-8] within general relativity.

This type of field unification is the result of the principle of general relativity, i.e., that all theories of physics must be theories of general relativity. (The MH theory is the archetypical theory of special relativity, and is not generally covariant. This leads to the fundamental problems summarized in Sec. 2.)

The method of interpreting Eq. (32) is found by considering integration around the transverse part of a helix. The transverse part of a helix is a position vector which coils around the Z axis as follows:

$$\mathbf{r} = (X\mathbf{i} - iY\mathbf{j})e^{i\phi} \tag{39}$$

and defines a non-Euclidean (curling) base-line in the non-Euclidean base manifold of general relativity. The product

$$r^2 = \mathbf{r} \cdot \mathbf{r}^* = X^2 + Y^2 \tag{40}$$

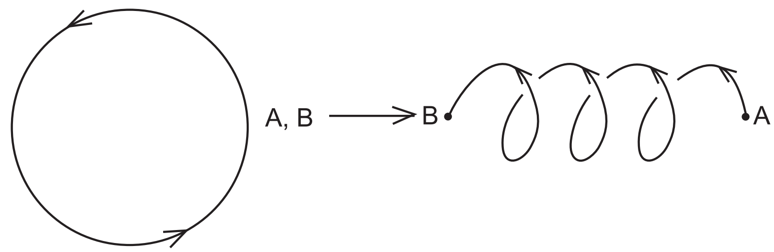

Fig. 1. Representation of the circle whose circumference is the same as the arc length of the helix AB.

for the helix is the same as for a circle of radius r. Therefore integration around the transverse part of a helix is the same as integration around a circle of circumference $2\pi r$ and area πr^2 provided that the arc length of the helix is also $2\pi r$. The arc length is the distance along the helix from A to B in Fig. 1. This diagram summarizes the process of taking a circle of diameter $2r$ and drawing it out into a helix coiled along the Z axis. The distance from A to B around the circumference of the circle is the same as the distance from A to B along the transverse part of the helix (its arc length [16]). The difference between the helix and the circle is that there is a longitudinal component of the helix, the distance along the Z axis from A to B. The arc length of the helix is along its transverse component from A to B, therefore the distance along Z from A to B is defined by

$$Z \leq 2\pi r. \tag{41}$$

When this distance is equal $2\pi r$, the helix becomes a straight line of length $2\pi r$ along the Z axis.

The difference between the helix and the circle illustrates the difference between the generally covariant Stokes theorem and the ordinary Stokes theorem, and is also the essential difference between generally covariant electrodynamics and MH electrodynamics in which radiation is the plane wave (35), with no longitudinal component. In generally covariant electrodynamics there is a longitudinal component, the Z component of the helix. The latter can be parameterized [16] by

$$x = x_0 \cos\theta, \quad y = y_0 \sin\theta, \quad z = z_0\theta, \tag{42}$$

and so contains both longitudinal and transverse components. The circle can be parameterized [16] by

$$x = x_0 \cos\theta, y = y_0 \sin\theta \tag{43}$$

and contains only components which are perpendicular, or transverse, to the Z axis. The latter is evidently undefined in the circle, but well defined in the helix.

It is well accepted that electromagnetic radiation propagates and so contains both rotational and translational components of emotion. These must be described by the helix, and not by the plane wave of the MH theory in special relativity. There must be longitudinal components of the free electromagnetic field, represented by $\mathbf{A}^{(3)}$ and $\mathbf{B}^{(3)}$.

The ordinary Stokes theorem cannot be applied to the helix, because the latter does not define a closed curve, i.e., the path from A to B of the helix is not closed. In the circle the path around the circumference from A to B brings us back to the starting point, and defines the area of the circle. In order to close the path in the helix the path from A to B along its transverse part must be followed by the path back from B to A along its Z axis, as in Fig. 2. The closed path on the right-hand side of this diagram defines the contour integral of the generally covariant Stokes theorem appearing on the left-hand side of Eq. (32). Furthermore, as we have seen, integration around the transverse part of the helix is equivalent to integration around a circle whose circumference is equal to the arc length of the helix. Therefore the plane wave contributes nothing to the generally covariant Stokes theorem because the integration of the plane wave around a circle is zero, as shown in Eq. (34) to (38).

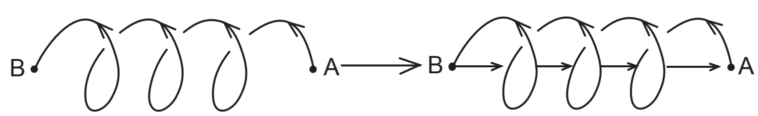

Fig. 2. Representation of the line over which the integral is evaluated in the non-Abelian Stokes theorem and Evans phase law.

It is concluded that the electromagnetic phase in generally covariant electrodynamics and physical optics is completely described by Eq. (32). The contour integral on the left-hand side of this equation is along the Z axis of the helix whose arc length is the same as the circumference $2\pi r$ of a circle of radius r. The area integral on the right-hand side of Eq. (32) is an integral around the area πr^2 of this circle. The contour integral is an integral over the irrotational and longitudinal potential field $\mathbf{A}^{(3)}$, and the area integral is an integral over the Evans-Vigier field $\mathbf{B}^{(3)}$, the spin invariant of the Einstein group. Therefore all of the physical optics and electrodynamics are described by a phase factor which is completely defined by $\mathbf{A}^{(3)}$ and $\mathbf{B}^{(3)}$.

In the following sections, applications of this law are given for reflection, interferometry, the Sagnac effect and the AB effect. The catastrophic failings of the MH field theory of special relativity (summarized in Sec. 2) are corrected in each case. The easiest way to see that the MH theory is not generally covariant is to note that the plane wave defines a finite scalar curvature

$$R = (\partial^2 \mathbf{A}^{(1)}/\partial Z^2)/|\mathbf{A}^{(1)}| = \kappa^2. \tag{44}$$

However, in special relativity, the scalar curvature $R = 0$ by definition, because the spacetime of a theory of special relativity is Euclidean (or "flat"). In the generally covariant description of electrodynamics, the scalar curvature κ^2 of the helix is consistent with the fact that we are reconsidering a baseline that coils around the Z axis in the base manifold.

4. REFLECTION AND MICHELSON INTERFEROMETRY

The generally covariant phase law (32) has the correct parity inversion symmetry for the description of reflection in physical optics, and thus of Michelson interferometry. Considering the Z axis as the propagation axis, the parity inversion symmetry is

$$\oint \kappa^{(3)} dZ \xrightarrow{\hat{p}} -\oint \kappa^{(3)} dZ, \tag{45}$$

and so parity inversion is equivalent to traversing the path along the Z axis in the opposite direction

$$\int_0^Z \kappa^{(3)} dZ \xrightarrow{\hat{p}} \int_Z^0 \kappa^{(3)} dZ. \tag{46}$$

Traversing the path in the opposite direction therefore produces the following change in phase factor:

$$\exp\left(i\int_0^Z \kappa^{(3)} dZ\right) \to \exp\left(i\int_Z^0 \kappa^{(3)} dZ\right), \tag{47}$$

producing the experimentally observed phase change upon normal reflection of electromagnetic radiation from a perfectly reflecting mirror:

$$\exp\left(i\int_0^Z \kappa^{(3)} dZ\right) \exp\left(i\int_Z^0 \kappa^{(3)} dZ\right) = \exp(2i\kappa^{(3)}Z). \tag{48}$$

In Michelson interferometry [14] the phase change (48) is observed as the cosinal interferogram for a monochromatic beam of radiation:

$$\Delta\gamma = \cos(2\kappa^{(2)}Z). \tag{49}$$

In MH electrodynamics (special relativity) normal reflection produces no phase change, because normal reflection, by definition, is equivalent to the parity inversion (4). The latter produces no change in phase in MH electrodynamics, and no Michelson interferogram, contrary to experimental observation. In generally covariant electrodynamics, the phase factor is chiral (or handed) in nature, because it is defined by the following cyclic equation:

$$\oint \mathbf{A}^{(3)} \cdot d\mathbf{r} = -ig \int \mathbf{A}^{(1)} \times \mathbf{A}^{(2)} \cdot \mathbf{k} dAr, \tag{50}$$

an expression of the generally covariant Stokes theorem.

5. THE SAGNAC EFFECT

The generally covariant phase law (32) can be exemplified by integration around a circle of radius r. The area on the right-hand side of Eq. (32) in this case is πr^2, so we obtain

$$\int \mathbf{B}^{(3)} \cdot \mathbf{k} dAr = \pi r^2 B_Z^{(3)} = \pi r^2 B^{(0)}. \tag{51}$$

The phase factor observed in the Sagnac effect therefore originates in the phase law (32) as follows:

$$\Phi = \exp(igB^{(0)}\pi r^2) = \exp(i\kappa^2 Ar) = \exp(i\omega^2/c^2 Ar), \tag{52}$$

where we have used

$$B^{(0)} = \kappa A^{(0)}, \quad g = \kappa/A^{(0)}, \quad Ar = \pi r^2. \tag{53}$$

The Sagnac effect with platform at rest is an interferogram formed from interference of clockwise and counter-clockwise waves, one with phase $e^{i\kappa^2 Ar}$ and one with phase factor $e^{-i\kappa^2 Ar}$ giving the difference in phase factor $e^{2i\kappa^2 Ar}$ and interferogram:

$$\Delta\gamma = \cos(2\kappa^2 Ar) = \cos(2\omega^2/c^2 Ar), \tag{54}$$

as observed experimentally (13) with great precision in the ring laser gyro and similar devices. In the MH theory there is no explanation for the Sagnac effect, as we have argued, because the MH phase is parity invariant with no sense of chirality. In MH theory there is no counterpart of the beam area, as on the right-hand side of the generally covariant phase law (32).

When the platform of the Sagnac effect is rotated at angular frequency Ω, there is a frequency shift

$$\omega \to \omega \pm \Omega, \tag{55}$$

giving rise to an extra interferogram from the phase law (32):

$$\Delta\Delta\gamma = \cos\left(4\omega\Omega/c^2\, Ar\right) \tag{56}$$

from the difference

$$(\omega + \Omega)^2 - (\omega - \Omega)^2 = 4\omega\Omega. \tag{57}$$

This is precisely as observed experimentally to great precision in the ring laser gyro [13].

The left-hand side of the phase law (32) for the Sagnac effect in a circle of radius r is

$$\int_0^Z A_Z^{(3)} dZ = A^{(0)} Z = \pi r^2 B^{(0)}, \tag{58}$$

where we have used

$$\int_0^Z dZ = Z, \quad A_Z^{(3)} = A^{(0)}. \tag{59}$$

From Eq. (53) and (58)

$$2\pi r \geq Z = \kappa\pi r^2, \quad \kappa r \geq 2. \tag{60}$$

This equation defines the distance from B to A in Fig. 2 in terms of the area πr^2 of a circle whose circumference $2\pi r$ is the same as the arc length of the helix. The latter must always be greater than or equal to the distance from A to B along Z, so:

$$\kappa r \geq 2. \tag{61}$$

Using

$$\kappa = \omega/c, \tag{62}$$

$$\omega = 2\pi f, \tag{63}$$

$$f\lambda = c, \tag{64}$$

we find that

$$\kappa = \frac{2\pi}{\lambda} \tag{65}$$

and in Eq. (60)

$$Z = \frac{2\pi}{\lambda} Ar. \tag{66}$$

This equation is a result of the phase law (32) and defines the propagation length of electromagnetic radiation in terms of its wavelength and area.

Equation (66) is the result of generally covariant electrodynamics and shows that a beam of light has a finite area. This result is obviously consistent with experimental data, but in special relativity (MH theory) the plane wave has infinite lateral extent, and the area of a beam of light is ill defined by the plane wave. In general relativity as we have seen the area is well defined by the phase law (32).

We can cross check this result for the Sagnac effect by integrating the wave number vector $\boldsymbol{\kappa}$ around the helix parameterized [16] by:

$$X = X_0 \cos\theta, \quad Y = Y_0 \sin\theta, \quad Z = Z_0\theta, \tag{67}$$

$$\frac{dX}{d\theta} = -X_0 \sin\theta, \quad \frac{dY}{d\theta} = Y_0 \cos\theta, \quad \frac{dZ}{d\theta} = Z_0. \tag{68}$$

The wave number vector is

$$\boldsymbol{\kappa} = \kappa_X \mathbf{i} + \kappa_Y \mathbf{j} + \kappa_Z \mathbf{k}, \tag{69}$$

and the contour integral on the left-hand side of the phase law (32) is

$$\oint \boldsymbol{\kappa} \cdot d\mathbf{r} = -\kappa_X X_0 \oint \sin\theta d\theta + \kappa_Y Y_0 \oint \cos\theta d\theta + \kappa_Z Z_0 \oint \theta d\theta. \tag{70}$$

The transverse part of the helix gives no contribution to this contour integral because:

$$\int_0^{2\pi} \sin\theta d\theta = \int_0^{2\pi} \cos\theta d\theta = 0. \tag{71}$$

The longitudinal component along the Z axis from B to A in Fig. 2 gives the only non-zero contribution to the contour integral, a contribution originating in

$$\int_0^{2\pi} \theta d\theta = 2\pi^2. \tag{72}$$

So,

$$\oint \boldsymbol{\kappa} \cdot d\mathbf{r} = 2\pi^2 \kappa_Z Z_0. \tag{73}$$

This result is Eq. (58) after identifying

$$Z = 2\pi^2 Z_0, \quad \kappa Z = gA^{(0)} Z. \tag{74}$$

The whole of Eq. (73) comes from the contribution along the Z axis in Fig. 2 through the axis of the helix. This contribution is not present in special relativity and the conventional Stokes theorem. Our generally covariant unified field theory [1-8] gives the observed phase factor of the Sagnac effect with platform at rest from

$$\Phi = \exp(i\kappa Z) = \exp(i\omega^2/c^2 Ar) \tag{75}$$

and also with platform in motion, as argued already.

The topological fundament of this result is that a circle can be shrunk continuously to a point and is simply connected, but a helix shrinks continuously to a line, and in this sense is not simply connected. This is the essential difference between special and general relativity in optics and electrodynamics.

6. THE AHARONOV-BOHM EFFECT

The AB effect in generally covariant electrodynamics is also described straightforwardly by the phase law (32):

$$\begin{aligned}\exp\left(ig\int \mathbf{B}^{(3)}\cdot\mathbf{k}dAr\right) &= \exp(ig\oint \mathbf{A}^{(3)}\cdot d\mathbf{r}) \\ &= \exp\left(g^2\int \mathbf{A}^{(1)}\times\mathbf{A}^{(2)}\cdot\mathbf{k}dAr\right)\end{aligned} \tag{76}$$

The interpretation of Eq. (76) for the AB effect is as follows:

(a) The magnetic field $\mathbf{B}^{(3)}$ is the static magnetic field (iron whisker or solenoid) placed between the two openings of a Young interferometer which measures the interferogram formed by two electron beams.

(b) Any magnetic field (including a static magnetic field) in generally covariant electrodynamics is the two-form [1-9]

$$B = d\wedge A + gA\wedge A, \tag{77}$$

and is also the torsion form or wedge product of tetrads:

$$B^c = -igA^a\wedge A^b. \tag{78}$$

Equation (78) follows from the fact that a magnetic field must be a component of a torsion two-form (or antisymmetric second rank tensor) that is the signature of spin in general relativity. The concept of spin is missing completely from Einstein's generally covariant theory of gravitation [9]. In the Evans unified field theory, spin (or torsion) gives rise to the generally covariant electromagnetic field. Curvature of spacetime gives rise to the gravitational field, and the latter is the symmetric product of tetrads [1-8]. In the complex circular basis [11],

$$c = (3), \quad a = (1), \quad b = (2). \tag{79}$$

(c) The factor g in Eq. (76) for the AB effect is

$$g = e/\hbar \tag{80}$$

and is the ratio of the modulus of the charge on the electron (situated in the electron beams) to the reduced Planck constant.

(d) The potentials $\mathbf{A}^{(1)} = \mathbf{A}^{(2)*}$ are defined by the cyclic relation (76) and extend outside the area of the solenoid because they are components of a tetrad multiplied by the scalar $A^{(0)}$. The tetrad is the metric tensor in differential geometry, so $\mathbf{A}^{(1)} = \mathbf{A}^{(2)*}$ are properties of non-Euclidean (i.e., spinning) spacetime. This spacetime, evidently, is not restricted to the solenoid.

(e) The area Ar is the area enclosed [12] by the electron beams of the Young interferometer.

The AB effect is therefore

$$\Delta\Phi = \exp\left(ie/\hbar \int \mathbf{B}^{(3)} \cdot \mathbf{k} dAr\right) = \exp(ie/\hbar\phi), \tag{81}$$

where ϕ is the magnetic flux produced by the solenoid of magnetic flux density $\mathbf{B}^{(3)}$. The magnetic flux (weber = volts s^{-1}) is $\mathbf{B}^{(3)}$ (tesla) multiplied by the area Ar enclosed by the electron beams.

The effect (81) is observed experimentally [12] as a shift in the interferogram of the Young interferometer, a shift caused by the iron whisker. It is seen that the AB effect is closely similar to the Sagnac effect and to Michelson interferometry and reflection in physical optics. All effects are described straightforwardly by the same phase law (76), which is therefore verified with great precision in these experiments. Recall that the MH, or U(1), phase factor (1) fails qualitatively in all four experiments (Sec. 2).

When we use the correctly covariant phase law of general relativity, Eq. (76), the AB effect becomes a direct interaction of the following potentials with the electron beam:

$$\mathbf{A}^{(1)} = \mathbf{A}^{(2)*} = \left(A^{(0)}/\sqrt{2}\right)(\mathbf{i} - i\mathbf{j})e^{i\omega t}. \tag{82}$$

These potentials define the static magnetic field of the iron whisker or solenoid as follows:

$$\mathbf{B}^{(3)} = -ig\mathbf{A}^{(1)} \times \mathbf{A}^{(2)}, \tag{83}$$

where ω is an angular frequency defining the rate at which the potentials spin around the Z axis of the solenoid. The potential

$$\mathbf{A}^{(1)} = \mathbf{A}^{(2)*} \tag{84}$$

is defined completely by the scalar magnitude $\mathbf{A}^{(0)}$, the frequency ω, and the complex unit vectors

$$\mathbf{e}^{(1)} = \mathbf{e}^{(2)*}. \tag{85}$$

The magnitude of the static magnetic field is

$$B^{(0)} = gA^{(0)2} \tag{86}$$

and

$$\mathbf{B}^{(3)} = B^{(0)}\mathbf{e}^{(3)}, \tag{87}$$

where

$$\mathbf{e}^{(1)} \times \mathbf{e}^{(2)} = i\mathbf{e}^{(3)*} = i\mathbf{k}, \quad \text{et cyclicum.} \tag{88}$$

Therefore the essence of the AB effect in general relativity is that it is an effect of spinning spacetime itself, the spinning potential $\mathbf{A}^{(1)} = \mathbf{A}^{(2)*}$ extends outside the solenoid (to whose Z axis $\mathbf{B}^{(3)}$ is confined) and $\mathbf{A}^{(1)} = \mathbf{A}^{(2)*}$ interacts directly with the electron beam, giving the observed phase shift (81). The spin of the spacetime is produced by the iron whisker, or static magnetic field. Conversely, a static magnetic field is spacetime spin as measured by the torsion form of the Evans theory [1-8] of generally covariant electrodynamics. This theory completes the earlier theory of gravitation [9].

This explanation has the advantage of simplicity (Ockham Razor), and there is no need for the obscure, and mathematically incorrect, assumptions used in the received view of the AB effect (Sec. 2). The explanation shows that the AB effect originates in general relativity with spin (the Evans unified field theory [1-8]). In analogy the AB effect is a whirlpool effect, the whirlpool is created by a stirring rod (the iron whisker) and the effect at the edges of the whirlpool is evidently measurable even though the stirring rod is not present there. The water of the whirlpool is the analogy to spinning spacetime. In the same type of analogy the Sagnac effect is a rotational effect caused by a physical rotation of the platform with respect to a fixed reference frame, whereas in the AB effect the magnetic field is a rotation or spin of spacetime (the reference frame itself). The Evans unified field theory gives a quantitative explanation of these whirlpool effects with a precision of up to 1 part on 10^{23} (contemporary ring laser gyro [11,13]).

7. GAUGE INVARIANCE OF THE PHASE LAW (32)

Finally in this paper a brief discussion is given of the gauge invariance of the generally covariant phase law (32). The Evans theory [1-8] is a theory of general relativity, in which the base manifold is non-Euclidean in nature, therefore covariant derivatives are always used from the outset instead of ordinary derivatives [9]. In both gravitation and electrodynamics the use of a covariant derivative implies that the Evans theory is intrinsically gauge invariant. The reason is that local gauge invariance in relativity theory is defined as the replacement of the ordinary derivative by a covariant derivative [12,13]. In the MH (or U(1)) electrodynamics, the covariant derivative is the *result* of a local gauge transformation of the generic gauge field ψ, a transformation under which the Lagrangian is invariant [12]. This local (or type two) gauge transformation is defined only in special relativity (flat spacetime) in MH theory and introduces the covariant derivative by changing the ordinary derivative to the U(1) covariant derivative:

$$\partial_\mu \rightarrow \partial_\mu - igA_\mu. \tag{89}$$

The introduction of the U(1) potential A_μ in this way is equivalent to the well-known minimal prescription in the received view [12], and so is equivalent to the interaction of field with matter (i.e., of A_μ with e). The shortcomings of this point of view, i.e., of the MH theory, are reviewed briefly in Sec. 2, and elsewhere [1-8,11]. One of several problems with the received view is that A_μ itself is not gauge invariant (i.e., assumed not to be a physical quantity), yet is used in the minimal prescription to represent the physical electromagnetic field. The same type of problem appears in the U(1) phase through the introduction of the random factor α (Sec. 2). In consequence there has been a long, confusing, and misleading debate within U(1) electrodynamics as to whether or not A_μ is physical [13]. This type of debate has also bedeviled the understanding of the AB effect.

In the Evans unified field theory the debate is resolved using general relativity by recognising that A^a_μ is a tetrad and therefore an element of a metric tensor in a spinning spacetime. The gauge invariance of the Evans theory is evident through the fact that it always uses covariant derivatives. In the language of differential geometry, this is the covariant exterior derivative $D\wedge$ [9], without which the geometry of spacetime is incorrectly defined. A magnetic field for example is always (as we have seen) the covariant derivative of a potential form, which is a vector valued one form. The correctly and generally covariant magnetic field is therefore a vector valued two-form. The latter is the torsion form of spacetime within a C negative scalar. This concept is missing both from the Einstein theory of gravitation and the MH theory of electrodynamics. It is the concept needed for a unified field theory of gravitation and electromagnetism in terms of general relativity and differential geometry.

In this paper we have developed the unified field theory into a novel phase law (32) that is gauge invariant and generally covariant. The phase law gives the first correct description of physical optics, interferometry, and related effects such as the Sagnac and AB effects. If we use many loops of the Sagnac effect (11), we obtain the Tomita-Chao effect; and applying the phase law (32) to matter waves, we obtain the Berry phase effects. This will be the subject of future communications.

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APPENDIX: THE SAGNAC EFFECT AS A CHANGE IN TETRAD

As argued in the text, the Sagnac effect is a change in frequency caused by rotating the platform of the interferometer:

$$\omega \rightarrow \omega \pm \Omega. \tag{A1}$$

It is a shift in wave number:

$$\kappa \rightarrow \kappa \pm \Omega / c \tag{A2}$$

and thus in the following component of the potential:

$$A_Z^{(3)} \rightarrow A_Z^{(3)} \pm (\Omega/\omega) A_Z^{(3)}. \tag{A3}$$

In the unified field theory,

$$A_\mu^a = A^{(0)} q_\mu^a, \tag{A4}$$

and in the Sagnac effect is a shift in a tetrad component:

$$q_Z^{(3)} \rightarrow (1 \pm \Omega/\omega) q_Z^{(3)} \tag{A5}$$

brought about by rotating the platform. The Sagnac effect is therefore one of general relativity applied to physical optics, and so cannot be explained by MH field theory, which is a theory of special relativity and so metric invariant [13]. Recall that the tetrad is the equivalent of the metric matrix in differential geometry. The components of the tetrad in the Evans unified field theory are clearly not those of special relativity.

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