7

A Review of Einstein Cartan Evans (ECE) Field Theory

by

Myron W. Evans,
Alpha Institute for Advanced Study, Civil List Scientist.
(emyrone@aol.com and www.aias.us)

Abstract

The development of ECE theory from Spring 2003 to present is reviewed in major themes, which include: geometrical principles, field and wave equations, phase theory and experimental effects, the unified laws of classical dynamics and electrodynamics, spin connection resonance and applications to new energy, experiments to detect the effects of gravitation in optics and electrodynamics, the theory of radiative corrections, the development of the fundamentals of general relativity, and technical appendices and equation flow charts.

Keywords: Review of ECE theory, major themes.

7.1 Introduction

The well accepted Einstein Cartan Evans (ECE) field theory [1–12] is reviewed in major themes of development from Spring 2003 to present in approximately 103 papers and volumes summarized on www.aias.us and www.atomicprecision.com. Recently a third website, www.telesio-galilei.com, has been associated with these two main websites of the theory. Additionally, these websites contain educational articles by members of the Alpha Institute for Advanced Study (AIAS) and the Telesio-Galilei Association, and also contain an Omnia Opera listing most of the collected works of the present author, including precursor theories to ECE theory from 1992 to present. Most original papers are available by hyperlink for scholarly study. It is seen in detail from the feedback activity sites of the three main sites that ECE theory is fully accepted. All the 103 papers to date are read by someone, somewhere every month, and detailed summaries of the feedback are available on www.aias.us. Additionally ECE theory has been published in the traditional manner: in four journals with anonymous reviewers, (three of them standard model journals), and is constantly internally refereed by AIAS staff. The latter are like minded professionals who have worked voluntarily on ECE theory and in the development of AIAS. Computer algebra (Maxima program) has been developed to check hand calculations of ECE theory and to perform calculations that are too complicated to carry out by hand. Therefore a review of the main themes of development and main discoveries of ECE theory is timely.

The ECE theory is a suggestion for the development of a generally covariant unified field theory based on the principles of general relativity, essentially that natural philosophy is geometry. This principle has been proposed since ancient times in many ways, but its most well known manifestation is probably the work of Albert Einstein from about 1906 to 1915, culminating in the proposal of the well known Einstein Hilbert (EH) field equation of gravitation. This work by Einstein and contemporaries is very well known, but a brief summary is given here. After several false starts Einstein proposed in 1915 that the so called "second Bianchi identity" of Riemann geometry be proportional to a form of the Noether Theorem in which the covariant derivative vanishes of the canonical energy-momentum tensor. It is much less well known that in so doing, Einstein used the only type of geometry then available to him: Riemann geometry without torsion. The EH field equation follows from this proposal by Einstein as a special case:

$$G_{\mu\nu} = kT_{\mu\nu} \tag{7.1}$$

where $G_{\mu\nu}$ is the Einstein tensor, k is the Einstein constant, and $T_{\mu\nu}$ is the canonical energy - momentum tensor. Eq. (7.1) is a special case of the Einstein proposal of 1915:

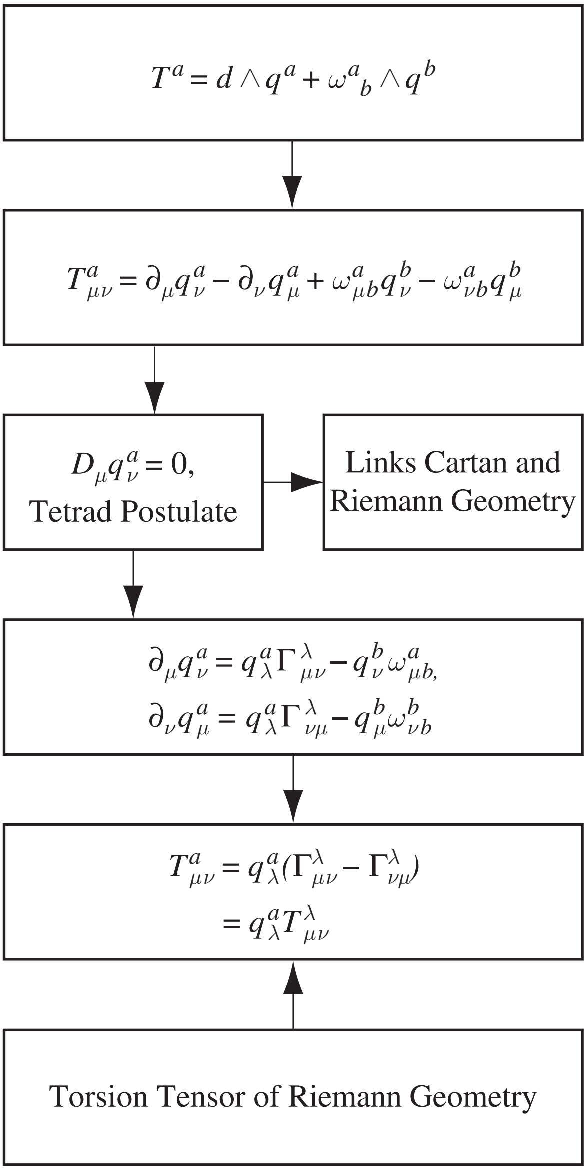
$$D^{\mu}G_{\mu\nu} = kD^{\mu}T_{\mu\nu} = 0 \tag{7.2}$$

where on the left hand side appears geometry, and on the right hand side appears natural philosophy. David Hilbert proposed the same equation at about the same time using Lagrangian principles, but Hilbert's work was motivated by Einstein's ideas, so the EH equation is usually attributed to Einstein. The EH equation applies however only to gravitation, whereas ECE has unified general relativity with the other fields of nature besides gravitation. The other fundamental fields are thought to be the electromagnetic, weak and strong fields. ECE has also unified general relativity with quantum mechanics by discarding the acausality and subjectivity of the Copenhagen School, and by deriving objective and causal wave equations from geometrical first principles. The two major and well accepted achievements of ECE theory are therefore the unification of fields using geometry, and the unification of relativity and quantum mechanics. This review is organized in sections outlining the main themes and discoveries of ECE theory, and into detailed technical appendices dealing with basics. These appendices include flow charts of the inter-relation of the main equations.

In Section 7.2 the geometrical first principles of ECE theory are summarized briefly, the theory is based on a form of geometry developed [13] by Cartan and first published in 1922. This geometry is fully self-consistent and well known - it can be regarded as the standard differential geometry taught in good universities. The dialogue between Einstein and Cartan on this geometry is perhaps not as well known as the dialogue between Einstein and Bohr, but is the basis for the development of ECE theory. It is named “Einstein Cartan Evans” field theory because the present author set out to suggest a completion of the Einstein Cartan dialogue. This dialogue was part of the attempt by Einstein and many others to complete general relativity by developing a generally covariant unified field theory on the principles of a given geometry. For many reasons this unification did not come about until Spring of 2003, when ECE theory was proposed. The main obstacles to unification were adherence in the standard model to a U(1) sector for electromagnetism, the neglect of the ECE spin field B(3), inferred in 1992, and adherence to the philosophy of the Copenhagen School. Standard model proponents adhere to these principles at the time of writing, but ECE proponents now adopt a different natural philosophy, since it may be claimed objectively from feedback data that ECE is a new school of thought.

In Section 7.3 the main field and wave equations of ECE are discussed in summary. They are derived from the well known principles of Cartan’s geometry. The gravitational, electromagnetic, weak and strong fields are unified by Cartan’s geometry, each is an aspect of the same geometry. The field equations are based on the one true Bianchi identity given by Cartan, using different representation spaces. The wave equations are derived from the tetrad postulate, the very fundamental requirement in natural philosophy and relativity theory that the complete vector field be invariant under the general transformation of coordinates. To translate Cartan to Riemann geometry requires use of the tetrad postulate. Therefore both the Bianchi identity and tetrad postulate are fundamentals of standard differential geometry and their use in ECE theory is entirely standard mathematics [13].

In Section 7.4 the unification of phase theory made possible by ECE is summarized in terms of the main discoveries and points of development. The main point of development in this context is the unification of apparently disparate phases such as the electromagnetic phase, the Dirac and Wu Yang phases, and the topological phases. ECE theory presents a unified geometrical approach to each phase, and this approach also gives a straightforward geometrical explanation of the Aharonov Bohm effects and “non-locality”. The



Flowchart 7.1. First Cartan Structure Equation.

electromagnetic phase for example is developed in terms of the B(3) spin field [14] and some glaring shortcomings of the standard model are corrected. Thus, apparently simple and well known effects such as reflection are developed self-consistently with ECE, while in the standard model they are at

$$D \wedge T := R \wedge q \neq 0$$

$$\begin{aligned}
&R^{\lambda}_{\rho\mu\nu} + R^{\lambda}_{\mu\nu\rho} + R^{\lambda}_{\nu\rho\mu} \\
&:= \partial_{\mu}\Gamma^{\lambda}_{\nu\rho} - \partial_{\nu}\Gamma^{\lambda}_{\mu\rho} + \Gamma^{\lambda}_{\mu\sigma}\Gamma^{\sigma}_{\nu\rho} - \Gamma^{\lambda}_{\nu\sigma}\Gamma^{\sigma}_{\mu\rho} \\
&+ \partial_{\nu}\Gamma^{\lambda}_{\rho\mu} - \partial_{\rho}\Gamma^{\lambda}_{\nu\mu} + \Gamma^{\lambda}_{\nu\sigma}\Gamma^{\sigma}_{\rho\mu} - \Gamma^{\lambda}_{\rho\sigma}\Gamma^{\sigma}_{\nu\mu} \\
&+ \partial_{\rho}\Gamma^{\lambda}_{\mu\nu} - \partial_{\mu}\Gamma^{\lambda}_{\rho\nu} + \Gamma^{\lambda}_{\rho\sigma}\Gamma^{\sigma}_{\mu\nu} - \Gamma^{\lambda}_{\mu\sigma}\Gamma^{\sigma}_{\rho\nu} \\
&\neq 0
\end{aligned}$$

$$T^{\lambda}_{\mu\nu} = \Gamma^{\lambda}_{\mu\nu} - \Gamma^{\lambda}_{\nu\mu}$$

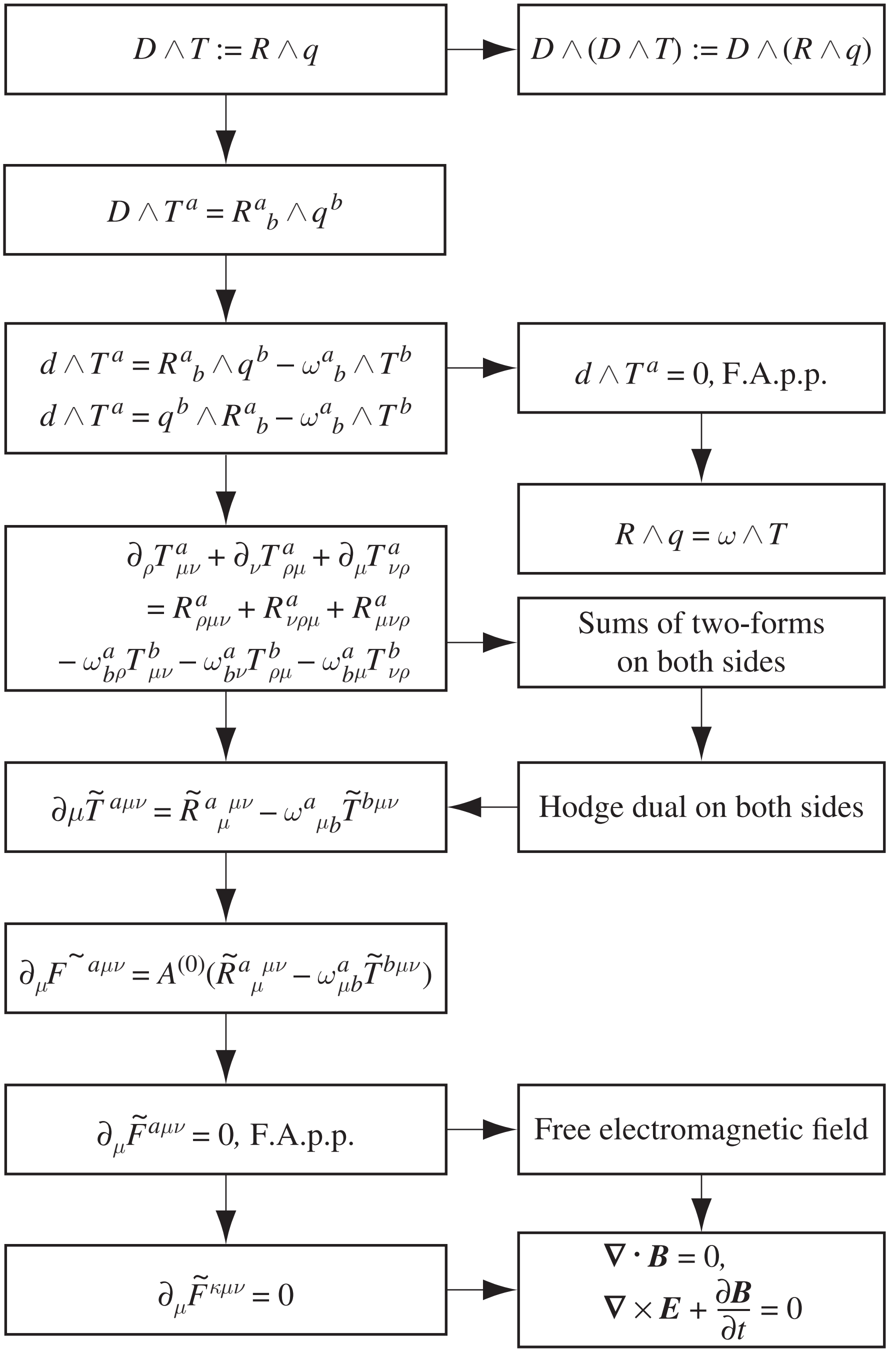
$$T^{a} = d \wedge q^{a} + \omega^{a}{}_{b} \wedge q^{b}$$

$$T = D \wedge q$$

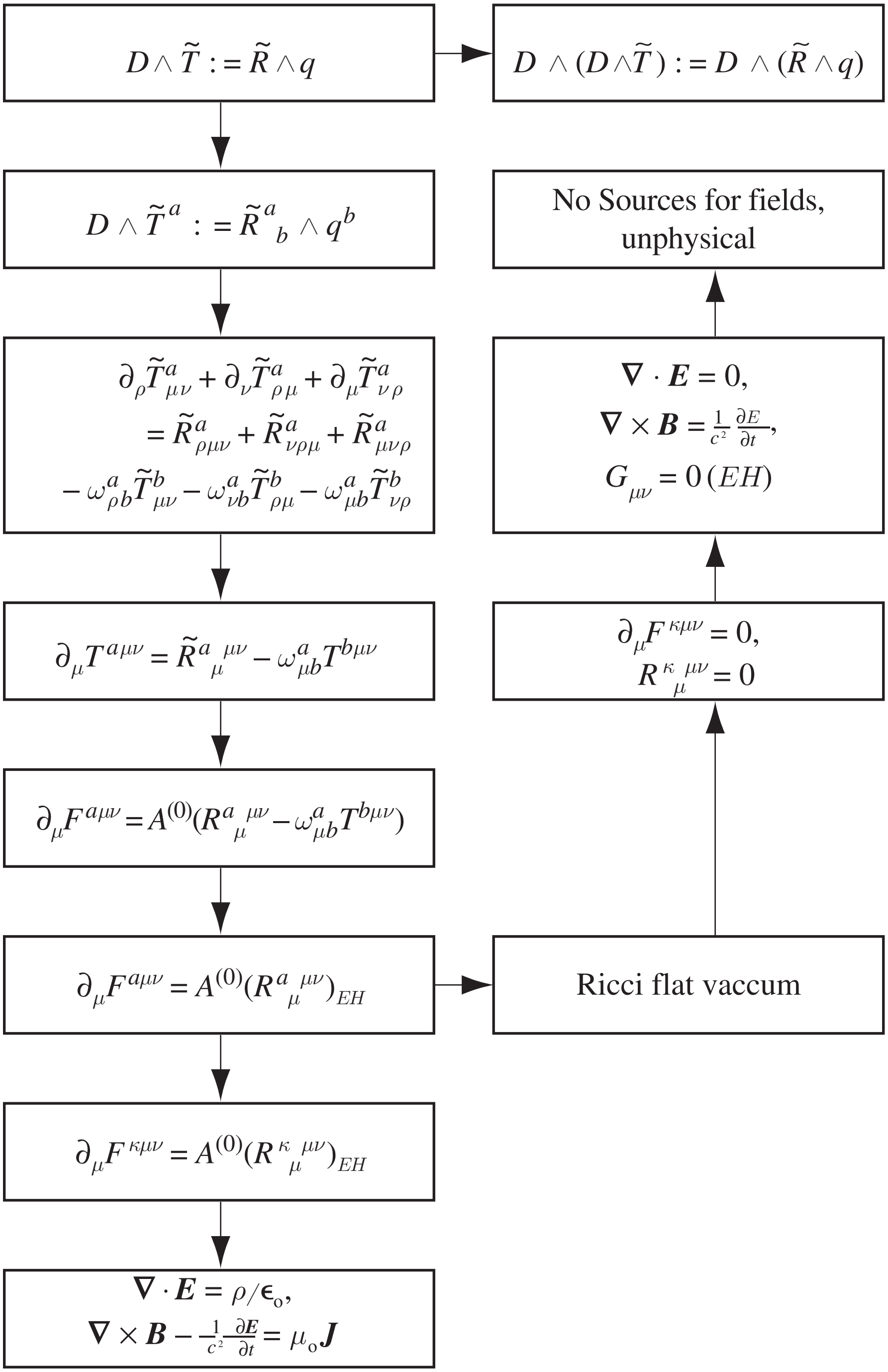
Flowchart 7.2. The Bianchi Identity.

odds with fundamental symmetry [1–12]. The standard model development of the Aharonov Bohm effects is also incorrect mathematically, obscure, controversial and convoluted, while in ECE theory it is straightforward.

In Section 7.5 the ECE laws of classical dynamics and electrodynamics are summarized in the language of vectors, the language used in electrical engineering. The equations of electrodynamics in ECE theory reduce to the four laws: Gauss law of magnetism, Faraday law of induction, Coulomb law and Ampère Maxwell law. In ECE theory they are the same in vector notation as in the familiar Maxwell Heaviside (MH) field theory, but in ECE are written in a different space-time. In ECE the electromagnetic field is the spinning of space-time, represented by the Cartan torsion, while in MH the field is a nineteenth century concept still used uncritically in the contemporary standard model of natural philosophy. The space-time of MH is the flat and static Minkowski space-time, while in ECE the space-time is dynamic with non-zero curvature and torsion. This difference manifests itself in the relation



Flowchart 7.3. Homogeneous ECE Field Equation.



Flowchart 7.4. In homogeneous ECE Field Equation.

Homogeneous

$$D_\mu \tilde{T}^{\kappa\mu\nu} = \tilde{R}^{\kappa}{}_{\mu}{}^{\mu\nu}$$

$$D_\mu \tilde{T}^{a\mu\nu} = \tilde{R}^{a}{}_{\mu}{}^{\mu\nu}$$

$$D_\rho T^a_{\mu\nu} + D_\nu T^a_{\rho\mu} + D_\mu T^a_{\nu\rho} = R^a_{\rho\mu\nu} + R^a_{\nu\rho\mu} + R^a_{\mu\nu\rho}$$

$$D \wedge T := q \wedge R$$

$$D \wedge \tilde{T} := q \wedge \tilde{R}$$

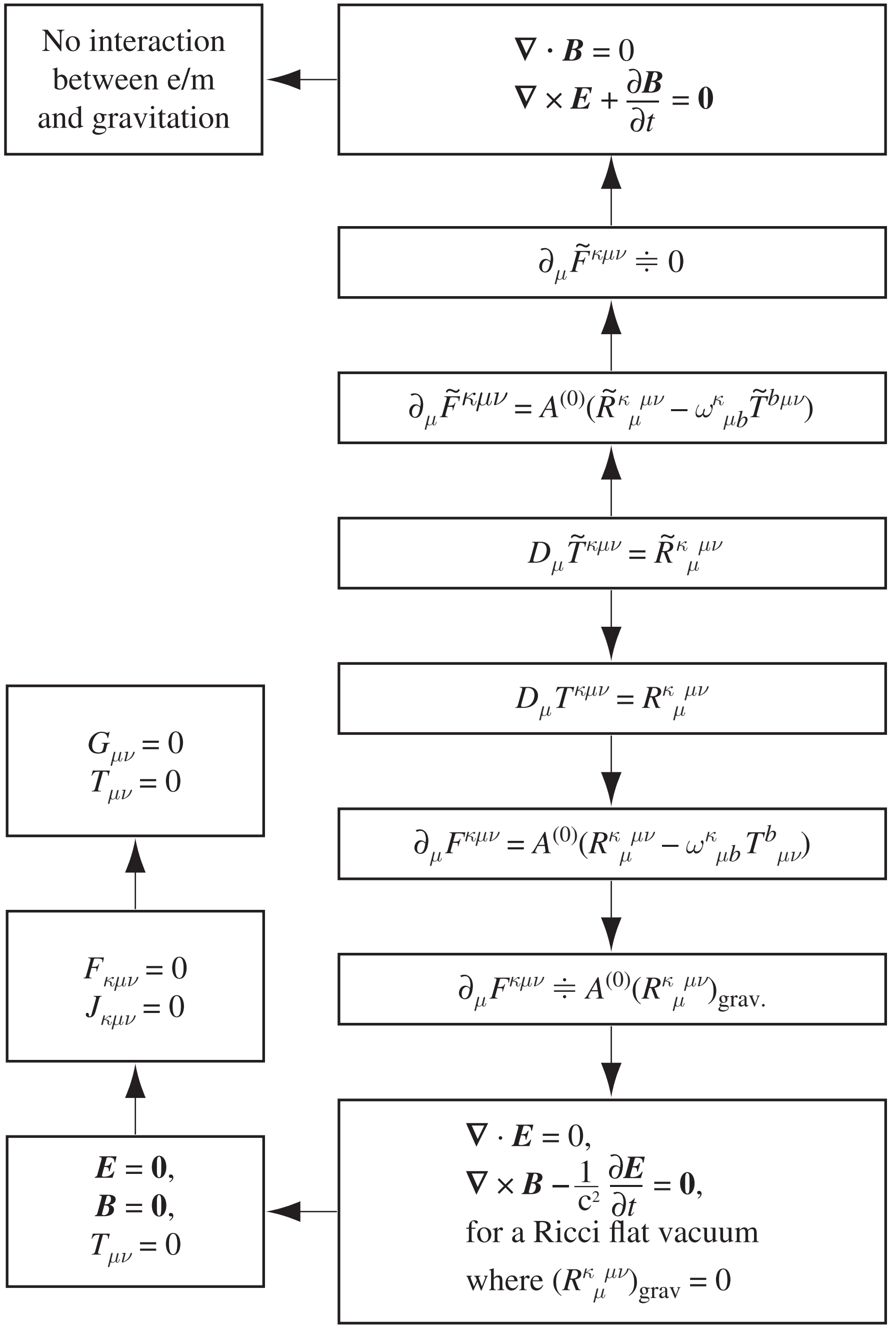
$$D_\rho \tilde{T}^a_{\mu\nu} + D_\nu \tilde{T}^a_{\rho\mu} + D_\mu \tilde{T}^a_{\nu\rho} = \tilde{R}^a_{\rho\mu\nu} + \tilde{R}^a_{\nu\rho\mu} + \tilde{R}^a_{\mu\nu\rho}$$

$$D_\mu T^{a\mu\nu} = R^{a}{}_{\mu}{}^{\mu\nu}$$

$$D_\mu T^{\kappa\mu\nu} = R^{\kappa}{}_{\mu}{}^{\mu\nu}$$

In Homogeneous

Flowchart 7.5. The Basic Field Equations.



Flowchart 7.6. Approximations to the Basic Field Equations.

$$[D_\mu, D_\nu]V^\rho = R^\rho{}_{\sigma\mu\nu}V^\sigma - T^\lambda_{\mu\nu}D_\lambda V^\rho$$

↓

$$D \wedge T := R \wedge q$$

↓

$$[D^\mu, D^\nu]_{HD}V^\rho = \tilde{R}^\rho{}_\sigma{}^{\mu\nu}V^\sigma - \tilde{T}^{\lambda\mu\nu}D_\lambda V^\rho$$

↓

$$g_{\mu\alpha}g_{\nu\beta}[D^\alpha, D^\beta]_{HD}V^\rho = g_{\mu\alpha}g_{\nu\beta}(\tilde{R}^\rho{}_\sigma{}^{\mu\nu}V^\sigma - \tilde{T}^{\lambda\mu\nu}D_\lambda V^\rho)$$

↓

$$g^{\mu\alpha}g^{\nu\beta}[D_\alpha, D_\beta]_{HD}V^\rho = g^{\mu\alpha}g^{\mu\beta}(\tilde{R}^\rho{}_{\sigma\alpha\beta}V^\sigma - \tilde{T}^\lambda_{\alpha\beta}D_\lambda V^\rho)$$

↓

$$[D_\mu, D_\nu]_{HD}V^\rho = \tilde{R}^\rho_{\sigma\mu\nu}V^\rho - \tilde{T}^\lambda_{\mu\nu}D_\lambda V^\rho$$

↓

$$D \wedge \tilde{T} := \tilde{R} \wedge q$$

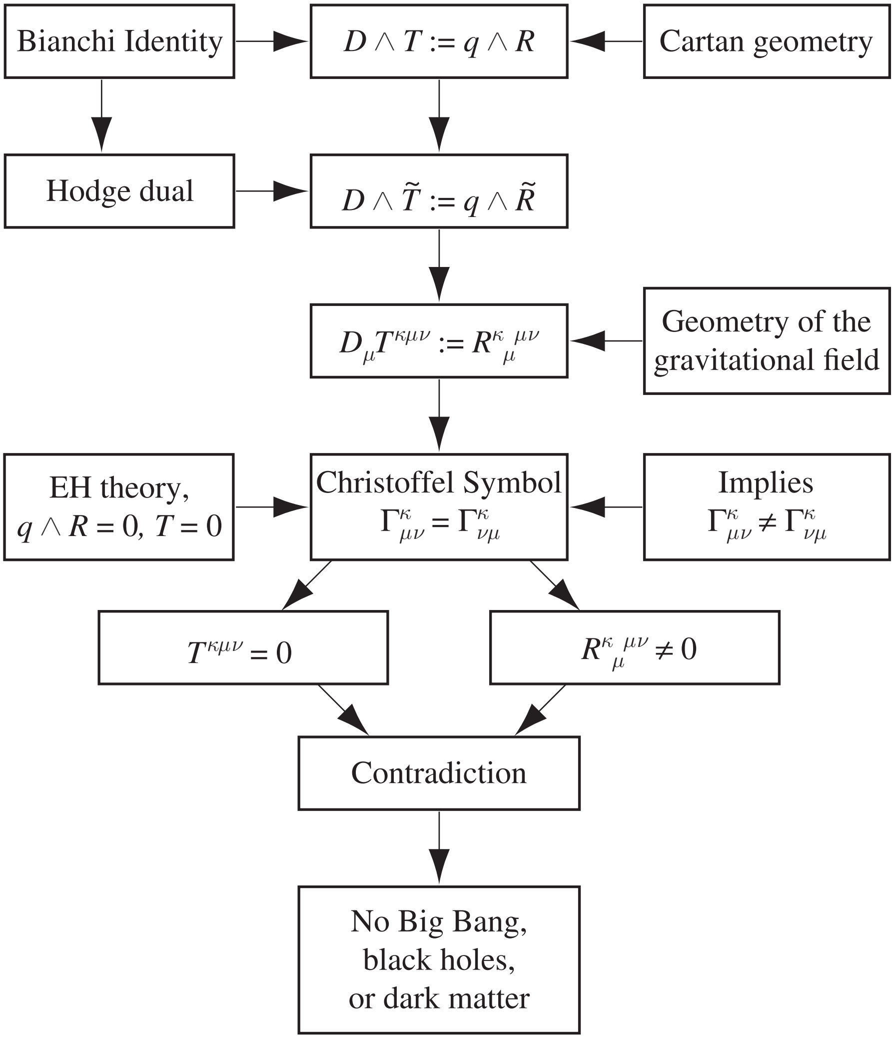
↓

$$D_\mu T^{\kappa\mu\nu} = R^\kappa{}_\mu{}^{\mu\nu}$$

↓

Einstein Hilbert equation is self-inconsistent

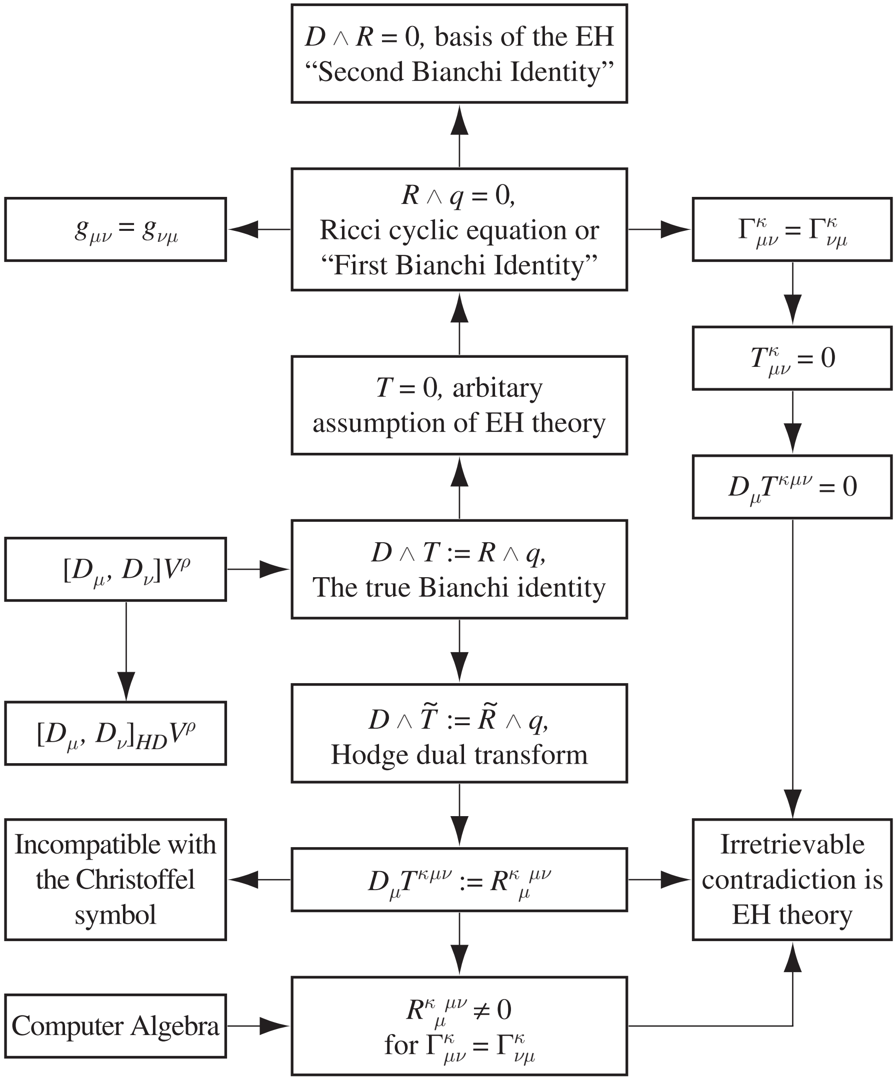
Flowchart 7.7. Hodge Dual of the Bianchi Identity.



Flowchart 7.8. Self Inconsistency of General Relativity.

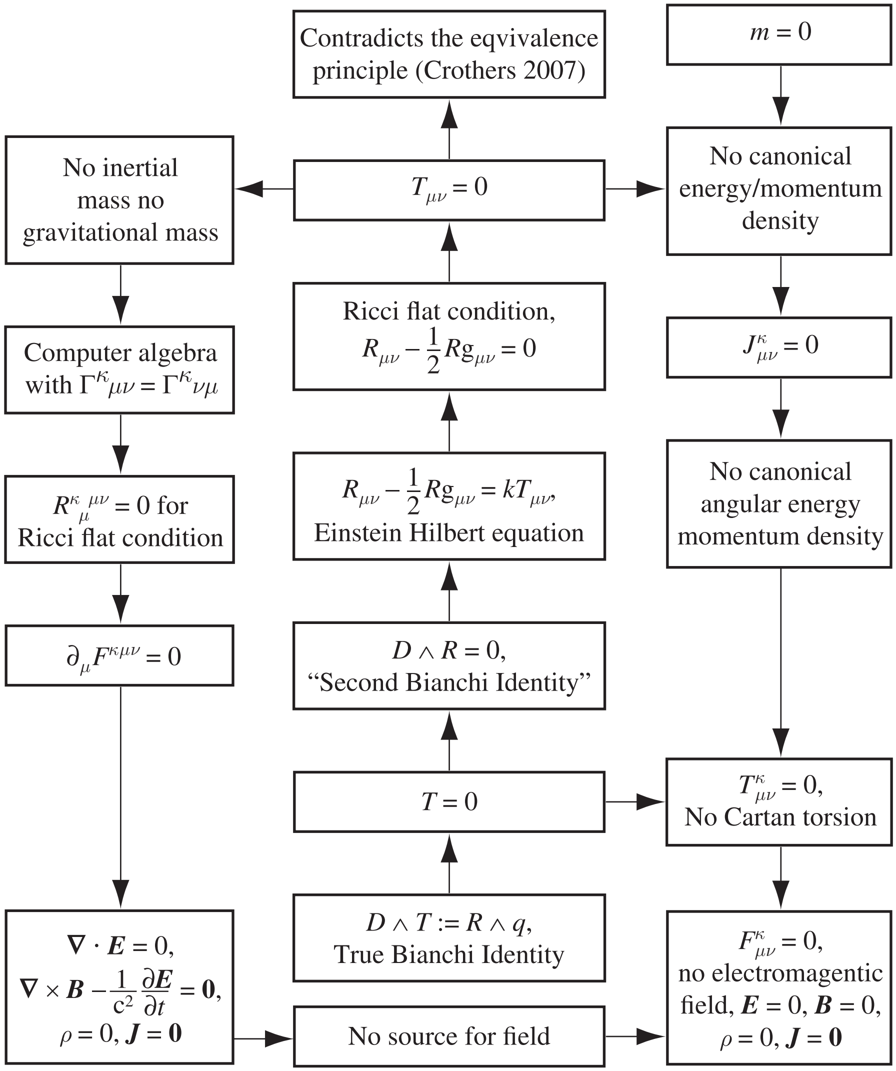
between the fields and potentials in ECE, a relation which includes the spin connection.

In Section 7.6, spin connection resonance (SCR) is discussed, concentrating as usual on the main discoveries and points of development of the ECE theory. In theory, SCR is of great practical utility because the equations of classical electrodynamics become resonance equations of the type first inferred by the Bernoulli's and Euler. Therefore a new source of electric power has been discovered in ECE theory - this source is the Cartan torsion of space-



Flowchart 7.9. IrretrievableFlaws in the Geometry of the Einstein Hilbert Field Theory.

time. Amplification occurs in principle through SCR, the spin connection itself being the property of the four-dimensional space-time with curvature and torsion which is the base manifold of ECE theory. It is well known [15] that these resonance equations are equivalent to circuits that can be used to amplify electric power. In all probability these circuits were the ones designed by Tesla empirically.



Flowchart 7.10. Irretrievable Contradiction in the Ricci Flat Condition.

In Section 7.7 the utility of ECE as a unified field theory is illustrated through the effects of gravitation in optics and spectroscopy. These are exemplified by the effect of gravitation on the ring laser gyro (Sagnac effect) and on radiatively induced fermion resonance (RFR). RFR itself is of great potential utility because it is a form of electron and proton spin resonance induced not by a permanent magnet, but by a circularly polarized electromagnetic field. This is known as the inverse Faraday effect (IFE) [16] from which the

ECE spin field B(3) was inferred in 1992 [17]. The spin field signals the fact that in a self consistent philosophy, classical electrodynamics must be part of a generally covariant field theory. This is incompatible with the U(1) sector of special relativity still used to describe electrodynamics in the standard model. Any proposal for a unified field theory based on U(1) cannot be generally covariant in all sectors, leaving ECE as the only satisfactory unified field theory at the time of writing.

In Section 7.8 the well known radiative corrections [18] are developed with ECE theory, and a summary of the main points of progress illustrated with the anomalous g factor of the electron and the Lamb shift. It is shown that claims to accuracy of standard model quantum electrodynamics (QED) are greatly exaggerated. The accuracy is limited by that of the Planck constant, the least accurately known fundamental constant appearing in the fine structure constant. There are glaring internal inconsistencies in standards laboratories tables of data on the fundamental constants, and QED is based on a number of what are effectively adjustable parameters introduced by ad hoc procedures such as dimensional renormalization The concepts used in QED are vastly complicated and are not used in the ECE theory of the experimentally known radiative corrections. The Feynman perturbation method is not used in ECE: it cannot be proven to converge as is well known, i.e. needs many terms of increasing complexity which must be evaluated by computer. So ECE is a fundamental theory of quantized electrodynamics from the first principles of general relativity, while QED is a theory of special relativity needing adjustable parameters, acausal and subjective concepts, and therefore of dubious validity.

In Section 7.9, finally, it is shown that EH theory has several fundamental shortcomings. As described on ww.telesio-galilei.com EH has been quite severely criticized down the years by several leading physicists. Notably, Crothers [19] has criticized the methods of solution of EH, and has shown that uncritically accepted concepts are in fact incompatible with general relativity. These include Big Bang, dark hole and dark matter theory and the concept of a Ricci flat space-time. He has also shown that the use of the familiar but mis-named “Schwarzschild metric” is due to lack of scholarship and understanding of Schwarzschild’s original papers of 1916. ECE has revealed that the use of the familiar Christoffel symbol is incompatible with the one true Bianchi identity of Cartan. This section suggests a development of the EH equation into one which is self consistent.

Several technical appendices give basic details which are not usually given in standard textbooks, but which are nevertheless important to the student. These appendices also contain flow charts inter-relating the main concepts and equations of ECE.

7.2 Geometrical Principles

The ECE theory is based on the two structure equations of Cartan, and the Bianchi identity of Cartan geometry. During the course of development of the theory a useful short-hand notation has been used in which the indices are removed in order to reveal the basic structure of the equations. In this notation the two Cartan structure equations are:

$$T = D \wedge q = d \wedge q + \omega \wedge q \tag{7.3}$$

and

$$R = D \wedge \omega = d \wedge \omega + \omega \wedge \omega \tag{7.4}$$

and the Bianchi identity is:

$$D \wedge T = d \wedge T + \omega \wedge T := R \wedge q. \tag{7.5}$$

In this notation T is the Cartan torsion form, ω is the spin connection symbol, q is the Cartan tetrad form, and R is the Cartan curvature form. The meaning of this symbolism is defined in all detail in the ECE literature [1–12], and the differential form is defined in the standard literature [13]. The purpose of this section is to summarize the main advances in basic geometry made during the development of ECE theory.

The Bianchi identity (7.5) is basic to the field equations of ECE, and its structure has been developed considerably [1–12]. It has been shown to be equivalent to the tensor equation:

$$\begin{aligned} &R^{\lambda}_{\rho\mu\nu} + R^{\lambda}_{\mu\nu\rho} + R^{\lambda}_{\nu\rho\mu} \\ &\quad := \partial_\nu \Gamma^{\lambda}_{\rho\mu} - \partial_\rho \Gamma^{\lambda}_{\nu\mu} + \Gamma^{\lambda}_{\nu\sigma}\Gamma^{\sigma}_{\rho\mu} - \Gamma^{\lambda}_{\rho\sigma}\Gamma^{\sigma}_{\nu\mu} \\ &\qquad + \partial_\rho \Gamma^{\lambda}_{\mu\nu} - \partial_\mu \Gamma^{\lambda}_{\rho\nu} + \Gamma^{\lambda}_{\rho\sigma}\Gamma^{\sigma}_{\mu\nu} - \Gamma^{\lambda}_{\mu\sigma}\Gamma^{\sigma}_{\rho\nu} \\ &\qquad + \partial_\mu \Gamma^{\lambda}_{\nu\rho} - \partial_\nu \Gamma^{\lambda}_{\mu\rho} + \Gamma^{\lambda}_{\mu\sigma}\Gamma^{\sigma}_{\nu\rho} - \Gamma^{\lambda}_{\nu\sigma}\Gamma^{\sigma}_{\mu\rho} \end{aligned} \tag{7.6}$$

in which a cyclic sum of three Riemann tensors is identically equal to the sum of three fundamental definitions of the same Riemann tensors. These fundamental definitions originate in the commutator of covariant derivatives acting on a four-vector in the base manifold. The latter is four dimensional space-time with BOTH curvature and torsion [1–13]. This operation produces:

$$[D_\mu, D_\nu] V^\rho = R^{\rho}{}_{\sigma\mu\nu} V^\sigma - T^{\lambda}_{\mu\nu} D_\lambda V^\rho \tag{7.7}$$

where the torsion tensor is:

$$T^{\lambda}_{\mu\nu} = \Gamma^{\lambda}_{\mu\nu} - \Gamma^{\lambda}_{\nu\mu}. \tag{7.8}$$

The curvature or Riemann tensor cannot exist without the torsion tensor, and the definition (7.7) has been shown to be equivalent to the Bianchi identity (7.6).

The second advance in basic geometry is the inference [1–12] of the Hodge dual of the Bianchi identity. In short-hand notation this is:

$$D \wedge \widetilde{T} := \widetilde{R} \wedge q \tag{7.9}$$

and is equivalent to:

$$[D_\mu, D_\nu]_{HD} V^\rho = \widetilde{R}^{\rho}{}_{\sigma\mu\nu} V^\sigma - \widetilde{T}^{\lambda}_{\mu\nu} D_\lambda V^\rho \tag{7.10}$$

where the subscript *HD* denotes Hodge dual. From these considerations it may be inferred that the Bianchi identity and its Hodge dual are the tensor equations:

$$D_\mu \widetilde{T}^{\kappa\mu\nu} = \widetilde{R}^{\kappa}{}_{\mu}{}^{\mu\nu} \tag{7.11}$$

and

$$D_\mu T^{\kappa\mu\nu} = R^{\kappa}{}_{\mu}{}^{\mu\nu} \tag{7.12}$$

in which the connection is NOT the Christoffel connection. Computer algebra [1–12] has shown that the tensor $R^{\kappa}{}_{\mu}{}^{\mu\nu}$ is not zero in general for line elements that use the Chrstoffel symbol, while $T^{\kappa\mu\nu}$ is always zero for the Christoffel symbol. So the use of the latter is inconsistent with the tensor equation (7.12). Therefore the neglect of torsion makes EH theory internally inconsistent, so standard model general relativity and cosmology are also internally inconsistent at a basic level. In short-hand notation the geometry used in EH is:

$$R \wedge q = 0 \tag{7.13}$$

which in tensor notation is known as "the first Bianchi identity":

$$R^{\kappa}{}_{\mu\nu\rho} + R^{\kappa}{}_{\rho\mu\nu} + R^{\kappa}{}_{\nu\rho\mu} = 0 \tag{7.14}$$

in the standard model literature. However, this is not an identity, because it conflicts with equation (7.5), and is true if and only if the Christoffel symbol and symmetric metric are used [1–13]. Eq. (7.14) was actually inferred by Ricci and Levi-Civita, not by Bianchi. So it is referred to in the ECE literature as the Ricci cyclic equation.

In the course of development of ECE theory a similar problem was found with what is referred to in the standard model literature as "the second Bianchi identity". In shorthand notation this is given [13] as:

$$D \wedge R = 0 \tag{7.15}$$

but again this neglects torsion. In tensor notation Eq. (7.15) is:

$$D_\rho R^\kappa{}_{\sigma\mu\rho} + D_\mu R^\kappa{}_{\sigma\nu\rho} + D_\nu R^\kappa{}_{\sigma\rho\mu} = 0. \tag{7.16}$$

It has been shown [1–12] that Eq. (7.15) should be:

$$D \wedge (D \wedge T) := D \wedge (R \wedge q) \tag{7.17}$$

which is found by taking $D\wedge$ on both sides of Eq. (7.15). Eq. (7.17) has been given in tensor notation [1–12], and reduces to Eq. (7.16) when:

$$T^\lambda_{\mu\nu} = 0. \tag{7.18}$$

However, Eq. (7.18) is inconsistent with the fundamental operation of the commutator of covariant derivatives on the four vector, Eq. (7.7). So in the ECE literature the torsion is always considered self-consistently. From the fundamentals [13] of Eq. (7.7) there is no a priori reason for neglecting torsion, and in fact the torsion tensor is always non-zero if the curvature tensor is non-zero. This fact precludes the use of the Christoffel symbol, making EH theory self-inconsistent.

These are the main geometrical advances made during the course of the development of ECE theory, which is the only self-consistent theory of general relativity. It has also been pointed out by Crothers [19] that methods of solution of the EH equation are geometrically incorrect, and must be discarded. It is thought that these errors have been repeated uncritically for ninety years because few have the necessary technical ability to understand the geometry of general relativity in sufficient depth, and that the prestige of Einstein has precluded or inhibited due criticism.

7.3 The Field and Wave Equations of ECE Theory

The wave equation of ECE was the first to be developed historically [1–12], and methods of derivation of the wave equation were subsequently simplified and clarified. The field equations were subsequently developed from the

Bianchi identity discussed in Section 7.2. This section summarizes the main equations and methods of derivation. More detail of the equations is given in technical appendices. The field equations are relevant to classical gravitation and electrodynamics, and the wave equation to causal and objective quantum mechanics. Full details of derivations are available in the literature [1–12], the aim of this section is to summarize the main inferences of ECE theory to date.

The Bianchi identity (7.5) and its Hodge dual (7.9) become the homogeneous and inhomogeneous field equations of ECE respectively. These field equations apply to the four fundamental fields of force: gravitational, electromagnetic, weak and strong and can be used to describe the interaction of the fundamental fields on the classical level. For example the electromagnetic field is described by making the fundamental hypothesis:

$$A = A^{(0)} q \tag{7.19}$$

where the shorthand (index-less) notation has been used. Here A represents the electromagnetic potential form and $cA^{(0)}$ is a primordial quantity with the units of volts, a quantity which is proportional to the charge, −e, on the electron. The hypothesis (7.19) implies that:

$$F = A^{(0)} T \tag{7.20}$$

where F is shorthand notation for the electromagnetic field form. The homogeneous ECE field equation of electrodynamics follows from the Bianchi identity (7.5):

$$d \wedge F + \omega \wedge F = A^{(0)} R \wedge q \tag{7.21}$$

and the inhomogeneous ECE field equation follows from the Hodge dual (7.9) of the Bianchi identity:

$$d \wedge \widetilde{F} + \omega \wedge \widetilde{F} = A^{(0)} \widetilde{R} \wedge q. \tag{7.22}$$

Therefore the ECE field equations are duality invariant, a basic symmetry which means that they transform into each other by means of the Hodge dual [1–12]. The Maxwell Heaviside (MH) field equations of the standard model do not have this fundamental symmetry and in differential form notation the MH equations are:

$$d \wedge F = 0 \tag{7.23}$$

and

$$d \wedge \widetilde{F} = \widetilde{J}/\epsilon_0 \tag{7.24}$$

where $\widetilde{J}$ denotes the inhomogeneous charge/current density and ϵ_0 is the S. I. vacuum permittivity. Duality symmetry is broken by the fact that there is no homogeneous charge current density (J) in MH theory (the right hand side of Eq. (7.23) is zero). The absence of J in the standard model is made the basis for gauge theory as is well known, and also made the basis for the absence of a magnetic monopole.

The ECE field equations (7.21) and (7.22) are re-arranged as follows in order to define the homogeneous (J) and inhomogeneous ($\widetilde{J}$) charge current densities of ECE theory:

$$d \wedge F = J/\epsilon_0 = A^{(0)}(R \wedge q - \omega \wedge T) \tag{7.25}$$

and

$$d \wedge \widetilde{F} = \widetilde{J}/\epsilon_0 = A^{(0)}(\widetilde{R} \wedge q - \omega \wedge \widetilde{T}). \tag{7.26}$$

Both equations are generally covariant because they originate in the Bianchi identity. The interaction of electromagnetism with gravitation occurs whenever J is non-zero. In MH theory such an interaction cannot be described, because MH theory is developed in Minkowski space-time. The latter has no curvature and in general relativity cannot describe gravitation at all. For all practical purposes in the laboratory there is no interaction of electromagnetism and gravitation, so Eq. (7.25) reduces to:

$$d \wedge F = 0. \tag{7.27}$$

Therefore ECE theory explains in this way why there is no magnetic monopole observable in the laboratory. The standard model has no physical explanation for this, and indeed asserts that gauge theory is mathematical in nature. ECE theory does not use gauge theory, and adopts Faraday's original point of view that the potential A is a physically effective entity. There are therefore important philosophical differences between ECE and the standard model of classical electrodynamics, in which the potential is mathematical in nature.

Therefore the structure of the ECE field equations is a simple one based directly on the Bianchi identity. The structure is seen the most clearly using the shorthand notation of Eqs. (7.25) and (7.26) where all indices are omitted. The notation of classical electrodynamics varies from subject to subject. In advanced field theory the elegant but concise differential form notation is used, and also the tensor notation. In electrical engineering the vector notation is used. In ECE theory all three notations have been developed [1–12] in all detail, and the ECE field equations developed into a vector form that is identical to the MH equations. The main differences between ECE and MH is firstly that the former is written in a four dimensional space-time with curvature and torsion both present. This is a dynamic space-time whose connection must be more general than the Christoffel connection. The MH equations, although having the same vector form as ECE, are written in the Minkowski space-time of special relativity. This is often referred to as "flat space-time", whose metric is time and space independent. Secondly the relation between the field and potential in ECE includes the connection, whereas in MH the connection is not present. The inclusion of the connection has the all important effect of making the equations of classical electrodynamics resonance equations of the Bernoulli/Euler type. This property means that it is possible to describe well known phenomena such as those first observed by Tesla, and to produce circuits that take electric power from a new source, the Cartan torsion.

The concise tensorial expression of the equations (7.25) and (7.26) is in general [1–12]

$$D_\mu \widetilde{F}^{a\mu\nu} = A^{(0)} \widetilde{R}^{a}{}_{\mu}{}^{\mu\nu} \tag{7.28}$$

and

$$D_\mu F^{a\mu\nu} = A^{(0)} R^{a}{}_{\mu}{}^{\mu\nu} \tag{7.29}$$

where the covariant derivative appears on one side and a Ricci type curvature tensor on the other. It has been shown [1–12] that these reduce in the laboratory, and for all practical purposes, to:

$$\partial_\mu \widetilde{F}^{a\mu\nu} = 0 \tag{7.30}$$

and

$$\partial_\mu F^{a\mu\nu} = A^{(0)} R^{a}{}_{\mu}{}^{\mu\nu}. \tag{7.31}$$

The index a in these equations comes from the well known [13] tangent space-time of Cartan geometry. However, it has been shown [1–12] that Eqs. (7.30) and (7.31) can be written in the base manifold as a special case of Eqs. (7.28) and (7.29), whereupon we arrive at:

$$\partial_\mu \widetilde{F}^{\kappa\mu\nu} = 0 \tag{7.32}$$

and

$$\partial_\mu F^{\kappa\mu\nu} = A^{(0)} R^{\kappa}{}_{\mu}{}^{\mu\nu}. \tag{7.33}$$

Therefore the electromagnetic field tensor in general relativity (ECE theory) develops into a three index tensor. In special relativity (MH theory) it is a two-index tensor as is well known. The equivalents of (7.32) and (7.33) in MH theory are the tensor equations:

$$\partial_\mu \widetilde{F}^{\mu\nu} = 0 \tag{7.34}$$

and

$$\partial_\mu F^{\mu\nu} = J^\nu / \epsilon_0. \tag{7.35}$$

The meaning of the three-index field tensor has been developed [1–12] in detail. It originates in the well known [18] three index angular energy/ momentum tensor density, $J^{\kappa\mu\nu}$ which is proportional to the three index Cartan torsion tensor. It is well known that the electromagnetic field carries angular momentum which in the Beth effect [20] is experimentally observable Therefore the Cartan torsion tensor is the expression of this well known angular energy/momentum density tensor of Minkowski space-time [18] in a more general manifold with curvature and torsion. The meaning of the vector form of the ECE field equations is further developed in Section 7.5.

The classical field equations of gravitation in ECE are also based directly on the Bianchi identity and its Hodge dual. The EH equation, as argued already, is incompatible with the Bianchi identity in its rigorously correct form, Eq. (7.5), so during the course of development of ECE theory the well known EH equation has been developed with the proportionalities:

$$T^{\kappa\mu\nu} = kJ^{\kappa\mu\nu} \tag{7.36}$$

and

$$R^{\kappa}{}_{\mu}{}^{\mu\nu} = kT^{\kappa}{}_{\mu}{}^{\mu\nu} \tag{7.37}$$

which give:

$$D_\mu J^{\kappa\mu\nu} = T^{\kappa}{}_{\mu}{}^{\mu\nu}. \tag{7.38}$$

This novel field equation of classical gravitation is based directly on the tensorial formulations (7.11) and (7.12) of the Bianchi identity. The Newton inverse square law for example has been derived straightforwardly from Eq. (7.38) in the limit where the connection goes to zero:

$$\partial_\mu J^{\kappa\mu\nu} \doteqdot T^{\kappa}{}_{\mu}{}^{\mu\nu} \tag{7.39}$$

whereupon we obtain:

$$\nabla \cdot \boldsymbol{g} = kc^2 \rho_m \tag{7.40}$$

an equation which is equivalent to the Newton inverse square law. Here $\boldsymbol{g}$ is the acceleration due to gravity, k is Einstein's constant, ρ_m and is the mass density in kilograms per cubic meter. Similarly the Coulomb inverse square law can be obtained straightforwardly [1–12] by considering the same type of limit of the inhomogeneous ECE field equation:

$$D_\mu F^{\kappa\mu\nu} = A^{(0)} R^{\kappa}{}_{\mu}{}^{\mu\nu}. \tag{7.41}$$

The appropriate limit in this case is:

$$\partial_\mu F^{\kappa\mu\nu} \doteqdot A^{(0)} R^{\kappa}{}_{\mu}{}^{\mu\nu} \tag{7.42}$$

and leads to the Coulomb inverse square law:

$$\nabla \cdot \boldsymbol{E} = \rho_e / \epsilon_0 \tag{7.43}$$

where ρ_e is the charge density in coulombs per cubic meter. These procedures illustrate one aspect of the unified nature of ECE, because both laws are obtained from the Bianchi identity. Many other examples of the unification properties of ECE have been discussed [1–12].

In order to unify the electromagnetic and weak fields in a field equation, the representation space is chosen to be SU(2) instead of O(3) and the parity violating nature of the weak field carefully considered. Similarly the electromagnetic and strong fields are unified with an SU(3) representation space, and we have already discussed the unification of the electromagnetic and gravitational fields. Any permutation or combination of fields may be unified, and several examples have been given [1–12] in various contexts. These are discussed further in Section 7.7.

The ECE wave equation was developed [1–12] from the tetrad postulate [13]:

$$D_\mu q^a_\nu = 0 \tag{7.44}$$

via the identity:

$$D^\mu (D_\mu q^a_\nu) := 0. \tag{7.45}$$

This was re-expressed as the ECE Lemma:

$$\Box q^a_\lambda = R q^a_\lambda \tag{7.46}$$

in which appears the scalar curvature:

$$R = q_a^\lambda \partial^\mu (\Gamma^\nu_{\mu\lambda} q^a_\nu - \omega^a_{\mu b} q^b_\lambda). \tag{7.47}$$

Here tensor notation is used, $\omega^a_{\mu b}$ being the spin connection and $\Gamma^\nu_{\mu\lambda}$ the general connection. The Lemma becomes the ECE wave equation using a generalization to all fields of the Einstein gravitational equation [1–13]:

$$R = -kT. \tag{7.48}$$

Here T is an index contracted energy momentum tensor. The main wave equations of physics were all obtained [1–12] as limits of Eq. (7.46), notably the Proca and Dirac wave equations. In so doing however the causal realist philosophy of Einstein and de Broglie was adhered to. This is the original philosophy of wave mechanics. The Schrödinger and Heisenberg equations were also obtained as non-relativistic quantum limits of the ECE wave equation, but the Heisenberg indeterminacy principle was not used in accord with the basic philosophy of relativity and with recent experimental data [21] which refute the uncertainty principle by as much as nine orders of magnitude.

7.4 Aharonov Bohm and Phase Effects in ECE Theory

The well known Aharonov Bohm (AB) effects have been observed using magnetic, electric and gravitational fields [1–12] and as shown by ECE theory are ubiquitous for ALL electromagnetic and optical effects, including phase effects: the subject of this section. These must all be explained by general relativity, and not by the obsolete special relativistic methods of the standard model. Therefore it is important to define the various AB conditions in ECE theory. In so doing a unified description of phase effects such as the electromagnetic, Dirac, Wu Yang and Berry phases may also be developed.

In general, the AB condition is defined in ECE theory by the first Cartan structure equation (adopting the index-less short-hand notation [1–12]):

$$T = D \wedge q := d \wedge q + \omega \wedge q. \tag{7.49}$$

Using the ECE hypothesis:

$$A = A^{(0)} q \tag{7.50}$$

this becomes:

$$F = D \wedge A := d \wedge A + \omega \wedge A \tag{7.51}$$

where F is short-hand for the electromagnetic field form and where A is short-hand for the electromagnetic potential form. The AB effects in ECE theory [1–12] were developed with the spin connection term $\omega \wedge A$ in Eq. (7.51). The accepted notation [13] of Cartan geometry uses the tangent space-time indices without the base manifold indices, because the latter are always the same on both sides of an equation of Cartan geometry. So in the standard notation Eq. (7.51) is:

$$F^a = d \wedge A^a + \omega^a{}_b \wedge A^b \tag{7.52}$$

This denotes that the electromagnetic field is a vector-valued two-form and the potential is a vector-valued one-form. In the standard model the spin connection is zero and the standard relation between field and potential is:

$$F = d \wedge A. \tag{7.53}$$

In Eq. (7.53), F is a scalar-valued two-form, and A is a scalar valued one-form [13] The spin connection is zero in Eq. (7.53) because the latter is written in a Minkowski space-time. In the standard model, classical electrodynamics is still represented by the MH equations, which are Lorentz covariant, but not generally covariant. In other words the MH equations are those of special relativity and not general relativity as required by the philosophy of relativity and objectivity. The latter demands that every equation of physics should be an equation of a generally covariant unified field theory. It is well known that the standard model complies with this only in its gravitational sector: the electro-weak and strong fields of the standard model are sectors of special relativity only. The standard model does not comply with general relativity, notably standard model quantum mechanics is philosophically different from relativity (Einstein Bohr dialogue). ECE complies rigorously with the philosophy of general relativity in all its sectors, and unifies all sectors with geometry as required. In ECE the spin connection is ALWAYS non-zero because the fundamental space-time being used is not a flat space-time, it always contains both torsion and curvature in all sectors of the generally covariant unified field theory [1–12]. Torsion and curvature are ineluctably inter-related in the Bianchi identity (Section 7.2), and during the course of development of ECE theory it was shown that there is only one true Bianchi identity, which always links torsion to curvature and vice versa. This is an important mathematical advance of ECE theory, another (Section 7.2) being the development of the Hodge dual of the Bianchi identity.

It has been shown [1–12] that there is a fundamental error in the standard model explanation of the magnetic AB effect [22]. In differential form notation the standard explanation is based on the two equations:

$$F = d \wedge A, d \wedge F = 0 \tag{7.54}$$

whose mathematical structure implies:

$$d \wedge (d \wedge A) = 0. \tag{7.55}$$

It is well known that this structure is invariant under the archetypical gauge transformation:

$$A \rightarrow A + d\chi \tag{7.56}$$

because of the Poincaré Lemma:

$$d \wedge d\chi := 0. \tag{7.57}$$

As explained in paper 56 of the ECE series (www.aias.us), the standard model uses the mathematical result (7.57) to claim that:

$$\oint d\chi = \int_s d \wedge d\chi \neq 0. \tag{7.58}$$

This claim is incorrect because it does not agree with the Stokes Theorem. The latter applies [23] in non simply connected spaces. The Poincaré Lemma (7.57) implies therefore that:

$$\oint d\chi = \int_s d \wedge d\chi := 0 \tag{7.59}$$

in all types of spaces, including non simply connected spaces and there cannot be an Aharonov Bohm effect due to the contour integral of $d\chi$. The incorrect claim of the standard model [22] is that non simply connected spaces allow $\oint d\chi$ to be non-zero. A counter example to this claim was given in paper 56 of www.aias.us. in full detail.

The explanation of the Aharonov Bohm (AB) effects in ECE theory is not based on the mathematical abstractions of gauge theory but on Einstein's philosophy of relativity and Faraday's philosophy of the potential as a physically effective entity (the electrotonic state). This philosophy of Faraday was also accepted by Maxwell and his followers. The idea that the potential is a mathematical abstraction is based on the perceived redundancy exemplified by Eq. (7.57), and this idea has been made into the basis of the mathematical gauge theory of the standard model, developed in the late twentieth century. It appears in standard model textbooks such as that of Jackson for example [1–24]. The idea of a mathematical potential and a physical field in classical electrodynamics is contradicted by the well known minimal prescription of field theory and quantum electrodynamics, where the PHYSICAL momentum eA is added to the momentum p. The idea of an abstract potential ran into trouble following the demonstration by Chambers of the first AB effect to be observed, the magnetic AB effect. It is well known that Chambers placed a magnetic iron whisker between the apertures of a Young interferometer and isolated the magnetic field from interfering electron beams. Therefore, if the potential is mathematical as claimed in gauge theory, it should have no effect on the electronic interference pattern. The experimental result showed a shift in the interference pattern, and so contradicts the standard model, meaning that Faraday was correct: the potential is a physically effective entity. The same results were later obtained experimentally in the electric and gravitational AB effects. As argued in this section, various phase effects also indicate the existence of an electromagnetic AB effect if interpreted by general relativity, of which ECE theory is an example.

The AB effect in ECE theory is summarized as follows:

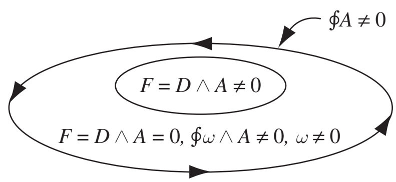


Fig. 7.1. ECE Explanation of the Aharonov Bohm Effect.

It has been shown [1–12] that the observable phase shift of the Chambers experiment in ECE theory is:

$$\Delta\phi = \frac{e}{\hbar}\Phi \tag{7.60}$$

where

$$\Phi = \oint A := -\int_s \omega \wedge A \tag{7.61}$$

in short-hand or index free notation. In the area between the inner and outer rings in Fig. (7.1):

$$F = D \wedge A = 0, A \neq 0, \omega \neq 0. \tag{7.62}$$

The electromagnetic field (F) is zero by experimental arrangement. However, the potential (A) and the spin connection (ω) are not zero in general in this same region between the inner and outer rings. The phase shift is due therefore to the contour integral around A in Eq. (7.61), as indicated in Fig. (7.1). Therefore ECE theory gives a simple explanation of the AB effects as being due to a physical A and a physical ω. The latter indicates that the

ECE space-time is not a Minkowski space-time as in the attempted standard model explanation of the AB effect. In the standard model the equivalent of Fig. (7.1) is:

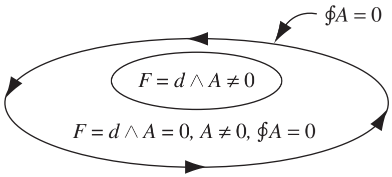


Fig. 7.2. Standard Attempt at Explaining the Aharonov Bohm Effect.

and the contour integral of A is zero. In the standard model the contour integral of the potential is zero in the area between the inner and outer rings of Fig. (7.2) because:

$$F = d \wedge A = 0, A \neq 0, \tag{7.63}$$

$$\int_s d \wedge A = \oint A = 0. \tag{7.64}$$

So when F is zero in the standard model, so is $d \wedge A$. It is possible therefore for A to be non-zero in the standard model while F is zero, but the incorrect twentieth century idea of a non-physical A means that in the standard model A must have no physical effect. In the end analysis this is pure obscurity and has caused great confusion. Such ideas are bad physics and must be discarded sooner or later. The only clear thing about the attempted standard model explanation of the magnetic AB effect is that in the area between the two rings of Fig. (7.2):

$$\int_s F = \int_s d \wedge A = \oint A = 0. \tag{7.65}$$

So the contour integral of A is zero by the Stokes Theorem and there is no AB effect contrary to experiment. Therefore in the standard model, when F is zero the contour integral of A is always zero even though A itself may be non-zero. In other words Stokes' Theorem implies that when F or $d \wedge A$ is zero in the standard model, the contour integral of A must vanish even though A itself may be non-zero. As we have seen, adding a $d\chi$ in an assumed non simply connected space-time does not solve this problem.

In ECE theory the presence of the spin connection ensures that when F is zero, $d \wedge A$ is not zero in general and the contour integral of A is not zero, meaning a phase shift as observed, Eq. (7.61). The way that such an ECE contour integral must be evaluated has been explained carefully [1–12]. Therefore the AB effects show that ECE is preferred experimentally over the standard model. This is one out of many experimental advantages of ECE theory over the standard model. A table of about thirty such advantages is available on the www.aias.us website and in the fourth volume of ref. (7.1). As argued already, the standard model has attempted to obfuscate its way out of the AB paradox by adding $d\chi$ to A and claiming that the AB effect is due to a non-zero contour integral of $d\chi$ when the contour integral of A is zero. Paper 56 of ECE (www.aias.us) shows that this claim is incorrect mathematically, and even if it were correct just leads to obscure ideas, notably that [22] space-time itself must be non-simply connected. This is typical of bad physics - the obscurantism of the twentieth century in natural philosophy with its plethora of nigh incomprehensible and unprovable ideas. In contrast, the twenty first century ECE theory explains the AB effect using the older but experimentally provable philosophy of Faraday, Maxwell and Einstein. Therefore one of the key philosophical advances of ECE theory is to discard standard model gauge theory as being obscurantist and meaningless. In so doing, ECE adheres to Baconian philosophy: the theory is fundamentally changed to successfully and simply explain data that clearly refute the old theory (in this case the old theory is gauge theory).

For self-consistency there should be an AB effect whenever there is present a field and its concomitant potential. So an electromagnetic AB effect should be ubiquitous throughout electrodynamics and optics. This is indeed the case, as manifested for example [1–12] in various well known phase effects interpreted according to general relativity (exemplified in turn by ECE theory). Therefore and in general the electromagnetic AB condition is:

$$\left.\begin{aligned} F = d \wedge A + \omega \wedge A = 0, \\ A \neq 0, \omega \neq 0, \end{aligned}\right\} \tag{7.66}$$

and for the gravitational field the AB condition is:

$$\left.\begin{aligned} T = d \wedge q + \omega \wedge q = 0, \\ q \neq 0, \omega \neq 0. \end{aligned}\right\} \tag{7.67}$$

This short-hand notation has been translated in full detail [1–12] into three other notations: differential form, tensor and vector because notation is not standardized and different subjects use different notations. In the vector notation of classical electrodynamics [24] and electrical engineering, Eq. (7.66) splits into two equations. The first defines the magnetic field in terms of the vector potential and the spin connection vector. This was developed further in paper 74 of ECE theory (www.aias.us) and published in a standard model journal, Physica B [25]. In paper 74 the context was a balance condition for magnetic motors, but the same equation is also an AB condition. It is:

$$\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A} - \boldsymbol{\omega} \times \boldsymbol{A} = \boldsymbol{0}. \tag{7.68}$$

For spin torsion [1–12] in gravitation the equivalent equation is:

$$\boldsymbol{T} = \boldsymbol{\nabla} \times \boldsymbol{q} - \boldsymbol{\omega} \times \boldsymbol{q} = \boldsymbol{0}. \tag{7.69}$$

In ECE every kind of magnetic field is defined by:

$$\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A} - \boldsymbol{\omega} \times \boldsymbol{A} \tag{7.70}$$

for self consistency. The spin connection vector is ubiquitous because it is a property of space-time itself. This is pure relativity of Einstein, but is still missing from the standard model of electrodynamics. The latter is still based on the well known vector development due to Heaviside of the original quaternionic Maxwell equations, and predates the philosophy of relativity.

If an electromagnetic AB effect is being considered the potential in Eq. (7.68) may be modeled by a plane wave as in paper 74 (www.aias.us). In that case the AB condition becomes a Beltrami condition:

$$\boldsymbol{\nabla} \times \boldsymbol{A}^{(1)} = -\kappa \boldsymbol{A}^{(1)} \tag{7.71}$$

$$\boldsymbol{\nabla} \times \boldsymbol{A}^{(2)} = \kappa \boldsymbol{A}^{(2)} \tag{7.72}$$

$$\boldsymbol{\nabla} \times \boldsymbol{A}^{(3)} = 0 \boldsymbol{A}^{(3)} \tag{7.73}$$

which can be developed in turn into a Helmholtz wave equation:

$$(\nabla^2 + \kappa^2)\boldsymbol{A}^{(1)} = \boldsymbol{0}. \tag{7.74}$$

Considering the X component for example:

$$\frac{\partial^2 A_X^{(1)}}{\partial Z^2} + \kappa^2 A_X^{(1)} = 0 \tag{7.75}$$

which is an undamped Bernoulli/Euler resonance equation without a driving force on the right hand side [1–12]. It is also a free space wave equation without a source. It is however a wave equation in the potential ONLY, there being no magnetic field present by Eq. (7.68). In other words there is no radiated electromagnetic field but there is a radiated potential field. This is an example of an electromagnetic AB effect. In ECE theory the radiated potential without field may have a physical effect, in this case an electrodynamic or optical effect.

These arguments of ECE theory go to the root of what is meant by a photon and what is meant by the electromagnetic field. In the standard model there are two approaches to electromagnetic phenomena. As argued already in this Section, the electromagnetic field F is physical but the electromagnetic potential A is unphysical in the standard model on the classical level, whereas in standard model quantum electrodynamics the minimal prescription is used with a physical potential. Also in the standard model there are other concepts such as virtual photons which occur in Feynman's version of quantum electrodynamics. During the course of ECE development however [1–12] the claimed accuracy of the Feynman type QED has been shown to be an exaggeration by several orders of magnitude. It is possible to see this through the fact that accuracy of the fine structure constant is limited by the accuracy of the Planck constant (paper 85 on www.aias.us). The standards laboratory data on fundamental constants were shown in this paper to be self-inconsistent. Finally, Feynman's QED method is based on what are essentially adjustable parameters, in other words it is based on obscurantist concepts such as dimensional renormalization, concepts which cannot be proven experimentally and so distill down to parameters that are adjusted to give a good fit of theory to experiment. It is also well known that the series summation used in the Feynman calculus cannot be proven a priori to converge, and thousands of terms have to be evaluated by computer even for the simplest of problems such as one electron interacting with one photon. The situation in quantum chromodynamics is much more complicated and much worse. In QCD it takes Nobel Prizes to prove renormalization, which is just an adjustable parameter. In a subject such as chemistry, such methods are impractical and are never used. They are therefore confined to ultra-specialist physics and even then are of dubious validity. This is typical of bad science, to claim that a theory is fundamental when it is not. It is well known [1–12] that there are many weaknesses in the standard model of electrodynamics, for example it is still not able to describe the Faraday disk generator of Dec. 26$^{\text{th}}$, 1831 whereas ECE has offered a straightforward explanation.

In ECE the field and potential are both physical [1–12] on both the classical and quantum levels, and in ECE there is no distinction between relativity and wave mechanics. These ideas of natural philosophy all become aspects of the same geometry, and in ECE this is the standard differential geometry of Cartan routinely taught in mathematics. The field, potential and photon are defined by this geometry. In the standard model there is also a distinction between locality and non-locality, a distinction which enters into areas such as quantum entanglement and one photon Young interferometry, in which one photon appears to self-interfere. In ECE [1–12] there is no distinction between locality and non-locality because of the ubiquitous spin connection of general relativity. Thus, in ECE theory, the AB effects are effects of general relativity, and the labels "local" and "non-local" becomes meaningless - all is geometry in four-dimensional space-time.

Having described the essentials of the AB effects, the various phase effects developed during the course of the development of ECE theory [1–12] have been understood by a similar application of the Stokes theorem:

$$\int_s F = \int_s D \wedge A = \oint A + \int_s \omega \wedge A \tag{7.76}$$

in which the covariant exterior derivative $D\wedge$ appears. The use of this type of Stokes Theorem has been exemplified in volume 1 of ref. (1) by integrating around a helix and by closing the contour in a well defined way. This type of integration was used in the development in ECE theory of the well known Dirac and Wu Yang phases, and in a generalization of the well known Berry phase as for example in the well studied paper 6 of the ECE series (www.aias.us). in which the origin of the Planck constant was discussed. (The extent to which the 103 or so individual ECE papers are studied is measured accurately through the feedback software of www.aias.us, and there can be no doubt that they are all well studied by a high quality of readership.) In the development of the electromagnetic phase with ECE theory [1–12] it has been demonstrated that the phase is due to the well known B(3) spin field of ECE theory, first inferred in 1992 from the inverse Faraday effect. This generally relativistic development of the electromagnetic phase is closely related to the AB effects and resolves basic problems in the standard model electromagnetic phase [1–12]. It has therefore been shown that the B(3) field is ubiquitous in optics and electrodynamics, because it derives from the ubiquitous spin connection of space-time itself.

These considerations have also been developed for the topological phases, such as that of Berry, using for self consistency the same methodology as for the electromagnetic, Dirac and Wu Yang phases [1–12]. These well known phases are again understood in ECE theory in terms of Cartan geometry by use of the Stokes Theorem with $D\wedge$ in place of $d\wedge$. All phase theory in physics becomes part of general relativity, and this methodology has been linked to traditional Lagrangian methods based on the minimization of action.

7.5 Tensor and Vector Laws of Classical Dynamics and Electrodynamics

The tensor law for the homogeneous field equation has been shown [1–12] to be:

$$\partial_\mu \widetilde{F}^{\kappa\mu\nu} = 0. \tag{7.77}$$

For each κ index the structure of the matrix is:

$$\widetilde{F}^{\mu\nu} = \begin{bmatrix} 0 & cB_X & cB_Y & cB_Z \\ -cB_X & 0 & -E_Z & E_Y \\ -cB_Y & E_Z & 0 & -E_X \\ -cB_Z & -E_Y & E_X & 0 \end{bmatrix} = \begin{bmatrix} 0 & \widetilde{F}^{01} & \widetilde{F}^{02} & \widetilde{F}^{03} \\ \widetilde{F}^{10} & 0 & \widetilde{F}^{12} & \widetilde{F}^{13} \\ \widetilde{F}^{20} & \widetilde{F}^{21} & 0 & \widetilde{F}^{23} \\ \widetilde{F}^{30} & \widetilde{F}^{31} & \widetilde{F}^{32} & 0 \end{bmatrix}. \tag{7.78}$$

The Gauss law of magnetism in ECE theory has been shown to be obtained from:

$$\kappa = \nu = 0 \tag{7.79}$$

and so:

$$\partial_1 \widetilde{F}^{010} + \partial_2 \widetilde{F}^{020} + \partial_3 \widetilde{F}^{030} = 0 \tag{7.80}$$

i.e.:

$$\boldsymbol{\nabla} \cdot \boldsymbol{B} = 0 \tag{7.81}$$

with:

$$\boldsymbol{B} = \boldsymbol{B}_X \boldsymbol{i} + B_Y \boldsymbol{j} + B_Z \boldsymbol{k} \tag{7.82}$$

and:

$$B_X = B^{001}, B_Y = B^{002}, B_Z = B^{003}. \tag{7.83}$$

These are orbital magnetic field components of the Gauss law of magnetism.

In ECE theory the Faraday law of induction is a spin law of electrodynamics defined by:

$$\left.\begin{aligned} \partial_0 \widetilde{F}^{\kappa 01} + \partial_2 \widetilde{F}^{\kappa 21} + \partial_3 \widetilde{F}^{\kappa 31} = 0 \\ \partial_0 \widetilde{F}^{\kappa 02} + \partial_1 \widetilde{F}^{\kappa 12} + \partial_3 \widetilde{F}^{\kappa 32} = 0 \\ \partial_0 \widetilde{F}^{\kappa 03} + \partial_1 \widetilde{F}^{\kappa 13} + \partial_2 \widetilde{F}^{\kappa 23} = 0 \end{aligned}\right\} . \tag{7.84}$$

The ECE Faraday law of induction for all practical purposes is [1–12]:

$$\boldsymbol{\nabla} \times \boldsymbol{E} + \frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{0} \tag{7.85}$$

where the spin electric and magnetic components are:

$$\left.\begin{array}{r}E_X = E^{332} = -E^{323},\ B_X = -B^{110} = B^{101},\\ E_Y = E^{113} = -E^{131},\ B_Y = -B^{220} = B^{202},\\ E_Z = E^{221} = -E^{112},\ B_Z = -B^{330} = B^{303}.\end{array}\right\} \tag{7.86}$$

The ECE Ampère Maxwell law is another spin law [1–12]:

$$\boldsymbol{\nabla} \times \boldsymbol{B} - \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t} = \mu_0 \boldsymbol{J} \tag{7.87}$$

where the components have been identified as:

$$\left.\begin{array}{r}E_X = E^{101},\ B_X = B^{332},\\ E_Y = E^{202},\ B_Y = B^{113},\\ E_Z = E^{303},\ B_Z = B^{221}.\end{array}\right\} \tag{7.88}$$

Therefore in these two spin laws different components appear in ECE theory of the electric and magnetic fields. In the MH theory of special relativity these components are not distinguishable.

Finally the Coulomb law has been shown to be [1–12]:

$$\boldsymbol{\nabla} \cdot \boldsymbol{E} = \rho/\epsilon_0 \tag{7.89}$$

and is an orbital law of electromagnetism as is the Gauss law of magnetism. In ECE theory these individual spin and orbital components are proportional to individual components of the three index Cartan torsion tensor and three index angular energy/momentum density tensor. Therefore ECE theory comes to the important conclusion that there are orbital and spin components of the electric field, and orbital and spin components of the magnetic field. The orbital components occur in the Gauss law of magnetism and Coulomb law and the spin components in the Faraday law of induction and the Ampère Maxwell law. This information, given by a generally covariant unified field theory, is not available in Maxwell Heaviside (MH) theory of the un-unified, special relativistic, field.

Therefore each law develops an internal structure which is summarized in Table 7.1. There are two orbital laws (Gauss and Coulomb) and two spin laws (Faraday law of induction and Ampère Maxwell law). In each law the components of the electric and magnetic fields are proportional to components of the well known [18] angular energy/momentum density tensor. Therefore for example the static electric field is distinguished form the radiated electric field. This is correct experimentally, it is well known that the static electric field exists between two static or unmoving charges, while the radiated

Table 7.1 Components of the Laws of Classical Electrodynamics

Law	Electric Field	Magnetic Field	Type
Gauss		$B_X = B^{001}$, $B_Y = B^{002}$, $B_Z = B^{003}$	orbital
Coulomb	$E_X = -E^{001}$, $E_Y = -E^{002}$, $E_Z = -E^{003}$		orbital
Faraday	$E_X = E^{332}$, $E_Y = E^{113}$, $E_Z = E^{221}$	$B_X = B^{101}$, $B_Y = B^{202}$, $B_Z = B^{303}$	spin
Ampère Maxwell	$E_X = -E^{101}$, $E_Y = -E^{202}$, $E_Z = -E^{303}$	$B_X = B^{332}$, $B_Y = B^{113}$, $B_Z = B^{221}$	spin

electric field requires accelerated charges for its existence. By postulate the components of the electric and magnetic fields are also proportional to components of the Cartan tensor, a rank three tensor in the base manifold (4-D space-time with torsion and curvature).

In tensor notation the inhomogeneous ECE field equation in the base manifold has been shown to be, for all practical purposes [1–12]:

$$\partial_\mu F^{\kappa\mu\nu} = \frac{1}{\epsilon_0} J^{\kappa\nu} = cA^{(0)} R^{\kappa}{}_{\mu}{}^{\mu\nu}. \tag{7.90}$$

The vacuum is defined by:

$$R^{\kappa}{}_{\mu}{}^{\mu\nu} = 0 \tag{7.91}$$

and is Ricci flat by construction. This result is consistent with the fact that the vacuum solutions of the EH equation are Ricci flat by construction. In a Ricci flat space-time there is no canonical energy momentum density [1–12] and so there are no electric and magnetic fields because there is no angular energy/momentum density. However, as in the theory of the Aharonov Bohm effects developed in Section 7.4, there may be non-zero potential and spin connection in a Ricci flat vacuum. Similarly, in the latter type of vacuum the Ricci tensor vanishes but the Christoffel connection and metric of EH theory do not vanish. Crothers has recently criticized the concept of the Ricci flat vacuum [19] as contradicting the Einstein equivalence principle. He has also shown that the mis-named Schwarzschild metric is inconsistent with the concept of a Ricci flat vacuum and with the geometry of the EH equation. Crothers has also argued that ideas such as Big Bang, black holes and dark matter are inconsistent with the EH equation.

The Coulomb law is the case:

$$\nu = 0 \tag{7.92}$$

of Eq. (7.90). During the course of development of ECE theory it has been shown by computer algebra that for all Ricci flat solutions of the EH equation:

$$R^{\kappa}{}_{\mu}{}^{\mu\nu} = 0 \tag{7.93}$$

but for all other solutions of the EH equation the right hand sides of Eq. (7.90) are non zero for the Christoffel connection. This result introduces a basic paradox in the EH equation as discussed already in this review paper.

The Ampère Maxwell law is the case:

$$\nu = 1, 2, 3 \tag{7.94}$$

in Eq. (7.90) and in tensor notation the ECE Ampère Maxwell law is:

$$\begin{aligned}
\partial_0 F^{\kappa 01} + \partial_2 F^{\kappa 21} + \partial_3 F^{\kappa 31} &= cA^{(0)}(R^{\kappa}{}_{0}{}^{01} + R^{\kappa}{}_{2}{}^{21} + R^{\kappa}{}_{3}{}^{31}) \\
\partial_0 F^{\kappa 02} + \partial_1 F^{\kappa 12} + \partial_3 F^{\kappa 32} &= cA^{(0)}(R^{\kappa}{}_{0}{}^{02} + R^{\kappa}{}_{1}{}^{12} + R^{\kappa}{}_{3}{}^{32}) \\
\partial_0 F^{\kappa 03} + \partial_1 F^{\kappa 13} + \partial_2 F^{\kappa 23} &= cA^{(0)}(R^{\kappa}{}_{0}{}^{03} + R^{\kappa}{}_{1}{}^{13} + R^{\kappa}{}_{2}{}^{23})
\end{aligned} \tag{7.95}$$

Therefore it is inferred that the time-like index is 0 and the space-like indices are 1, 2 and 3. The left hand side of Eq. (7.89) is a scalar and so

$$\kappa = 0 \tag{7.96}$$

is identified with a scalar index. So Eq. (7.89) of the Coulomb law is:

$$\partial_1 F^{010} + \partial_2 F^{020} + \partial_3 F^{030} = cA^{(0)}(R^{0}{}_{1}{}^{10} + R^{0}{}_{2}{}^{20} + R^{0}{}_{3}{}^{30}) \tag{7.97}$$

and is the orbital ECE Coulomb law. In vector notation this law is:

$$\boldsymbol{\nabla} \cdot \boldsymbol{E} = \frac{\rho}{\epsilon_0} \tag{7.98}$$

where:

$$\begin{aligned} E_X = E^{010}, E_Y = E^{020}, E_Z = E^{030}, \\ \rho = \epsilon_0 c A^{(0)} (R^0{}_1{}^{10} + R^0{}_2{}^{20} + R^0{}_3{}^{30}). \end{aligned} \tag{7.99}$$

The S.I. units of this law are:

$$A^{(0)} = JsC^{-1}m^{-1}, R = m^{-2}, \epsilon_0 = J^{-1}c^2 m^{-1}, \rho = Cm^{-3}. \tag{7.100}$$

In Eq. (7.90):

$$\left.\begin{aligned} cA^{(0)} = JC^{-1} = \text{volts}, \\ \boldsymbol{E} = \text{volt } m^{-1}, \boldsymbol{\nabla} \cdot \boldsymbol{E} = \text{volt } m^{-2}, \\ cA^{(0)} R = \text{volt } m^{-2}, \\ \rho / \epsilon_0 = JC^{-1}m^{-2} = \text{volt } m^{-2}, \end{aligned}\right\} \tag{7.101}$$

thus checking the S. I. units for self consistency. In the Ricci flat vacuum:

$$\boldsymbol{\nabla} \cdot \boldsymbol{E} = 0 \tag{7.102}$$

which is consistent with:

$$R^0{}_1{}^{10} + R^0{}_2{}^{20} + R^0{}_3{}^{30} = 0 \tag{7.103}$$

for vacuum solutions of the EH equation as argued already. However, for complete internal consistency the Christoffel symbol cannot be used, because it is not internally consistent with the Bianchi identity as argued already in this review paper.

It is possible to define a curvature scalar of the Coulomb law as:

$$R_{(0)} := R^0{}_1{}^{10} + R^0{}_2{}^{20} + R^0{}_3{}^{30} \tag{7.104}$$

so that:

$$\boldsymbol{\nabla} \cdot \boldsymbol{E} = \frac{\rho}{\epsilon_0} = cA^{(0)} R_{(0)} \tag{7.105}$$

and that the charge density of the Coulomb law becomes:

$$\rho = cA^{(0)} \epsilon_0 R_{(0)} \tag{7.106}$$

in coulombs per cubic meter. In the Cartesian system of coordinates the electric field components of the Coulomb law are:

$$E_X = E^{010}, E_Y = E^{020}, E_Z = E^{030} \tag{7.107}$$

and are proportional to these same components of the three index angular energy momentum density tensor. They are anti-symmetric in their last two indices:

$$E^{010} = -E^{001} \text{etc.} \tag{7.108}$$

In tensor notation the ECE Ampère Maxwell law is given by Eq. (7.95), i.e.:

$$\left.\begin{aligned} \partial_\mu F^{\kappa\mu\nu} = cA^{(0)} R^{\kappa}{}_{\mu}{}^{\mu\nu}, \\ \nu = 1, 2, 3 \end{aligned}\right\} \tag{7.109}$$

and in vector notation by:

$$\boldsymbol{\nabla} \times \boldsymbol{B} - \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t} = \mu_0 \boldsymbol{J}. \tag{7.110}$$

In the Cartesian system:

$$\boldsymbol{J} = J_X \boldsymbol{i} + J_Y \boldsymbol{j} + J_Z \boldsymbol{k} \tag{7.111}$$

where:

$$\left.\begin{aligned} J_X &= \frac{A^{(0)}}{\mu_0}(R^{1}{}_{0}{}^{01} + R^{1}{}_{2}{}^{21} + R^{1}{}_{3}{}^{31}), \\ J_Y &= \frac{A^{(0)}}{\mu_0}(R^{2}{}_{0}{}^{02} + R^{2}{}_{1}{}^{12} + R^{3}{}_{2}{}^{32}), \\ J_Z &= \frac{A^{(0)}}{\mu_0}(R^{3}{}_{0}{}^{03} + R^{3}{}_{1}{}^{13} + R^{3}{}_{2}{}^{23}), \end{aligned}\right\} \tag{7.112}$$

and self consistently in the vacuum:

$$J_X = J_Y = J_Z = 0 \tag{7.113}$$

for Ricci flat space-times. As argued this result has been demonstrated by computer algebra [1–12]. In the Ampère Maxwell law the electric and magnetic field components are proportional to spin angular energy momentum density tensor components of the electromagnetic field as follows:

$$\left.\begin{aligned} E^{\kappa\mu\nu} &= \frac{c^2}{e\omega} J^{\kappa\mu\nu}, \\ B^{\kappa\mu\nu} &= \frac{c}{e\omega} J^{\kappa\mu\nu}. \end{aligned}\right\} \tag{7.114}$$

The electric field components of the Coulomb law and the magnetic field components of the Gauss law are all orbital angular energy density tensor components of the electromagnetic field. The angular energy momentum density tensor may be defined as [18]:

$$J^{\kappa\mu\nu} = -\frac{1}{2}(T^{\kappa\mu}x^{\nu} - T^{\kappa\nu}x^{\mu}) \tag{7.115}$$

using the symmetric canonical energy momentum density tensor:

$$T^{\kappa\mu} = T^{\mu\kappa} \tag{7.116}$$

and the components of the electric and magnetic fields are components of $J^{\kappa\mu\nu}$ as follows:

$$\begin{aligned} E^{00i} &= \frac{c^2}{e\omega} J^{00i}, i = 1, 2, 3, \text{(orbital)}, \\ E^{ii0} &= \frac{c^2}{e\omega} J^{ii0}, i = 1, 2, 3, \text{(spin)}, \\ B^{112} = \frac{c}{e\omega} J^{112}, B^{221} = \frac{c}{e\omega} J^{221}, B^{331} &= \frac{c}{e\omega} J^{331}. \end{aligned} \tag{7.117}$$

The two index angular energy/momentum tensor of the electromagnetic field is an integral over the three index density tensor. Ryder gives one example of such an integral in Minkowski space-time [18]:

$$M^{\mu\nu} = \int M^{0\mu\nu} d^3x. \tag{7.118}$$

Therefore the four laws of electrodynamics in ECE theory are:

$$\boldsymbol{\nabla} \cdot \boldsymbol{B} = 0, \tag{7.119}$$

$$\boldsymbol{\nabla} \times \boldsymbol{E} + \frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{0}, \tag{7.120}$$

$$\nabla \cdot \boldsymbol{E} = \rho/\epsilon_0, \tag{7.121}$$

$$\nabla \times \boldsymbol{B} - \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t} = \mu_0 \boldsymbol{J}, \tag{7.122}$$

and therefore have the same vector structure as the familiar MH equations. However, as argued in this section, the ECE theory gives additional information. In the four laws the components of the magnetic and electric fields are as follows. The Gauss law of magnetism in ECE theory is, for all practical purposes (FAPP):

$$\nabla \cdot \boldsymbol{B} = 0 \tag{7.123}$$

which is an orbital law in which the components of the magnetic field are proportional to orbital components of the angular momentum/energy density tensor and are:

$$\boldsymbol{B} = B^{001}\boldsymbol{i} + B^{002}\boldsymbol{j} + B^{003}\boldsymbol{k}. \tag{7.124}$$

The Faraday law of induction in ECE is a spin law with electric and magnetic field components as follows:

$$\boldsymbol{E} = E^{332}\boldsymbol{i} + E^{113}\boldsymbol{j} + E^{221}\boldsymbol{k}, \tag{7.125}$$

$$\boldsymbol{B} = B^{101}\boldsymbol{i} + B^{202}\boldsymbol{j} + B^{303}\boldsymbol{k}. \tag{7.126}$$

The Coulomb law in ECE is an orbital law with electric field components as follows:

$$\boldsymbol{E} = E^{010}\boldsymbol{i} + E^{020}\boldsymbol{j} + E^{030}\boldsymbol{k}, \tag{7.127}$$

Finally the Ampère Maxwell law in ECE is a spin law with electric and magnetic field components as follows:

$$\boldsymbol{E} = E^{110}\boldsymbol{i} + E^{220}\boldsymbol{j} + E^{330}\boldsymbol{k}, \tag{7.128}$$

$$\boldsymbol{B} = B^{332}\boldsymbol{i} + B^{113}\boldsymbol{j} + B^{221}\boldsymbol{k}. \tag{7.129}$$

As argued in Section 7.4 of this review paper, the relation between field and potential is different in ECE theory and contains the spin connection [1–12]. The various notations for the relation between field and potential in ECE theory are collected here for convenience. In the index-less notation:

$$F = d \wedge A + \omega \wedge A \tag{7.130}$$

which is based on the first Cartan structure equation:

$$T = d \wedge q + \omega \wedge q. \tag{7.131}$$

In the standard notation of differential geometry:

$$F^a = d \wedge A^a + \omega^a{}_b \wedge A^b. \tag{7.132}$$

In tensor notation from differential geometry:

$$F^a_{\mu\nu} = (d \wedge A^a)_{\mu\nu} + (\omega^a{}_b \wedge A^b)_{\mu\nu}. \tag{7.133}$$

In the base manifold Eq. (7.133) becomes:

$$F^{\kappa\mu\nu} = \partial^\mu A^{\kappa\nu} - \partial^\nu A^{\kappa\mu} + (\omega^{\kappa\mu}{}_\lambda A^{\lambda\nu} - \omega^{\kappa\nu}{}_\lambda A^{\lambda\mu}) \tag{7.134}$$

In vector notation Eq. (7.134) splits into two equations, one for the electric field and one for the magnetic field:

$$\boldsymbol{E} = -\boldsymbol{\nabla}\phi - \frac{\partial \boldsymbol{A}}{\partial t} + \phi\boldsymbol{\omega} - \omega\boldsymbol{A} \tag{7.135}$$

and

$$\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A} - \boldsymbol{\omega} \times \boldsymbol{A} \tag{7.136}$$

For the orbital electric field component of the Coulomb law Eq. (7.135) has the following internal structure:

$$\phi = cA^{00}, \boldsymbol{A} = A^{01}\boldsymbol{i} + A^{02}\boldsymbol{j} + A^{03}\boldsymbol{k}, \tag{7.137}$$

$$\omega = c\omega^{00}{}_0, \boldsymbol{\omega} = (\omega^{01}{}_0\boldsymbol{i} + \omega^{02}{}_0\boldsymbol{j} + \omega^{03}{}_0\boldsymbol{k}). \tag{7.138}$$

This result illustrates that the internal structure of the relation between field and potential is different for each law of electrodynamics in ECE theory. Therefore in a GCUFT such as ECE different types of field and potential exist for each law, and also different types of spin connection.

For the orbital Gauss law of magnetism the internal structure of Eq. (7.136) is:

$$\boldsymbol{A} = A^{01}\boldsymbol{i} + \boldsymbol{A}^{02}\boldsymbol{j} + A^{03}\boldsymbol{k}, \tag{7.139}$$

$$\boldsymbol{\omega} = -(\omega^{01}{}_0\boldsymbol{i} + \omega^{02}{}_0\boldsymbol{j} + \omega^{03}{}_0\boldsymbol{k}). \tag{7.140}$$

For the Ampère Maxwell law, a spin law, the internal structure of Eqs. (7.135) and (7.136) are again different, and are defined as follows. The structure of Eq. (7.135) is:

$$\begin{aligned}\phi &= cA^{00} = cA^{01} = cA^{02} = cA^{03},\\ A_X &= A^{01} = A^{11} = A^{21} = A^{31},\\ A_Y &= A^{02} = A^{12} = A^{22} = A^{32},\\ A_Z &= A^{03} = A^{13} = A^{23} = A^{33},\\ \omega_X &= \omega^{11}{}_0 = \omega^{11}{}_1 = \omega^{11}{}_2 = \omega^{11}{}_3,\\ \omega_Y &= \omega^{22}{}_0 = \omega^{22}{}_1 = \omega^{22}{}_2 = \omega^{22}{}_3,\\ \omega_Z &= \omega^{33}{}_0 = \omega^{33}{}_1 = \omega^{33}{}_2 = \omega^{33}{}_3,\\ \omega &= c\omega^{10}{}_0 = c\omega^{10}{}_1 = c\omega^{10}{}_2 = c\omega^{10}{}_3\\ &= c\omega^{20}{}_0 = c\omega^{20}{}_1 = c\omega^{20}{}_2 = c\omega^{20}{}_3\\ &= c\omega^{30}{}_0 = c\omega^{30}{}_1 = c\omega^{30}{}_2 = c\omega^{30}{}_3\end{aligned} \tag{7.141}$$

and the structure of Eq. (7.136) is:

$$\begin{aligned}B_X = B^{332} &= \frac{\partial A_Z}{\partial Y} - \frac{\partial A_Y}{\partial Z} + \omega_Z A_Y - \omega_Y A_Z,\\ B_Y = B^{113} &= \frac{\partial A_X}{\partial Z} - \frac{\partial A_Z}{\partial X} + \omega_X A_Z - \omega_Z A_X,\\ B_Z = B^{221} &= \frac{\partial A_Y}{\partial X} - \frac{\partial A_X}{\partial Y} + \omega_Y A_X - \omega_X A_Y.\end{aligned} \tag{7.142}$$

Finally the internal structures are again different for the Faraday law of induction. In arriving at these conclusions the relation between field and potential in the base manifold is:

$$F^{\kappa\mu\nu} = \partial^\mu A^{\kappa\nu} - \partial^\nu A^{\kappa\mu} + (\omega^{\kappa\mu}{}_\lambda A^{\lambda\nu} - \omega^{\kappa\nu}{}_\lambda A^{\lambda\mu}). \tag{7.143}$$

The Hodge dual of this equation is:

$$\widetilde{F}^{\kappa\mu\nu} = (\partial^\mu A^{\kappa\nu} - \partial^\nu A^{\kappa\mu} + (\omega^{\kappa\mu}{}_\lambda A^{\lambda\nu} - \omega^{\kappa\nu}{}_\lambda A^{\lambda\mu}))_{HD} \tag{7.144}$$

and this is needed to give the results for the homogenous laws. An example of taking the Hodge dual is given below:

$$\begin{aligned}\widetilde{F}^{001} &= (\partial^0 A^{01} - \partial^1 A^{00} + (\omega^{00}{}_\lambda A^{\lambda 1} - \omega^{01}{}_\lambda A^{\lambda 0}))_{HD}\\ &= \partial^2 A^{03} - \partial^3 A^{02} + (\omega^{02}{}_\lambda A^{\lambda 3} - \omega^{03}{}_\lambda A^{\lambda 2}).\end{aligned} \tag{7.145}$$

With these rules the overall conclusion is that in a generally covariant unified field theory (GCUFT) such as ECE the four laws of classical electrodynamics can be reduced to the same vector form as the MH laws of un-unified special relativity (nineteenth century), but the four laws are no longer written in a flat, Minkowski spacetime. They are written in a four dimensional space-time with torsion and curvature. This procedure reveals the internal structure of the electric and magnetic fields appearing in each law, for example correctly makes the distinction between a static and radiated electric field, and a static and radiated magnetic field. The relation between field and potential also develops an internal structure which is different for each law, but for each law, the vector relation can be reduced to:

$$\boldsymbol{E} = -\boldsymbol{\nabla}\phi - \frac{\partial \boldsymbol{A}}{\partial t} + \phi\boldsymbol{\omega} - \omega\boldsymbol{A} \tag{7.146}$$

and

$$\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A} - \boldsymbol{\omega} \times \boldsymbol{A}. \tag{7.147}$$

In a GCUFT, gauge theory is not used because the potential has a physical effect as in the electrotonic state of Faraday. The ECE theory is developed entirely in four dimensions, is entirely self-consistent, and reproduces a range of experimental data [1–12] which the MH theory cannot explain. The ECE theory is also philosophically consistent with the need to apply the philosophy of relativity to the whole of physics. The latter becomes a unified field theory based on geometry. The first attempts by Einstein to develop general relativity were based on Riemann geometry and restricted to the theory of gravitation. In the philosophy of relativity, however, the basic idea that physics is geometry must be used for every equation of physics, and each equation must be part of the same geometrical framework. This is achieved in a GCUFT such as ECE theory by using Cartan's standard differential geometry. This is a self-consistent geometry that recognizes the existence of space-time torsion in the first Cartan structure equation, and space-time curvature in the second. It is also recognized that there is only one Bianchi identity, and that this must always inter-relate torsion and curvature, both are fundamental to the structure of space-time.

7.6 Spin Connection Resonance

One of the most important consequences of general relativity applied to electrodynamics is that the spin connection enters into the relation between the field and potential as described in Section 7.5. The equations of electrodynamics as written in terms of the potential can be reduced to the form of Bernoulli Euler resonance equations. These have been incorporated during the course of development of ECE theory into the Coulomb law, which is the basic law used in the development of quantum chemistry in for example density functional code. This process has been illustrated [1–12] with the hydrogen and helium atoms. The ECE theory has also been used to design or explain circuits which use spin connection resonance to take power from space-time, notably papers 63 and 94 of the ECE series on www.aias.us. In paper 63, the spin connection was incorporated into the Coulomb law and the resulting equation in the scalar potential shown to have resonance solutions using an Euler transform method. In paper 94 this method was extended and applied systematically to the Bedini motor. The method is most simply illustrated by considering the vector form of the Coulomb law deduced in Section 7.5:

$$\boldsymbol{\nabla} \cdot \boldsymbol{E} = \rho/\epsilon_0 \tag{7.148}$$

and assuming the absence of a vector potential (absence of a magnetic field). The electric field is then described by:

$$\boldsymbol{E} = -(\boldsymbol{\nabla} + \boldsymbol{\omega})\phi \tag{7.149}$$

rather than the standard model's:

$$\boldsymbol{E} = -\boldsymbol{\nabla}\phi. \tag{7.150}$$

Therefore Eq. (7.149) in (7.148) produces the equation

$$\nabla^2\phi + \boldsymbol{\omega} \cdot \boldsymbol{\nabla}\phi + (\boldsymbol{\nabla} \cdot \boldsymbol{\omega})\phi = -\frac{\rho}{\epsilon_0} \tag{7.151}$$

which is capable of giving resonant solutions as described in paper 63. The equivalent equation in the standard model is the Poisson equation, which is a limit of Eq. (7.151) when the spin connection is zero. The Poisson equation does not give resonant solutions. It is known from the work of Tesla for example that strong resonances in electric power can be obtained with suitable apparatus, and such resonances cannot be explained using the standard model. A plausible explanation of Tesla's well known results is given by the incorporation of the spin connection into classical electrodynamics. Using spherical polar coordinates and restricting consideration to the radial component:

$$\nabla^2\phi = \frac{\partial^2\phi}{\partial r^2} + \frac{2}{r}\frac{\partial\phi}{\partial r}, \tag{7.152}$$

$$\boldsymbol{\omega} \cdot \boldsymbol{\nabla}\phi = \omega_r\frac{\partial\phi}{\partial r}, (\boldsymbol{\nabla} \cdot \boldsymbol{\omega})\phi = \frac{\phi}{r^2}\frac{\partial}{\partial r}(r^2\omega_r), \tag{7.153}$$

so that Eq. (7.151) becomes:

$$\frac{\partial^2 \phi}{\partial r^2} + \left(\frac{2}{r} + \omega\right)\frac{\partial \phi}{\partial r} + \frac{\phi}{r^2}\left(2r\omega_r + r^2\frac{\partial \omega_r}{\partial r}\right) = \frac{-\rho}{\epsilon_0} \tag{7.154}$$

In paper 63 a spin connection was chosen of the simplest type compatible with its dimensions of inverse meters:

$$\omega_r = -\frac{1}{r} \tag{7.155}$$

and thus giving the differential equation:

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r}\frac{\partial \phi}{\partial r} - \frac{1}{r^2}\phi = \frac{-\rho}{\epsilon_0} \tag{7.156}$$

as a function of r. Eq. (7.156) becomes a resonance equation if the driving term on the right hand side is chosen to be oscillatory, in the simplest instance:

$$\rho = \rho(0)\cos(\kappa_r r). \tag{7.157}$$

To obtain resonance solutions from Eq. (7.156), an Euler transform [1–12] is needed as follows:

$$\kappa_r r = \exp(i\kappa_r R). \tag{7.158}$$

This is a standard Euler transform extended to a complex variable. This simple change of variable transforms Eq. (7.156) into:

$$\frac{\partial^2 \phi}{\partial R^2} + \kappa_r^2 \phi = \frac{\rho(0)}{\epsilon_0}\mathrm{Real}(e^{2i\kappa_r R}\cos(e^{i\kappa_r R})) \tag{7.159}$$

which is an undamped oscillator equation as demonstrated in detail in Eq. (7.63), where the domain of validity of the transformed variable was discussed in detail. It is seen from feedback software to www.aias.us that paper 63 has been studied in great detail by a high quality readership, so we may judge that its impact has been extensive. The concept of spin connection resonance has been extended to gravitational theory and magnetic motors and the theory published in standard model journals [25–27]. In paper 63 the simplest possible form of the spin connection was used, Eq. (7.155) and the resulting Eq. (7.156) was shown to have resonance solutions using a change of variable. There is therefore resonance in the variable R. In paper 90 of www.aias.us this method was made more general by considering the equation

$$\frac{\partial^2 \phi}{\partial r^2} + \left(\frac{2}{r} + \omega_r\right)\frac{\partial \phi}{\partial r} + \frac{\phi}{r^2}\left(2r\omega_r + r^2\frac{\partial \omega_r}{\partial r}\right) = \frac{-\rho}{\epsilon_0} \tag{7.160}$$

which is a more general form of Eq. (7.156). When the spin connection is defined as:

$$\omega_r = \omega_0^2 r - 4\beta \log_e r - \frac{4}{r}. \tag{7.161}$$

Eq. (7.160) becomes a simple resonance equation in r itself:

$$\frac{\partial^2 \phi}{\partial r^2} + 2\beta\frac{\partial \phi}{\partial r} + \omega_0^2 \phi = \frac{-\rho}{\epsilon_0}. \tag{7.162}$$

There is freedom of choice of the spin connection. The latter was unknown in electrodynamics prior to ECE theory and must ultimately be determined experimentally. An example of this procedure is given in paper 94, where spin connection resonance (SCR) theory is applied to a patented device. One of the papers published in the standard model literature [26] applies SCR to magnetic motors that are driven by space-time. It is probable that SCR was also discovered and demonstrated by Tesla [28], but empirically before the emergence of relativity theory. SCR has also been applied to gravitation and published in the standard model literature [27]. So a gradual loosening of the ties to the standard model is being observed at present.

In paper 92 of the ECE series (www.aias.us), Eq. (7.160) was further considered and shown to reduce to an Euler Bernoulli resonance equation of the general type:

$$\frac{d^2 x}{dr^2} + 2\beta\frac{dx}{dr} + \kappa_0^2 x = A\cos(\kappa r) \tag{7.163}$$

in which β plays the role of friction coefficient, κ_0 is a Hooke's law wave-number and in which the right hand side is a cosinal driving term. Eq. (7.160) reduces to Eq. (7.163) when:

$$\omega_r = 2\left(\beta - \frac{1}{r}\right), \kappa_0^2 = \frac{4}{r}\left(\beta - \frac{1}{r}\right) + \frac{\partial \omega_r}{\partial r} \tag{7.164}$$

Therefore the condition udner which the spin connection gives the simple resonance Eq. (7.163) is defined by:

$$\omega_r = \kappa_0^2 - 4\beta \log_e r - \frac{4}{r}. \tag{7.165}$$

Reduction to the standard model Coulomb law occurs when:

$$\beta = \frac{1}{r} \tag{7.166}$$

when

$$\omega_r = 0, \kappa_0^2 = 0. \tag{7.167}$$

In general there is no reason to assume that condition (7.166) always holds. The reason why the standard model Coulomb law is so accurate in the laboratory is that it is tested off resonance. In this off resonant limit the ECE theory has been shown [1–12] to give the Standard Coulomb law as required by a vast amount of accumulated data of two centuries since Coulomb first inferred the law. In general, ECE theory has been shown to reduce to all the known laws of physics, and in addition ECE gives new information. This is a classic hallmark of a new advance in physics. It is probable that Tesla inferred methods of tuning the Coulomb law (and other laws) to spin connection resonance. Many other reports of such surges in power have been made, and it is now known and accepted by the international community of scientists that they come from general relativity applied to classical electrodynamics.

Paper 94 of the ECE series is a pioneering paper in which the theory of SCR is applied to a patented device in order to explain in detail how the patented device takes energy from space-time. No violation of the laws of conservation of energy and momentum occurs in ECE theory or in SCR theory.

7.7 Effects of Gravitation on Optics and Spectroscopy

In the standard model of electrodynamics the electromagnetic sector is described by the nineteenth century Maxwell Heaviside (MH) field theory, which in gauge theory is U(1) invariant and Lorentz covariant in a Minkowski space-time. As such MH theory cannot describe the effect of gravitation on optics and spectroscopy because gravitation requires a non-Minkowski space-time. In ECE theory on the other hand all sectors are generally covariant, and during the course of development of ECE theory several effects of gravitation on optics and spectroscopy have been inferred, notably the effect of gravitation on the Sagnac effect, RFR and on the polarization of light grazing a white dwarf. An explanation for the well known Faraday disk generator has also been given in terms of spinning space-time, an explanation which illustrates the fact that the torsion of space-time produces effects not present in the standard model. Gravitation is the curvature of space-time and in ECE theory the interaction of torsion and curvature is determined by Cartan geometry.

The Faraday disk generator has been explained in ECE theory from the basic assumption that the electromagnetic field is the Cartan torsion within a factor:

$$\boldsymbol{F}_{\text{mech}} = A^{(0)} T_{\text{mech}} \tag{7.168}$$

where $cA^{(0)}$ is the primordial voltage. The factor $A^{(0)}$ is considered to originate in the magnet of the Faraday disk generator. The Faraday disk generator consists essentially of a spinning disk placed on a magnet, without the magnet no induction is observed, i.e. no p.d.f. is generated between the center and rim of the disk without a magnet being present. The original experiment by Faraday on 26^{th} Dec. 1831 consisted of spinning a disk on top of a static magnet, but an e.m.f. is also observed if both the disk and the magnet are spun about their common vertical axis. There continues to be no explanation for the Faraday disk generator in the standard model, because in the latter there is no connection between the electromagnetic field and mechanical spin, angular momentum and torsion, while ECE makes this connection in Eq. (7.168). The standard model law of induction of Faraday is:

$$\boldsymbol{\nabla} \times \boldsymbol{E} + \frac{\partial \boldsymbol{B}}{\partial t} = \mathbf{0} \tag{7.169}$$

and spinning the magnetic field about its own axis does not produce a nonzero curl of the electric field as required. Clearly, a static magnetic field will not cause induction from Eq. (7.169). So this is a weak point of the standard model, in which induction is caused in the classical textbook description by moving a bar magnet inside a coil, causing a current to appear. In ECE it has been shown [1–12] that the explanation of the Faraday disk generator is simply:

$$\boldsymbol{F} = \boldsymbol{F}_{e/m} + \boldsymbol{F}_{\text{mech}} \tag{7.170}$$

which in vector notation (section 7.5) produces the law of induction:

$$\boldsymbol{\nabla} \times \boldsymbol{E}_{\text{mech}} + \frac{\partial \boldsymbol{B}_{\text{mech}}}{\partial t} = \mathbf{0}. \tag{7.171}$$

Spinning the disk has the following effect in ECE theory.

In the complex circular basis [1–12] the magnetic flux density in ECE theory is defined by:

$$\boldsymbol{B}^{(1)*} = \boldsymbol{\nabla} \times \boldsymbol{A}^{(1)*} - i\frac{\kappa}{A^{(0)}}\boldsymbol{A}^{(2)} \times \boldsymbol{A}^{(3)} \tag{7.172}$$

$$\boldsymbol{B}^{(2)*} = \boldsymbol{\nabla} \times \boldsymbol{A}^{(2)*} - i\frac{\kappa}{A^{(0)}}\boldsymbol{A}^{(3)} \times \boldsymbol{A}^{(1)} \tag{7.173}$$

$$\boldsymbol{B}^{(3)*} = \boldsymbol{\nabla} \times \boldsymbol{A}^{(3)*} - i\frac{\kappa}{A^{(0)}}\boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)} \tag{7.174}$$

where

$$\kappa = \frac{\Omega}{c} \tag{7.175}$$

is a wave-number and Ω is an angular frequency in radians per second. When the disk is stationary the ECE vector potential is [1–12] proportional by fundamental hypothesis to the tetrad:

$$\boldsymbol{A}^{(1)} = A^{(0)}\boldsymbol{q}^{(1)} \tag{7.176}$$

$$\boldsymbol{A}^{(2)} = A^{(0)}\boldsymbol{q}^{(2)} \tag{7.177}$$

$$\boldsymbol{A}^{(3)} = A^{(0)}\boldsymbol{q}^{(3)}. \tag{7.178}$$

In the complex circular basis the tetrads are:

$$\boldsymbol{q}^{(1)} = \frac{1}{\sqrt{2}}(\boldsymbol{i} - i\boldsymbol{j}), \tag{7.179}$$

$$\boldsymbol{q}^{(2)} = \frac{1}{\sqrt{2}}(\boldsymbol{i} + i\boldsymbol{j}), \tag{7.180}$$

$$\boldsymbol{q}^{(3)} = \boldsymbol{k}, \tag{7.181}$$

and have O(3) symmetry as follows:

$$\boldsymbol{q}^{(1)} \times \boldsymbol{q}^{(2)} = i\boldsymbol{q}^{(3)*}, \tag{7.182}$$

$$\boldsymbol{q}^{(2)} \times \boldsymbol{q}^{(1)} = i\boldsymbol{q}^{(1)*}, \tag{7.183}$$

$$\boldsymbol{q}^{(3)} \times \boldsymbol{q}^{(1)} = i\boldsymbol{q}^{(2)*}. \tag{7.184}$$

In the absence of rotation about Z:

$$\boldsymbol{\nabla} \times \boldsymbol{A}^{(1)} = \boldsymbol{\nabla} \times \boldsymbol{A}^{(2)} = \boldsymbol{0}, \tag{7.185}$$

$$\boldsymbol{A}^{(3)} = A^{(0)}\boldsymbol{k}. \tag{7.186}$$

In the complex circular basis:

$$\boldsymbol{\nabla} \times \boldsymbol{E}^{(1)} + \partial \boldsymbol{B}^{(1)}/\partial t = \boldsymbol{0}, \tag{7.187}$$

$$\boldsymbol{\nabla} \times \boldsymbol{E}^{(2)} + \partial \boldsymbol{B}^{(2)}/\partial t = \boldsymbol{0}, \tag{7.188}$$

$$\boldsymbol{\nabla} \times \boldsymbol{E}^{(3)} + \partial \boldsymbol{B}^{(3)}/\partial t = \boldsymbol{0}. \tag{7.189}$$

Therefore from Eqs. (7.176) to (7.189) the only field present is:

$$\begin{aligned} \boldsymbol{B}^{(3)*} &= \boldsymbol{B}^{(3)} = -iB^{(0)}\boldsymbol{q}^{(1)} \times \boldsymbol{q}^{(2)} \\ &= B_z^{(3)}\boldsymbol{k} = B_z\boldsymbol{k} \end{aligned} \tag{7.190}$$

which is the static magnetic field of the bar magnet.

Now mechanically rotate the disk at an angular frequency Ω to produce:

$$\boldsymbol{A}^{(1)} = \frac{A^{(0)}}{\sqrt{2}}(\boldsymbol{i} - i\boldsymbol{j})\exp(i\Omega t), \tag{7.191}$$

$$\boldsymbol{A}^{(2)} = \frac{A^{(0)}}{\sqrt{2}}(\boldsymbol{i} + i\boldsymbol{j})\exp(-i\Omega t). \tag{7.192}$$

From Eqs. (7.176) to (7.192) electric and magnetic fields are induced in a direction transverse to Z, i.e. in the XY plane of the spinning disk as observed experimentally. However, the Z axis magnetic flux density is unchanged by physical rotation about the same Z axis. This is again as observed experimentally. The (2) component of the transverse electric field spins around the rim of the disk and is defined from Eq. (7.151) as:

$$\boldsymbol{E}^{(2)} = \boldsymbol{E}^{(1)*} = -\left(\frac{\partial}{\partial t} + i\Omega\right)\boldsymbol{A}^{(2)}. \tag{7.193}$$

It can be seen from section 7.5 that $i\Omega$ is a type of spin connection. The latter is caused by mechanical spin, which in ECE is a spinning of space-time itself. The real and physical part of the induced $E^{(1)}$ is:

$$\text{Real}(\boldsymbol{E}^{(1)}) = \frac{2}{\sqrt{2}}A^{(0)}\Omega(\boldsymbol{i}\sin\Omega t - \boldsymbol{j}\cos\Omega t) \tag{7.194}$$

and is proportional to the product of $A^{(0)}$ and Ω, again as observed experimentally. An electromotive force is set up between the center of the disk and the rotating rim, as first observed experimentally by Faraday. This e. m. f. is measured experimentally with a voltmeter at rest with respect to the rotating disk.

The homogeneous law (7.120) of ECE theory is generally covariant [1–12] by construction, so retains its form in any frame of reference. ECE therefore produces a simple and complete description of the Faraday disk generator in terms of the spinning of space-time, and concomitant spin connection. The latter is therefore demonstrated in classical electrodynamics by the generator. All known experimental features are explained straightforwardly by ECE theory, but cannot be explained by MH theory, in which the spin connection is missing because Minkowski space-time has no connection by construction - it is a "flat" space-time. It is relatively easy to think of electrodynamics as spinning space-time if we think of gravitation as curving space-time. This analysis also gives confidence in the arguments of Section 7.6, where power is obtained from space-time with spin connection resonance.

The same ECE concept just used to explain the Faraday disk generator has been used [1–12] to give a simple explanation of the Sagnac effect (ring laser gyro). Again, the standard model has no satisfactory explanation for the Sagnac effect [1–12]. Consider the rotation of a beam of light of any polarization around a circle of area πr^2 in the XY plane at an angular frequency ω_1. The rotation is a rotation of space-time itself in ECE theory, described by the rotating tetrad:

$$\boldsymbol{q}^{(1)} = \frac{1}{\sqrt{2}}(\boldsymbol{i} - i\boldsymbol{j})e^{i\omega_1 t}. \tag{7.195}$$

This is rotation around the static platform of the Sagnac interferometer. The fundamental ECE assumption means that this rotation produces the electromagnetic vector potential:

$$\boldsymbol{A}_L^{(1)} = A^{(0)}\boldsymbol{q}^{(1)} \tag{7.196}$$

for left rotation and:

$$\boldsymbol{A}_R^{(1)} = \frac{A^{(0)}}{\sqrt{2}}(\boldsymbol{i} + i\boldsymbol{j})e^{i\omega_1 t} \tag{7.197}$$

for right rotation. When the platform is at rest a beam going around left-wise takes the same time to reach its starting point as a beam going around right-wise. The time delay is zero:

$$\Delta t = 2\pi\left(\frac{1}{\omega_1} - \frac{1}{\omega_1}\right) = 0. \tag{7.198}$$

Eqs. (7.196) and (7.197) do not exist in special relativity because in the MH theory electromagnetism is a nineteenth century entity superimposed on a space-time that is flat and static and never rotates.

Now consider the left - wise rotating beam (7.196) and spin the platform mechanically in the same left-wise direction at an angular frequency Ω. The result is an increase in the angular frequency of the rotating tetrad as follows:

$$\omega_1 \to \omega_1 + \Omega. \tag{7.199}$$

Similarly consider the left wise rotating beam (7.196) and spin the platform right-wise. The result is a decrease in the angular frequency of the rotating tetrad:

$$\omega_1 \to \omega_1 - \Omega. \tag{7.200}$$

The time delay between a beam circling left-wise with the platform, and one circling left-wise against the platform is therefore:

$$\Delta t = 2\pi \left(\frac{1}{\omega_1 - \Omega} - \frac{1}{\omega_1 + \Omega} \right) \tag{7.201}$$

which is the Sagnac effect. The angular frequency ω_1 can be calculated from the experimental result [1–12]:

$$\Delta t = \frac{4\Omega}{c^2} Ar = \frac{4\pi\Omega}{\omega_1^2 - \Omega^2} \tag{7.202}$$

If

$$\Omega \ll \omega_1 \tag{7.203}$$

it is found that

$$\omega_1 = \frac{c}{r} = c\kappa \tag{7.204}$$

Q.E.D. Therefore the Sagnac effect is another result of a spin connection, which in this case can be thought of as the potential (7.196) itself.

Similarly, phase effects such as the Tomita Chao effect were also described straightforwardly with the same basic concept during the development of ECE theory.

In order to describe the effects of gravitation on optics and spectroscopy a dielectric version of the ECE theory was developed and implemented to find that the polarization of light is changed by light grazing a very massive object such as a white dwarf, and the dielectric theory was also used to demonstrate the effect of gravitation on the Sagnac effect [1–12]. The standard model is not capable of such descriptions without the use of adjustable parameters in such transient twentieth century artifacts as superstring theory, now being essentially discarded as being untestable experimentally. ECE is far simpler and is also capable of describing data such as the Faraday disk generator and the Sagnac effect straightforwardly. During the course of its development the ECE theory has also been applied to the interaction of three fields [27] and the effect of gravitation on the inverse Faraday effect and its resonance counterpart, known as radiatively induced fermion resonance (RFR).

The interaction of fields in ECE theory is controlled by Cartan geometry, in the particular case of the interaction of gravitation and electromagnetism, there is a very small homogeneous charge current density in the Gauss law and in the Faraday law of induction. For all practical purpose in the laboratory this is not observable. However, it has been shown in ECE theory to result in changes of polarization and other optical properties of light grazing a white dwarf, which is an object many times heavier than the sun. Such changes of polarization are not described by the Einstein Hilbert equation.

7.8 Radiative Corrections in ECE Theory

During the course of development of ECE theory the anomalous g factor of the electron and Lamb shifts in hydrogen and helium have been explained satisfactorily in a far simpler manner than the standard model and using the causal and objective principles of Einsteinian relativity. The usual approach to the radiative corrections in quantum electrodynamics (QED) has been criticized [1–12], especially its claim to accuracy. The QED method of the standard model relies on assumptions that are not present in Einsteinian relativity, and also on adjustable parameters. The Feynman method consists of assuming the existence of virtual particles and on a perturbation method of quantum mechanics which sums thousands of terms of increasing complexity. There is no proof that this sum converges. It is also claimed in standard model QED that the accuracy of the fine structure constant is reproduced theoretically to high precision. However the fine structure constant in S.I. units is:

$$\alpha = \frac{e^2}{4\pi\epsilon\hbar c} \tag{7.205}$$

and its accuracy is limited by the experimental accuracy of the Planck constant. There is no way that a theory can produce a higher accuracy than experiment, and the theoretical value of the g factor of the electron is based on the value of the fine structure constant. Thus g cannot be known with greater accuracy than that of the fine structure constant. These surprising inconsistencies in the standard model data were discussed in detail [1–12] and a brief summary is given here.

The fundamental constants of physics are agreed upon by treaty and are given on sites such as that of the National Institute for Standards and Technology (www.nist.gov). This site gives:

$$g(\text{exptl.}) = 2.0023193043718 \pm 0.0000000000075 \tag{7.206}$$

$$\hbar(\text{exptl}) = (6.6260693 \pm 0.0000011) \times 10^{-34} Js \tag{7.207}$$

$$e(\text{exptl.}) = (1.60217653 \pm 0.00000014) \times 10^{-19} C \tag{7.208}$$

$$c(\text{exact}) = 2.99792458 \times 10^{8} ms^{-1} \tag{7.209}$$

$$\epsilon_0(\text{exact}) = 8.854187817 \times 10^{-12} J^{-1} C^2 m^{-1} \tag{7.210}$$

$$\mu_0(\text{exact}) = 4\pi \times 10^{-7} Js^2 C^{-2} m^{-1} \tag{7.211}$$

with relative standard uncertainties. With a sufficiently precise value of:

$$\pi = 3.141592653590 \tag{7.212}$$

gives, from these data:

$$\alpha = 0.007297(34) \tag{7.213}$$

where the result has been rounded off to the relative standard uncertainty of the Planck constant h. This is an experimentally determined uncertainty. The theoretical value of g from ECE theory was found by using Eq. (7.213) in

$$g = 2\left(1 + \frac{\alpha}{4\pi}\right)^2 \tag{7.214}$$

and gives:

$$g(\text{ECE}) = 2.002323(49). \tag{7.215}$$

The experimental value of g is known to a much higher precision than the experimental value of h, and is:

$$g(\text{exptl.}) = 2.0023193043718 \pm 0.0000000000075. \tag{7.216}$$

It is seen that:

$$g(\text{ECE}) - g(\text{exptl.}) = 0.000004 \tag{7.217}$$

which is about the same order of magnitude as the experimental uncertainty of h. Therefore it was shown that ECE theory gives g as precisely as the experimental uncertainty in h will allow. The standard model literature was found to be severely self-inconsistent. For example a much used text by Atkins [29] gives h as:

$$h\,(\text{Atkins}) = 6.62818 \times 10^{-34} Js \tag{7.218}$$

without uncertainty estimates. This is different in the fourth decimal place from the NIST value given above, a discrepancy of four orders of magnitude. Despite this, Atkins gives:

$$\alpha(\text{Atkins}) = 0.00729351 \tag{7.219}$$

which claims to be different from Eq. (7.213) only in the sixth decimal place. Atkins gives the g factor of the electron as:

$$g(\text{Atkins}) = 2.002319314 \tag{7.220}$$

which is different from the NIST value in the eighth decimal place, while it is claimed at NIST that g(exp) from Eq. (7.216) is accurate to the twelfth decimal place. So there is another discrepancy of four orders of magnitude. Ryder on the other hand [18] gives:

$$g(\text{Ryder}) = 2.0023193048 \tag{7.221}$$

which is different from the NIST value in the tenth decimal place, a discrepancy of two orders of magnitude. One could try to explain these discrepancies by increasing accuracy of experimental method over the years, but there is no way in which QED can reproduce g to the tenth decimal place as claimed by Ryder. This is easily seen from the fact that g is calculated theoretically in QED from the fine structure constant, whose accuracy is limited by h as we have argued. There is also no way in which QED can be a fundamental theory as is often claimed in the standard model literature. This is again easily seen from the fact that QED has several assumptions extraneous to the theory of relativity [1–12]. Examples are virtual particles, acausality (the electron can do what it likes, g backwards in time and so on), dimensional regularization, re-normalization and the hugely elaborate perturbation method known as the Feynman calculus. It is not known whether the series expansion used in the Feynman calculus converges. Its thousands of terms are just worked out by computer in the hope that it converges. In summary:

$$g(\text{Schwinger}) = 2 + \alpha/\pi = 2.002322(8) \tag{7.222}$$

$$g(\text{ECE}) = 2 + \alpha/\pi + \frac{\alpha^2}{8\pi^2} = 2.002323(49) \tag{7.223}$$

$$g(\text{exptl.}) = 2.0023193043718 \pm 0.0000000000075 \tag{7.224}$$

$$g(\text{Atkins}) = 2.002319314 \pm (?) \tag{7.225}$$

$$g(\text{Ryder}) = 2.0023193048 \pm (?) \tag{7.226}$$

and there is little doubt that other textbooks and sources give further different values of g to add to the confusion in the standard model literature. So where does this analysis leave the claims of QED? The Wolfram site claims that QED gives g using the series

$$g = 2\left(1 + \frac{\alpha}{2\pi} - 0.328\left(\frac{\alpha}{2\pi}\right)^2 + 1.181\left(\frac{\alpha}{\pi}\right)^3 - 1.510\left(\frac{\alpha}{\pi}\right)^4 + \ldots + 4.393 \times 10^{-12}\right) \tag{7.227}$$

which is derived from thousands of Feynman diagrams (sic). However, the numbers in Eq. (7.227) all come from the various assumptions of QED, none of which are present in Einsteinian relativity. The latter is causal and objective by construction. An even worse internal inconsistency emerges within the NIST site itself, because the fine structure constant is claimed to be:

$$\alpha(\text{NIST}) = (7.297352560 \pm 0.000000024) \times 10^{-3} \tag{7.228}$$

both experimentally and theoretically. This cannot be true because Eq. (7.228) is different in the eighth decimal place from Eq. (7.213), which is calculated with NIST's OWN data, Eqs. (7.206) to (7.211). So the NIST site is internally inconsistent to several orders of magnitude, because it is at the same time claimed that Eq. (7.228) is accurate to the tenth decimal place. From Eq. (7.207) however it is seen that h at NIST is accurate only to the sixth decimal place, which limits the accuracy of α to this, i.e. four orders of magnitude less precise than claimed.

The theoretical claim for the fine structure constant at NIST comes from QED, which his described as a theory in which an electron emits a virtual photon, which in turn emits virtual electron positron pairs. The virtual positron is attracted and the virtual electron is repelled from the real electron. This process results in a screened charge, a mathematical concept with a limiting value defined as the limit of zero momentum transfer or infinite distance. At high energies the fine structure constant drops to 1/128, and so is not a constant at all. It cannot therefore be claimed to be precise to the relative standard uncertainty of Eq. (7.228), taken directly from the NIST website itself. There is therefore no direct way of proving experimentally the existence of virtual electron positron pairs, or of virtual photons. The experimental claim for the fine structure constant at NIST comes from the quantum Hall effect combined with a calculable cross capacitor to measure standard resistance. The von Klitzing constant:

$$R_\kappa = \frac{\hbar}{e^2} = \frac{\mu_0 c}{2}(\text{sic}) \tag{7.229}$$

is used in this experimental determination. However, this method is again limited by the experimental accuracy of h. The accuracy of e is only ten times better than h from NIST's own data, and R_κ cannot be more accurate than h. If α were really as accurate as claimed in Eq. (7.228), both h and e would have to be this accurate experimentally, and this is obviously not true.

In view of these severe inconsistencies in the standard model and in view of the many ad hoc and indeed unprovable assumptions of QED, it is considered that the so called "precision tests" of QED are of no utility and no meaning. These include the g factor of the electron, the Lamb shift, the Casimir effect, positronium, and so forth.

The ECE theory of these radiative corrections therefore set out to reproduce what is really known experimentally in the simplest way. These methods are of course those of William of Ockham and Francis Bacon. In the non-relativistic quantum approximation to ECE theory the Schrödinger equation was modified as follows [1–12]:

$$-\frac{\hbar^2}{2m}\nabla^2\left(\frac{\alpha}{2\pi}+\frac{\alpha^2}{16\pi^2}\right)\psi = \frac{e^2}{4\pi\epsilon_0}\left(\frac{1}{r}-\frac{1}{r+r(\text{vac})}\right)\psi \tag{7.230}$$

in which the effect of the vacuum potential is considered to be a shift in the electron to proton distance for each orbital of an atom or molecule, in the simplest case atomic hydrogen (H). Computer algebra was used to show that:

$$\frac{r(\text{vac})(2s)}{r+(r+r(\text{vac}))} - \frac{r(\text{vac})(2\rho_z, \cos\theta = 1)}{r(r+r(\text{vac}))} = \frac{1}{4\pi}\frac{\hbar}{mc}\frac{1}{r^2} \tag{7.231}$$

so that the simple ECE method of Eq. (7.230) gives the correct qualitative result observed first by Lamb in atomic H. This is known as the Lamb shift. Computer algebra was used to show that the ECE Lamb shift is:

$$\Delta E = \left(\frac{1}{16\pi^{3/2}}\frac{\alpha}{a}\frac{\hbar}{mc}\right)\frac{1}{r} = 0.0353\ cm^{-1} \tag{7.232}$$

in the approximation in which the angular dependence if the Lamb shift is not considered.

The potential energy of the unperturbed H atom in wave-numbers is:

$$V_0 = -\frac{\alpha}{r} \tag{7.233}$$

and the vacuum perturbs this as follows:

$$V = -\frac{\alpha}{r + r(\mathrm{vac})}. \tag{7.234}$$

So the change in potential energy due to the vacuum (i.e. the radiative correction) is positive valued as follows:

$$\Delta V = |V - V_0| = \alpha \left(\frac{1}{r} - \frac{1}{r + r(\mathrm{vac})} \right). \tag{7.235}$$

This equation was obtained by assuming that the Schrödinger equation of H in the presence of the radiative correction due to the vacuum is, to first order in α:

$$-\frac{\hbar^2}{2m}\left(1 + \frac{\alpha}{2\pi}\right)\nabla^2\psi - \frac{e^2}{4\pi\epsilon_0 r}\psi = E\psi \tag{7.236}$$

and that this is equivalent to:

$$-\frac{\hbar^2}{2m}\nabla^2\psi - \frac{e^2}{4\pi\epsilon_0(r + r(\mathrm{vac}))}\psi = E\psi. \tag{7.237}$$

It was assumed that $r(\mathrm{vac})$ is small enough to justify using the analytically known unperturbed wave-functions of H (ψ_0) to a good approximation. So:

$$\psi \sim \psi_0 \tag{7.238}$$

and:

$$\nabla^2\psi_0 = -\frac{4\pi mc}{\hbar}\left(\frac{1}{r} - \frac{1}{r + r(\mathrm{vac})}\right)\psi_0. \tag{7.239}$$

Using computer algebra this approximation gives [1–12]:

$$\frac{1}{r + r_{2p}(\mathrm{vac})} - \frac{1}{r + r_{2s}(\mathrm{vac})} = \frac{1}{2\pi^{3/2}}\frac{\hbar}{mc}\frac{1}{r^2}. \tag{7.240}$$

The change in potential energy due to the radiative correction of the vacuum is thus:

$$\Delta V = \frac{\alpha}{2\pi^{3/2}} \frac{\hbar}{mc} \frac{1}{r^2} \tag{7.241}$$

and the change in total energy is:

$$\Delta E = \frac{r}{2n^2 a} \Delta V = \left(\frac{1}{16\pi^{3/2}} \frac{\alpha}{a} \frac{\hbar}{mc} \right) \frac{1}{r} = 0.0353\ cm^{-1} \tag{7.242}$$

which is the Lamb shift of atomic H. Here:

$$r = 1.69 \times 10^{-7} m \tag{7.243}$$

From Eq. (240):

$$\frac{r_{2s}(\text{vac}) - r_{2p}(\text{vac})}{(r + r_{2p}(\text{vac}))(r + r_{2s}(\text{vac}))} = \frac{1}{2\pi^{3/2}} \frac{\hbar}{mc} \frac{1}{r^2}. \tag{7.244}$$

Eq. (238) implies:

$$r \gg r_{2s}(\text{vac}) \sim r_{2p}(\text{vac}) \tag{7.245}$$

so in this approximation Eq. (7.244) becomes:

$$r_{2s}(\text{vac}) - r_{2p}(\text{vac}) = \frac{1}{2\pi^{3/2}} \frac{\hbar}{mc} \tag{7.246}$$

i.e.

$$r_{2s}(\text{vac}) - r_{2p}(\text{vac}) = \frac{1}{4\pi^{5/2}} \frac{\hbar}{mc} \tag{7.247}$$

where the standard Compton wavelength is:

$$\frac{h}{mc} = 2.426 \times 10^{-12} m. \tag{7.248}$$

Thus we arrive at:

$$r_{2s}(\text{vac}) - r_{2p}(\text{vac}) = 3.48 \times 10^{-13} m. \tag{7.249}$$

This is a plausible result because the classical electron radius is:

$$r(\text{classical}) = \frac{1}{4\pi\epsilon_0}\frac{e^2}{mc^2} = 2.818 \times 10^{-15} m \tag{7.250}$$

and the Bohr radius is:

$$a = 5.292 \times 10^{-11} m. \tag{7.251}$$

So the radiative correction perturbs the electron orbitals by about ten times the classical radius of the electron and by orders less than the Bohr radius. The ECE theory also shows why the Lamb shift is constant as observed because for a given orientation:

$$\cos\theta = 1 \tag{7.252}$$

the shift is determined completely by 1/r within a constant of proportionality defined by:

$$\zeta = \frac{1}{32\pi^{3/2}}\frac{\alpha}{a}\frac{\hbar}{mc}. \tag{7.253}$$

The angular dependence of the Lamb shift in H was also considered [1–12] and the method extended to the helium atom. Finally, consideration was given to how radiative corrections may be amplified by spin connection resonance.

Therefore in summary, the accuracy of the fine structure constant is determined experimentally by that of the Planck constant h. The LEAST accurately known constant determines the accuracy of the fine structure constant, as should be well known. There is no way that any theory can determine the fine structure constant more accurately than it is known experimentally. Therefore ECE theory sets out to use the experimental accuracy in α. The latter is determined by the accuracy in h as argued. This was done as simply as possible in accordance with Ockham's Razor. QED on the other hand is hugely elaborate, and its claims to be an accurate fundamental theory are unjustifiable. There can be no experimental justification for the existence of virtual particle pairs because of the gross internal inconsistencies in data reviewed in this section. Additionally, there are several ad hoc assumptions in the theory of QED itself.

7.9 Summary of Advances Made by ECE Theory, and Criticisms of the Standard Model

In this section a summary is given of the main advances of ECE theory over the past five years since inception in Spring 2003, and also a summary of implied criticisms of the current model of physics known as the standard model.

The major advantage of ECE theory is that it relies on the original principles of the theory and philosophy of relativity, without any extraneous input. This approach adheres therefore to the Ockham Razor of philosophy, the simpler the better. It also adheres to the principles of Francis Bacon, that every theory is tested experimentally, and not against another theory.

1. The inverse Faraday effect. This is described by the spinning of space-time and the B(3) field (see www.aias.us Omnia Opera) from first principles. In the standard model the effect cannot be described self consistently and cannot be described without an ad hoc conjugate product of non-linear optics. The latter is introduced phenomenologically in the standard model of non-linear optics, a theory of special relativity. In ECE theory the B(3) spin field indicates that optics and spectroscopy are parts of a generally covariant unified field theory (GCUFT).

2. The Aharonov Bohm effects. These are described self consistently in ECE through the spin connection using the principles of general relativity. As shown in this review paper, the standard model description of the Aharonov Bohm (AB) effects is at best controversial and at worst erroneous. A satisfactory description of the AB effects in ECE leads to a new understanding of quantum entanglement and one photon interferometry.

3. The polarization change in light deflected by gravitation. This is not described in the Einstein Hilbert (EH) equation of the standard model because it is a purely kinematic equation relying on the gravitational attraction between a photon and a mass M, for example the solar mass. In ECE all the optical effects of gravitation are developed self consistently from the Bianchi identity of Cartan geometry.

4. The Faraday disk generator. This is described in ECE through the Cartan torsion of space-time introduced by mechanical spin, this concept is missing entirely from the standard model, which still cannot describe the 1831 Faraday disk generator.

5. The Sagnac effect and ring laser gyro. These are described again by the Cartan torsion of space-time introduced by spinning the platform of the Sagnac interferometer. The Sagnac effect is very difficult to understand using Maxwell Heaviside theory, but is easily described in ECE theory. The latter offers a far simpler description than other available attempts at explaining the Sagnac effect of 1913.

6. The velocity curve of a spiral galaxy. This is described straightforwardly and simply in ECE theory by introducing again the concept of constant space-time torsion. The spiral galaxies main features cannot be

described at all in the standard model. This is because the latter relies on an ad hoc “dark matter” that originates in the EH equation. The latter is self inconsistent as argued in this review paper.

7. The topological phases such as the Berry phase. These are derived in ECE from first principles, and are rigorously inter-related. In the standard model their description is incomplete, and in the case of the electromagnetic phase, erroneous.

8. The electromagnetic phase. This is described self consistently in terms of the B(3) spin field of ECE theory using general relativity. In the standard model the phase is incompletely determined mathematically, and violates parity in simple effects such as reflection.

9. Snell’s law, reflection, refraction, diffraction, interferometry and related optical effects. These can be described correctly only in a GCUFT such as ECE. In the standard model the theory of reflection for example, does not fit with parity inversion symmetry due to the neglect of the B(3) spin field.

10. Improvements to the Heisenberg Uncertainty Principle. Various experiments have shown that the principle is incorrect by orders of magnitude, in ECE theory it is developed with causal and objective general relativity and the concept of quantum of action density.

11. The unification of wave mechanics and general relativity. This has been achieved straightforwardly in ECE theory through the use of Cartan geometry. In the standard model it is still not possible to make this basic unification. The Dirac, Proca and other wave equations are limits of the ECE wave equation, which is derived easily from the tetrad postulate of Cartan. So ECE allows the description of the effect of gravitation on such equations, and on such phenomena as the Sagnac effect. This is again not possible in the standard model.

12. Description of particle interaction. This description is achieved with simultaneous ECE equations without assuming the existence either of virtual particles or of the Higgs mechanism. The Higgs boson still has not been verified experimentally, and its energy is not defined theoretically.

13. The photon mass. The Proca equation is derived easily from Cartan geometry using the simple hypothesis that the potential is proportional to the Cartan tetrad. In the standard model the Proca equation is directly incompatible with gauge invariance, a fundamental self-inconsistency of the standard model, one of many self - inconsistencies.

14. Replacement of the gauge principle. The gauge principle is not tenable in a GCUFT such as ECE because the potential in ECE is physically meaningful as in Faraday's original electrotonic state. Abandonment of the gauge principle allows a return to the earlier concepts of relativity without introducing an ad hoc and abstract internal space as in Yang Mills theory. In ECE theory the tetrad postulate is invariant under the general coordinate transform, and this is the principle that governs the potential field in ECE.

15. Description of the electro-weak field without the Higgs mechanism. This becomes possible in a relatively straightforward manner by using simultaneous ECE equations. The Higgs mechanism is ad hoc, and to date unproven experimentally, indeed it is unprovable because an energy cannot be assigned to the Higgs boson. The Higgs boson, having no well defined energy, cannot be proven experimentally by particle collision methods, however powerful the accelerator. No sign of a Higgs boson was found at LEP, and to date no sign at the CERN heavy hadron collider.

16. Description of neutrino oscillations. This is a relatively simple exercise in ECE theory but in the standard model neutrino oscillations remained deeply controversial for years because of adherence to the assumption that the neutrino had no mass. In ECE all particles have mass - a fundamental requirement of relativity.

17. The generally covariant description of the laws of classical electrodynamics. These laws become laws of general relativity and a unified field theory, they are no longer laws of a Minkowski space-time as in the standard model. The concept of spin connection and spin connection resonance make important advances and potentially open up new sources of energy.

18. Derivation of the quark model from general relativity. This has been achieved in ECE theory by using an SU(n) representation space in the wave and field equations. In the standard model the quark theory is one of special relativity. QCD relies on ad hoc concepts such as renormalization, which as argued in section 7.8, are not internally consistent with data. The situation in QCD is worse than that in QED.

19. Derivation of the quantum theory of electrodynamics. This is achieved using the wave equation and the ECE hypothesis, resulting in a generally covariant version of the Proca equation with non-zero photon mass. In so doing a minimum particle volume is always present, so there are no point particles and no need for re-normalization. Feynman's QED is abandoned as described in Section 7.8.

20. The origin of particle spin. This is traced to geometry and particle spins of all kinds are successfully incorporated into general relativity. This is not possible with the EH equation, which has been shown to be fundamentally flawed.

21. Development of cosmology. The major advantage of considering the Cartan torsion becomes abundantly clear in cosmology, in particular the explanation of the spiral galaxies. Cosmology based on the EH equation has been shown to be meaningless in several different ways.

22. No Singularities. This is a flawed concept introduced by incorrect solutions of the EH equation. The latter is itself inconsistent with the Bianchi identity. In ECE theory the concept of Big Bang is replaced with the steady state universe with local oscillations. Similarly there are no black holes and no dark matter. Applications of experimentally untestable string theory to these concepts multiplies the heavily criticized obscurantism of modern physics.

23. Explanation of the red shift. This is a simple optical effect in ECE theory, there can be different red shifts in equidistant objects. ECE also offers a new explanation of the background radiation if indeed it is not an artifact of the Earth's atmosphere as some scholars now think.

24. Spin connection resonance. This concept is made possible in ECE and has been offered as an explanation of Tesla's well known giant resonances and similar reports of over a century of work. The latter cannot be explained in the standard model yet is potentially a source of major new energy.

25. Spinning Space-time. This is a key new concept of electrodynamics, akin to curving space-time in gravitation. ECE has made the major discovery that the two concepts are linked ineluctably in relativity, and this has led to the abandonment of the EH equation. A suggested replacement of the equation has been made in recent work.

26. Counter gravitation. It has been shown that this is feasible only by using resonance methods based again on the spin connection and the interaction of gravitation and electromagnetism. It needs a GCUFT such as ECE to begin to describe this interaction of the fundamental fields of force.

27. Gravitational Dynamics. These are developed in ECE in the same way as electrodynamics. For example it is relatively easy to show that there is a gravitational equivalent of the Faraday law of induction, as indeed observed recently. A new approach to the derivation of the acceleration due to gravity has also been made possible, an approach based on the rigorous Bianchi identity given by Cartan.

28. Quantum Entanglement. These well known quantum effects can be understood using the spin connection of ECE in a similar way to the AB effects. Similarly the argument can be extended to such phenomena as one photon Young interferometry. In the standard model they are very difficult to understand because of the use of a Minkowski space-time with no connection. In the standard model these are mysterious effects with many offered explanations, none convincing.

29. Superconductivity and related fields. The equations governing the behavior of classes of materials are all derived in ECE from geometry, so there is an overall self-consistency which is often missing in the standard model. For example plasma, semiconductors, superconductors, and so forth.

30. Quantum Field Theory. This is developed in ECE entirely without he use of string theory or super-symmetry. String theory in particular has been heavily criticized because it cannot be tested experimentally and makes no new predictions at all. Such matters as photon mass theory, canonical quantization, and creation annihilation operator theory are all improved by ECE theory.

31. Radiative Corrections. These are understood in a far simpler way in ECE theory as discussed in Section 7.8. The claims of QED theory have been shown to be false by several orders of magnitude, and the complacency of the standard physics community heavily criticized thereby.

32. Fermion Resonance. New methods of detecting and developing fermion resonance have been developed and it is shown that such resonance can be induced without the use of magnets. This method is known as radiatively induced fermion resonance (RFR). It has been clearly understood to be due to the B(3) field.

33. Ubiquitous B(3) Field. It has been shown that the B(3) field is the one responsible for the general relativistic description of the electromagnetic phase, so it occurs throughout optics and spectroscopy, in everyday phenomena such as reflection.

34. Fundamental Advances in Geometry. In the course of developing ECE theory it has been shown that there is only one Bianchi identity, not two unrelated identities used in the standard model. It has also been shown rigorously in many ways that the Bianchi identity has a Hodge dual. These properties lead to field equations with duality symmetry. Such a symmetry is not present in the standard model.

35. Self Consistency of Cartan's geometry. This has been tested in many ways, and it has been shown that the tetrad postulate is rigorously self consistent and fundamental to physics. Numerous tests of self consistency have been made.

36. Development of Gravitational Relativity. It has been shown that the correct description of gravitation requires the Bianchi identity of Cartan, which links torsion to curvature. The Bianchi identity used by Einstein has been shown to be incomplete, and using computer algebra, it has been shown that the EH equation is inconsistent with the use of a Christoffel connection and symmetric metric. It has also been shown that claimed solutions of the EH equation are often incorrect mathematically. Finally it has been shown that the Ricci flat space-time is incompatible with the Einsteinian equivalence principle. Therefore the standard model literature has to be read with considerable caution. Many claims of the standard model have not stood up to scrutiny, whereas ECE has developed strongly.

Acknowledgments

The British Government is thanked for the award of a Civil List Pension, and the staff of AIAS for five years of voluntary work on ECE theory and websites. The staff of the Telesio-Galilei Association are thanked for appointments and logistical support.

A

Appendix 1: Homogeneous Maxwell Heaviside Equations

In the first of several technical appendices it is shown how to translate the homogeneous Maxwell Heaviside (MH) from tensor to vector notation, giving details that are rarely found in textbooks. In tensor notation the equation is:

$$\partial_\mu \widetilde{F}^{\mu\nu} = 0 \tag{A.1}$$

and involves the Hodge dual of the 4 x 4 field tensor, defined as follows:

$$\widetilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}. \tag{A.2}$$

Indices are raised using the Minkowski metric:

$$\widetilde{F}^{\mu\nu} = g^{\mu\kappa}g^{\nu\rho}\widetilde{F}_{\kappa\rho} \tag{A.3}$$

where:

$$g_{\mu\nu} = g^{\mu\nu} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}. \tag{A.4}$$

Therefore the Hodge dual is:

$$\widetilde{F}^{\mu\nu} = \begin{bmatrix} 0 & cB^1 & cB^2 & cB^3 \\ -cB^1 & 0 & -E^3 & E^2 \\ -cB^2 & E^3 & 0 & -E^1 \\ -cB^3 & -E^2 & E^1 & 0 \end{bmatrix} \tag{A.5}$$

For example:

$$\widetilde{F}_{01} = \frac{1}{2}(\epsilon_{0123}F^{23} + \epsilon_{0132}F^{32}) = F^{23} \tag{A.6}$$

and

$$\widetilde{F}^{01} = g^{00}g^{11}\widetilde{F}_{01} = -\widetilde{F}_{10}. \tag{A.7}$$

The homogeneous laws of classical electrodynamics are the Gauss law and Faraday law of induction. They are obtained as follows by choice of indices. The Gauss law is obtained by choosing:

$$\nu = 0 \tag{A.8}$$

and so

$$\partial_1\widetilde{F}^{10} + \partial_2\widetilde{F}^{20} + \partial_3\widetilde{F}^{30} = 0. \tag{A.9}$$

In vector notation this is

$$\boldsymbol{\nabla} \cdot \boldsymbol{B} = 0. \tag{A.10}$$

The Faraday law of induction is obtained by choosing:

$$\nu = 1, 2, 3 \tag{A.11}$$

and is three component equations:

$$\partial_0\widetilde{F}^{01} + \partial_2\widetilde{F}^{21} + \partial_3\widetilde{F}^{31} = 0 \tag{A.12}$$

$$\partial_0\widetilde{F}^{02} + \partial_1\widetilde{F}^{12} + \partial_3\widetilde{F}^{32} = 0 \tag{A.13}$$

$$\partial_0\widetilde{F}^{03} + \partial_1\widetilde{F}^{13} + \partial_2\widetilde{F}^{23} = 0. \tag{A.14}$$

These can be condensed into one vector equation, which is

$$\boldsymbol{\nabla} \times \boldsymbol{E} + \frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{0}. \tag{A.15}$$

The differential form, tensor and vector notations are summarized as follows:

$$d \wedge F = 0 \rightarrow \partial_\mu \widetilde{F}^{\mu\nu} = 0 \rightarrow \boldsymbol{\nabla} \cdot \boldsymbol{B} = 0 \tag{A.16}$$
$$\boldsymbol{\nabla} \times \boldsymbol{E} + \frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{0}$$

The homogeneous laws of classical electrodynamics are most elegantly represented by the differential form notation, but most usefully represented by the vector notation.

B

Appendix 2: The Inhomogeneous Equations

The inhomogeneous laws are the Coulomb law and the Ampère Maxwell law. In tensor notation they are condensed into one equation:

$$\partial_\mu F^{\mu\nu} = \frac{1}{\epsilon_0} J^\nu \tag{B.1}$$

where the charge current density is:

$$J^\nu = \left(\rho, \frac{\boldsymbol{J}}{c}\right) \tag{B.2}$$

and where the partial derivative is:

$$\partial_\mu = \left(\frac{1}{c}\frac{\partial}{\partial t}, \frac{\partial}{\partial X}, \frac{\partial}{\partial Y}, \frac{\partial}{\partial Z}\right) \tag{B.3}$$

The field tensor is:

$$F^{\mu\nu} = \begin{bmatrix} 0 & -E^1 & -E^2 & -E^3 \\ E^1 & 0 & -cB^3 & cB^2 \\ E^2 & cB^3 & 0 & -cB^1 \\ E^3 & -cB^2 & cB^1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & F^{01} & F^{02} & F^{03} \\ F^{10} & 0 & F^{12} & F^{13} \\ F^{20} & F^{21} & 0 & F^{23} \\ F^{30} & F^{31} & F^{32} & 0 \end{bmatrix} \tag{B.4}$$

and in S.I. units:

$$\epsilon_0 \mu_0 = \frac{1}{c^2}. \tag{B.5}$$

In this notation:

$$\left.\begin{aligned} E_X &= E^1 = F^{10}, \\ E_Y &= E^2 = F^{20}, \\ E_Z &= E^3 = F^{30}, \end{aligned}\right\} \tag{B.6}$$

and so on. The Coulomb law is obtained from choosing:

$$\nu = 0 \tag{B.7}$$

so that:

$$\partial_1 F^{10} + \partial_2 F^{20} + \partial_3 F^{30} = \frac{1}{\epsilon_0} J^0. \tag{B.8}$$

In vector component notation this is:

$$\frac{\partial E_X}{\partial X} + \frac{\partial E_Y}{\partial Y} + \frac{\partial E_Z}{\partial Z} = \frac{1}{\epsilon_0}\rho \tag{B.9}$$

which in vector notation is:

$$\boldsymbol{\nabla} \cdot \boldsymbol{E} = \frac{\rho}{\epsilon_0}. \tag{B.10}$$

The Ampère Maxwell law is obtained from choosing

$$\nu = 1, 2, 3 \tag{B.11}$$

which gives three equations:

$$\partial_0 F^{01} + \partial_2 F^{21} + \partial_3 F^{31} = \frac{1}{\epsilon_0} J^1 \tag{B.12}$$

$$\partial_0 F^{02} + \partial_1 F^{12} + \partial_3 F^{32} = \frac{1}{\epsilon_0} J^2 \tag{B.13}$$

$$\partial_0 F^{03} + \partial_1 F^{13} + \partial_2 F^{23} = \frac{1}{\epsilon_0} J^3. \tag{B.14}$$

In vector component notation these are:

$$-\frac{1}{c}\frac{\partial E_X}{\partial t} + c\left(\frac{\partial B_Z}{\partial Y} - \frac{\partial B_Y}{\partial Z}\right) = \frac{1}{\epsilon_0} J_X \tag{B.15}$$

$$-\frac{1}{c}\frac{\partial E_Y}{\partial t} + c\left(\frac{\partial B_X}{\partial Z} - \frac{\partial B_Z}{\partial X}\right) = \frac{1}{\epsilon_0} J_Y \tag{B.16}$$

$$-\frac{1}{c}\frac{\partial E_Z}{\partial t} + c\left(\frac{\partial B_Y}{\partial X} - \frac{\partial B_X}{\partial Y}\right) = \frac{1}{\epsilon_0} J_Z. \tag{B.17}$$

The definition of the vector curl is

$$\boldsymbol{\nabla} \times \boldsymbol{B} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ \partial/\partial Z & \partial/\partial Y & \partial/\partial Z \\ B_X & B_Y & B_Z \end{vmatrix} \tag{B.18}$$

$$= \left(\frac{\partial B_Z}{\partial Y} - \frac{\partial B_Y}{\partial Z}\right)\boldsymbol{i} - \left(\frac{\partial B_Z}{\partial X} - \frac{\partial B_X}{\partial Z}\right)\boldsymbol{j} + \left(\frac{\partial B_Y}{\partial X} - \frac{\partial B_X}{\partial Y}\right)\boldsymbol{k},$$

so it is seen that the three equations (B.15) to (B.17) can be condensed into one vector equation:

$$\boldsymbol{\nabla} \times \boldsymbol{B} - \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t} = \mu_0 \boldsymbol{J} \tag{B.19}$$

which is the Ampère Maxwell Law. The differential form, tensor and vector formulations of the inhomogeneous laws of standard model classical electrodynamics are summarized as follows:

$$d \wedge \widetilde{F} = \frac{J}{\epsilon_0} \rightarrow \partial_\mu F^{\mu\nu} = \frac{J^\nu}{\epsilon_0} \rightarrow \boldsymbol{\nabla} \cdot \boldsymbol{E} = \frac{\rho}{\epsilon_0}, \tag{B.20}$$

$$\boldsymbol{\nabla} \times \boldsymbol{B} - \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t} = \mu_0 \boldsymbol{J}.$$

C

Appendix 3: Some Examples of Hodge Duals in Minkowski Space-Time

In Minkowski space-time the Hodge dual of a rank two anti-symmetric tensor (two-form) in four dimensions is defined by:

$$\widetilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}. \tag{C.1}$$

For example, the B(3) field is defined by:

$$F^{\mu\nu} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -cB^{(3)} & 0 \\ 0 & cB^{(3)} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{C.2}$$

so its Hodge dual is:

$$\widetilde{F}^{\mu\nu} = \begin{bmatrix} 0 & 0 & 0 & cB^{(3)} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -cB^{(3)} & 0 & 0 & 0 \end{bmatrix}. \tag{C.3}$$

It can be seen that the Hodge dual of the B(3) field does not imply the existence of an E(3) field, it is a re-arrangement of matrix elements. There appears to be no experimental evidence for the existence of a radiated E(3) field. In other words there is no electric equivalent of the inverse Faraday effect, and there is no electric equivalent of the Faraday effect.

The radiated B(3) field is generated by the spin connection, the static magnetic field of the standard model is defined without the spin connection as follows:

$$\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A}. \tag{C.4}$$

In tensor form the static magnetic field is:

$$F^{\mu\nu} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -cB_Z & cB_Y \\ 0 & cB_Z & 0 & -cB_X \\ 0 & -cB_Y & cB_X & 0 \end{bmatrix} \tag{C.5}$$

whose Hodge dual is:

$$\widetilde{F}^{\mu\nu} = \begin{bmatrix} 0 & cB_X & cB_Y & cB_Z \\ -cB_X & 0 & 0 & 0 \\ -cB_Y & 0 & 0 & 0 \\ -cB_Z & 0 & 0 & 0 \end{bmatrix}. \tag{C.6}$$

Again, the Hodge dual does not generate an electric field. In ECE theory the magnetic field in vector notation always includes the spin connection vector as follows:

$$\boldsymbol{B} = \boldsymbol{\nabla} \times \boldsymbol{A} - \boldsymbol{\omega} \times \boldsymbol{A} \tag{C.7}$$

and this is true for all types of magnetic field.

D

Appendix 4: Standard Tensorial Formulation of the Homogeneous Maxwell Heaviside Field Equations

The standard tensorial formulation developed in this appendix is:

$$\partial_\mu \widetilde{F}^{\mu\nu} = \partial^\mu \widetilde{F}_{\mu\nu} = 0 \tag{D.1}$$

and is needed as a baseline for the development of ECE theory. The field tensor is defined as:

$$F^{\mu\nu} = \begin{bmatrix} 0 & cB^1 & cB^2 & cB^3 \\ -cB^1 & 0 & -E^3 & E^2 \\ -cB^2 & E^3 & 0 & -E^1 \\ -cB^3 & -E^2 & E^1 & 0 \end{bmatrix}. \tag{D.2}$$

where, in standard covariant - contravariant notation and in S.I. units:

$$\partial_\mu = \left(\frac{1}{c}\frac{\partial}{\partial t}, \frac{\partial}{\partial X}, \frac{\partial}{\partial Y}, \frac{\partial}{\partial Z} \right), \tag{D.3}$$

$$\partial^\mu = \left(\frac{1}{c}\frac{\partial}{\partial t}, -\frac{\partial}{\partial X}, -\frac{\partial}{\partial Y}, -\frac{\partial}{\partial Z} \right), \tag{D.4}$$

$$x^\mu = (ct, X, Y, Z), \tag{D.5}$$

$$x_\mu = (ct, -X, -Y, -Z). \tag{D.6}$$

The metric and inverse metric tensors in Minkowski space-time are equal, and are given by:

$$g_{\mu\nu} = g^{\mu\nu} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}. \tag{D.7}$$

Indices are raised and lowered with the metric, for example:

$$\widetilde{F}_{\mu\nu} = g_{\mu\rho}\ g_{\nu\sigma}\ \widetilde{F}^{\rho\sigma} \tag{D.8}$$

where

$$g_{00} = 1, g_{11} = g_{22} = g_{33} = -1 \tag{D.9}$$

and so on. Therefore:

$$\widetilde{F}_{01} = g_{00}\ g_{11} F^{01} = -\widetilde{F}^{01},\ \widetilde{F}_{02} = -\widetilde{F}^{02}, \widetilde{F}_{03} = -\widetilde{F}^{03} \tag{D.10}$$

and so on. Therefore:

$$\widetilde{F}^{\mu\nu} = \begin{bmatrix} 0 & cB_X & cB_Y & cB_Z \\ -cB_x & 0 & -E_Z & E_Y \\ -cB_Y & E_Z & 0 & -E_X \\ -cB_Z & -E_Y & E_X & 0 \end{bmatrix}, \widetilde{F}_{\mu\nu} = \begin{bmatrix} 0 & -cB_X & -cB_Y & -cB_Z \\ cB_X & 0 & -E_Z & E_Y \\ cB_Y & E_Z & 0 & -E_X \\ cB_Z & -E_Y & E_X & 0 \end{bmatrix}. \tag{D.11}$$

If the field tensor is defined with raised indices then the Gauss law is given by:

$$\partial_1 \widetilde{F}^{10} + \partial_2 \widetilde{F}^{20} + \partial_3 \widetilde{F}^{30} = 0 \tag{D.12}$$

i.e.:

$$-\boldsymbol{\nabla} \cdot \boldsymbol{B} = 0 \tag{D.13}$$

and the Faraday law of induction is given by

$$\partial_0 \widetilde{F}^{01} + \partial_2 \widetilde{F}^{21} + \partial_3 \widetilde{F}^{31} = 0 \tag{D.14}$$

$$\partial_0 \widetilde{F}^{02} + \partial_1 \widetilde{F}^{12} + \partial_3 \widetilde{F}^{32} = 0 \tag{D.15}$$

$$\partial_0 \widetilde{F}^{03} + \partial_1 \widetilde{F}^{13} + \partial_2 \widetilde{F}^{23} = 0 \tag{D.16}$$

i.e.

$$\boldsymbol{\nabla} \times \boldsymbol{E} + \frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{0}. \tag{D.17}$$

In almost all textbooks the Gauss law is written as:

$$\boldsymbol{\nabla} \cdot \boldsymbol{B} = 0, \tag{D.18}$$

but the above is the rigorously correct result.

Similarly if the field tensor is written with lowered indices, : i.e.:

$$\partial^\mu \widetilde{F}_{\mu\nu} = 0 \tag{D.19}$$

the rigorously correct result is:

$$\begin{aligned} -\boldsymbol{\nabla} \cdot \boldsymbol{B} &= 0 \\ -\left(\boldsymbol{\nabla} \times \boldsymbol{E} + \frac{\partial \boldsymbol{B}}{\partial t}\right) &= \boldsymbol{0} \end{aligned} \tag{D.20}$$

The minus signs are always omitted in textbook material.

If the field tensor is defined with indices raised:

$$\partial_\mu F^{\mu\nu} = \frac{J^\nu}{\epsilon_0} \tag{D.21}$$

where:

$$F^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\widetilde{F}_{\rho\sigma}. \tag{D.22}$$

The totally anti-symmetric unit tensor in four-dimensions has elements:

$$\begin{aligned} \epsilon^{0123} &= -\epsilon^{1230} = \epsilon^{2301} = -\epsilon^{3012} = 1 \\ \epsilon^{1023} &= -\epsilon^{2130} = \epsilon^{3201} = -\epsilon^{0312} = -1 \\ \epsilon^{1032} &= -\epsilon^{2103} = \epsilon^{3210} = -\epsilon^{0321} = 1 \\ \epsilon^{1302} &= -\epsilon^{2013} = \epsilon^{3120} = -\epsilon^{0231} = -1 \end{aligned} \tag{D.23}$$

So for example:

$$\begin{aligned} F^{01} &= \frac{1}{2}\left(\epsilon^{0123}\widetilde{F}_{23} + \epsilon^{0132}\widetilde{F}_{32}\right) = \widetilde{F}_{23} = -E_X \\ F^{02} &= \frac{1}{2}\left(\epsilon^{0231}\widetilde{F}_{31} + \epsilon^{0213}\widetilde{F}_{13}\right) = \widetilde{F}_{31} = -E_Y \\ F^{03} &= \frac{1}{2}\left(\epsilon^{0312}\widetilde{F}_{12} + \epsilon^{0321}\widetilde{F}_{21}\right) = \widetilde{F}_{12} = -E_Z \\ F^{23} &= \frac{1}{2}\left(\epsilon^{2301}\widetilde{F}_{01} + \epsilon^{2310}\widetilde{F}_{10}\right) = \widetilde{F}_{01} = -cB_X \\ F^{13} &= \frac{1}{2}\left(\epsilon^{1302}\widetilde{F}_{02} + \epsilon^{1320}\widetilde{F}_{20}\right) = -\widetilde{F}_{02} = cB_Y \\ F^{12} &= \frac{1}{2}\left(\epsilon^{1230}\widetilde{F}_{30} + \epsilon^{1203}\widetilde{F}_{03}\right) = \widetilde{F}_{03} = -cB_Z \end{aligned} \tag{D.24}$$

Therefore:

$$F^{\mu\nu} = \begin{bmatrix} 0 & -E_X & E_Y & -E_Z \\ E_X & 0 & -cB_Z & cB_Y \\ E_Y & cB_Z & 0 & -cB_X \\ E_Z & -cB_Y & cB_X & 0 \end{bmatrix} = \begin{bmatrix} 0 & -E^1 & -E^2 & -E^3 \\ E^1 & 0 & -cB^3 & cB^2 \\ E^2 & cB^3 & 0 & -cB^1 \\ E^3 & -cB^2 & cB^1 & 0 \end{bmatrix}. \tag{D.25}$$

The charge current density is:

$$J^\nu = \left(\rho, \frac{J}{c}\right). \tag{D.26}$$

The Coulomb law is:

$$\partial_1 F^{10} + \partial_2 F^{20} + \partial_3 F^{30} = \frac{1}{\epsilon_0} J^0 = \frac{\rho}{\epsilon_0} \tag{D.27}$$

which in vector notation is:

$$\boldsymbol{\nabla} \cdot \boldsymbol{E} = \frac{\rho}{\epsilon_0}. \tag{D.28}$$

The Ampère Maxwell law is:

$$\partial_0 F^{01} + \partial_2 F^{21} + \partial_3 F^{31} = J^1/\epsilon_0 \tag{D.29}$$

$$\partial_0 F^{02} + \partial_1 F^{12} + \partial_3 F^{32} = J^2/\epsilon_0 \tag{D.30}$$

$$\partial_0 F^{03} + \partial_1 F^{13} + \partial_2 F^{23} = J^3/\epsilon_0 \tag{D.31}$$

i.e.:

$$-\frac{1}{c}\frac{\partial E_X}{\partial t} + c\left(\frac{\partial B_Z}{\partial Y} - \frac{\partial B_Y}{\partial Z}\right) = \frac{1}{\epsilon_0} J_X \tag{D.32}$$

$$-\frac{1}{c}\frac{\partial E_Y}{\partial t} + c\left(\frac{\partial B_X}{\partial Z} - \frac{\partial B_Z}{\partial X}\right) = \frac{1}{\epsilon_0} J_Y \tag{D.33}$$

$$-\frac{1}{c}\frac{\partial E_Z}{\partial t} + c\left(\frac{\partial B_Y}{\partial X} - \frac{\partial B_X}{\partial Y}\right) = \frac{1}{\epsilon_0} J_Z \tag{D.34}$$

which is:

$$\boldsymbol{\nabla} \times \boldsymbol{B} - \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t} = \mu_0 \, \boldsymbol{J}. \tag{D.35}$$

Therefore the standard adopted is:

$$\partial_\mu F^{\mu\nu} = \frac{1}{\epsilon_0} J^\nu \rightarrow \boldsymbol{\nabla} \cdot \boldsymbol{E} = \rho/\epsilon_0 \qquad \text{(D.36)}$$

$$\boldsymbol{\nabla} \times \boldsymbol{B} - \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t} = \mu_0 \, \boldsymbol{J}.$$

To be precisely correct therefore, the tensorial formulation of the four laws of electrodynamics is:

$$\partial_\mu F^{\mu\nu} = \frac{1}{\epsilon_0} J^\nu \qquad \text{(D.37)}$$

$$-\partial^\mu \widetilde{F}_{\mu\nu} = 0 \qquad \text{(D.38)}$$

where:

$$F^{\mu\nu} = \begin{bmatrix} 0 & -E^1 & -E^2 & -E^3 \\ E^1 & 0 & -cB^3 & cB^2 \\ E^2 & cB^3 & 0 & -cB^1 \\ E^3 & -cB^2 & cB^1 & 0 \end{bmatrix} \qquad \text{(D.39)}$$

and

$$\widetilde{F}^{\mu\nu} = \begin{bmatrix} 0 & -cB^1 & -cB^2 & -cB^3 \\ cB^1 & 0 & -E^3 & E^2 \\ cB^2 & E^3 & 0 & -E^1 \\ -cB^3 & -E^2 & E^1 & 0 \end{bmatrix}. \qquad \text{(D.40)}$$

In free space:

$$\partial_\mu F^{\mu\nu} = 0, \qquad \text{(D.41)}$$

$$-\partial^\mu \widetilde{F}_{\mu\nu} = 0. \qquad \text{(D.42)}$$

The free space equations are duality invariant under:

$$F^{\mu\nu} \leftrightarrow \widetilde{F}_{\mu\nu} \qquad \text{(D.43)}$$

i.e.:

$$E_X \leftrightarrow cB_X, E_Y \leftrightarrow cB_Y, E_Z \leftrightarrow cB_Z. \qquad \text{(D.44)}$$

The Hodge dual transform is:

$$F^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\widetilde{F}_{\rho\sigma} \tag{D.45}$$

and can be summarized as:

$$\boxed{-\partial^{\mu}\widetilde{F}_{\mu\nu} = 0} \longleftrightarrow \boxed{\partial_{\mu}F^{\mu\nu} = 0}$$

$$\boxed{\begin{gathered}\nabla \cdot \boldsymbol{B} = 0 \\ \frac{\partial \boldsymbol{B}}{\partial t} + \nabla \times \boldsymbol{E} = \boldsymbol{0}\end{gathered}} \longleftrightarrow \boxed{\begin{gathered}\nabla \cdot \boldsymbol{E} = 0 \\ \nabla \times \boldsymbol{B} - \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t} = \boldsymbol{0}\end{gathered}}$$

The presence of matter and charge-current density breaks the duality symmetry, or duality invariance.

E

Appendix 5: Illustrating the Meaning of the Connection with Rotation in a Plane

Consider the clockwise rotation in a plane of a vector V^1 to V^2 as in Fig. (7.E1). This rotation is carried out by moving the vector and keeping the frame of reference fixed. This process is equivalent to keeping the vector fixed and rotating the frame of reference anti-clockwise through an equal angle θ. In Cartesian coordinates:

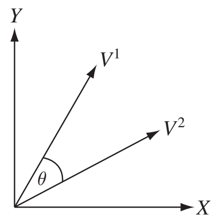


Fig. E.1. Rotation of a Vector in a Plane.

and

$$\boldsymbol{V}^1 = V_X^1 \boldsymbol{i} + V_Y^1 \boldsymbol{j} \tag{E.1}$$

$$\boldsymbol{V}^2 = V_X^2 \boldsymbol{i} + V_Y^2 \boldsymbol{j} \tag{E.2}$$

where:

$$|\boldsymbol{V}^1| = |\boldsymbol{V}^2|, \tag{E.3}$$

$$|\boldsymbol{V}^1| = (V_X^{1\ 2} + V_Y^{1\ 2})^{\frac{1}{2}}, \tag{E.4}$$

$$|\boldsymbol{V}^2| = (V_X^{2\ 2} + V_Y^{2\ 2})^{\frac{1}{2}}. \tag{E.5}$$

This is a rotation in which the frame is fixed, i.e. the Cartesian unit vectors i and j do not change. The rotation could equally well be represented by:

$$\boldsymbol{V}^1 = V_X \boldsymbol{i}_1 + V_Y \boldsymbol{j}_1, \tag{E.6}$$

$$\boldsymbol{V}^2 = V_X \boldsymbol{i}_2 + V_Y \boldsymbol{j}_2, \tag{E.7}$$

and in this case the vector is fixed and the frame rotated anti-clockwise. We now have:

$$|\boldsymbol{V}^1| = |\boldsymbol{V}^2| = (V_X^2 + V_Y^2)^{\frac{1}{2}} \tag{E.8}$$

because:

$$\left.\begin{array}{r} \boldsymbol{i}_1 \cdot \boldsymbol{i}_1 = \boldsymbol{i}_2 \cdot \boldsymbol{i}_2 = 1 \\ \boldsymbol{j}_1 \cdot \boldsymbol{j}_1 = \boldsymbol{j}_2 \cdot \boldsymbol{j}_2 = 1 \end{array}\right\}. \tag{E.9}$$

The invariance under rotation of the complete vector field is true in both cases:

$$\begin{array}{ll} \text{a)} & V^{12} = V_X^{12} + V_Y^{12} = V_X^{22} + V_Y^{22} = V^{22}, \\ \text{b)} & V^{12} = V_X^2 + V_Y^2 = V^{22}. \end{array} \tag{E.10}$$

The rotation can also be represented by:

$$\begin{bmatrix} V_X^1 \\ V_Y^1 \\ V_Z^1 \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_X^2 \\ V_Y^2 \\ V_Z^2 \end{bmatrix} \tag{E.11}$$

i.e.:

$$V_X^1 = V_X^2 \cos\theta + V_Y^2 \sin\theta \tag{E.12}$$

$$V_Y^1 = -V_X^2 \sin\theta + V_Y^2 \cos\theta \tag{E.13}$$

$$V_Z^1 = V_Z^2. \tag{E.14}$$

These equations are usually interpreted as the vector rotated clockwise with fixed frame. However they are also true for a fixed vector and frame rotated anti-clockwise. So this is an example of the frame itself moving. Therefore a connection can be defined because the connection determines how the frame itself moves. The general rule for covariant derivative is:

$$D_\nu V^\mu = \partial_\nu V^\mu + \Gamma^\mu{}_{\lambda\nu} V^\lambda. \tag{E.15}$$

This equation means that D_ν acting on V^μ is the four derivative ∂_ν plus the term $\Gamma^\mu{}_{\lambda\nu} V^\lambda$. The three index symbol is referred to as "the connection", and describes the movement of the frame itself. The latter produces, for a given ν:

$$U^\mu = \Gamma^\mu{}_\lambda V^\lambda. \tag{E.16}$$

It is seen that Eq. (E.11) is an example of Eq. (E.16) in three dimensions, X, Y, and Z. So for a rotation of the frame anti-clockwise in three dimensions about the Z axis the matrix is the rotation matrix:

$$\Gamma^\mu{}_\lambda = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{E.17}$$

Thus:

$$\left.\begin{aligned} \Gamma^1{}_1 = \cos\theta, \Gamma^1{}_2 = \sin\theta, \Gamma^1{}_3 = 0, \\ \Gamma^2{}_1 = -\sin\theta, \Gamma^2{}_2 = \cos\theta, \Gamma^2{}_3 = 0, \\ \Gamma^3{}_1 = 0, \Gamma^3{}_2 = 0, \Gamma^3{}_3 = 1 \end{aligned}\right\}. \tag{E.18}$$

for each ν. Summation over repeated indices is used in Eq. (E.16) so:

$$\left.\begin{aligned} U^1 = \Gamma^1{}_1 V^1 + \Gamma^1{}_2 V^2 + \Gamma^1{}_3 V^3, \\ U^2 = \Gamma^2{}_1 V^1 + \Gamma^2{}_2 V^2 + \Gamma^2{}_3 V^3, \\ U^3 = \Gamma^3{}_1 V^1 + \Gamma^3{}_2 V^2 + \Gamma^3{}_3 V^3, \end{aligned}\right\} \tag{E.19}$$

for each ν. These equations (E.19) are the same as Eqs. (E.12) to (E.14).

The covariant derivative of Eq. (E.15) in this case is therefore:

$$D_\nu V^\mu = (\partial + \Gamma^\mu{}_\lambda)_\nu V^\lambda. \tag{E.20}$$

For example:

$$\begin{aligned} D_\nu V^1 &= (\partial + \Gamma^1{}_1)_\nu V^1 + \Gamma^1{}_{2\nu} V^2 \\ D_\nu V^1 &= (\partial + \cos\theta)_\nu V^1 + (\sin\theta)_\nu V^2 \\ D_\nu V^1 &= \partial_\nu V^1 + (\cos\theta)_\nu V^1 + (\sin\theta)_\nu V^2 \end{aligned} \tag{E.21}$$

Thus:

$$\Gamma^1{}_{1\nu} = (\cos\theta)_\nu, \Gamma^1{}_{2\nu} = (\sin\theta)_\nu. \tag{E.22}$$

These connections must have the units of inverse meters and must operate in the same way as the four derivative ∂_ν. So it is reasonable to assume:

$$\Gamma^1_{1\nu} = \frac{1}{2}\cos\theta\ \partial_\nu,\ \Gamma^1_{2\nu} = \frac{1}{2}\sin\theta\ \partial_\nu \tag{E.23}$$

and

$$D_\nu V^1 = \frac{1}{2}((1+\cos\theta)\partial_\nu V^1 + \sin\theta\partial_\nu V^2) \tag{E.24}$$

If there is no frame rotation:

$$\theta = 0 \tag{E.25}$$

and

$$D_\nu V^1 = \partial_\nu V^1. \tag{E.26}$$

This method regards the connection as an operator. It is well known that the set is a basis set in Riemann geometry. Others possibilities consistent with the correct dimensions of the connection are

$$(\cos\theta)_\nu = \frac{\cos\theta}{r}, (\sin\theta)_\nu = \frac{\sin\theta}{r}. \tag{E.27}$$

References

[1] M. W. Evans, “Generally Covariant Unified Field Theory: the Geometrization of Physics” (Abramis Academic, 2005 to present), vols. 1 to 4, vol. 5 in prep. (Papers 71 to 93 on www.aias.us).

[2] L. Felker, “The Evans Equations of Unified Field Theory” (Abramis Academic, 2007).

[3] K. Pendergast, “Crystal Spheres” (preprint on www.aias.us, Abramis to be published).

[4] Omnia Opera Section of www.aias.us.

[5] H. Eckardt, L. Felker, D. Indranu and S. Crothers, articles on www.aias.us.

[6] M. W. Evans, (ed.), “Modern Non-linear Optics”, a special topical issue of I. Prigogine and S. A. Rice, “Advances in Chemical Physics” (Wiley Interscience, New York, 2001, second edition), vols. 119(1) to 119(3).

[7] M. W. Evans and S. Kielich (eds.), ibid., first edition (Wiley Interscience, New York, 1992, 1993, 1997), vols. 85(1) to 85(3).

[8] M. W. Evans and L. D. Crowell, “Classical and Quantum Electrodynamics and the B(3) Field” (World Scientific, 2001).

[9] M. W. Evans and J.-P. Vigier, “The Enigmatic Photon” (Kluwer, 1994 to 2002), in five volumes.

[10] M. W. Evans and A. A. Hasanein, “The Photomagneton in Quantum Field Theory” (World Scientific, 1994).

[11] M. W. Evans, “The Photon’s Magnetic Field, Optical NMR” (World Scientific, 1992).

[12] M. W. Evans, Physica B, **182**, 227 and 237 (1992).

[13] S. P. Carroll, “Spacetime and Geometry, an Introduction to General Relativity” (Addison Wesley, New York, 2004), chapter three.

[14] M. W. Evans, Omnia Opera Section of www.aias.us (1992 to present).

[15] J. B. Marion and S. T. Thornton, “Classical Dynamics of Particles and Systems” (Harcourt Brace College Publishers, 1988, third edition).

[16] See for example a review by Zawodny in ref. (7).

[17] The first papers on the B(3) field are ref. (12), available in the Omnia Opera section of www.aias.us.

[18] L. H. Ryder, “Quantum Field Theory” (Cambridge University Press, 1996, 2^{nd} ed.).

[19] S. Crothers, papers and references therein on the www.aias.us site and papers 93 of the ECE series on www.aias.us.
[20] P. W. Atkins, "Molecular Quantum Mechanics" (Oxford University Press, 1983, 2^{nd} edition and subsequent editions).
[21] J. R. Croca, "Towards a Non-linear Quantum Physics" (World Scientific, 2001).
[22] This claim is made in ref. (18) and corrected in ref. (1).
[23] E. G. Milewski (Chief Editor), "The Vector Analysis Problem Solver" (Research and Education Association, New York, 1987, revised printing).
[24] J. D. Jackson, "Classical Electrodynamics" (Wiley, New York, 1999, third edition).
[25] M. W. Evans, Acta Phys. Polonica, **38**, 2211 (2007).
[26] M. W. Evans and H. Eckardt, Physica B, **400**, 175 (2007).
[27] M. W. Evans, Physica B, **403**, 517 (2008).
[28] Michael Krause (Director), "All About Tesla" (a film available on DVD that premiered in 2007).
[29] P. W. Atkins, frontispiece of ref. (20).