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The Fundamental Origin of the Bianchi Identity of Cartan Geometry and ECE Theory

by

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Abstract

The fundamental origin of the Bianchi identity of Cartan geometry is shown to be the commutator of covariant derivatives acting on a four vector in a space-time with curvature and torsion. The Hodge dual of this operation results in a Hodge dual Bianchi identity. In tensorial form the Bianchi identity and its Hodge dual are the field equations of electrodynamics in Einstein Cartan Evans (ECE) theory. These field equations reduce to the same vector formulation as the well known Maxwell Heaviside field equations, but are generally covariant and unified with other fundamental force fields.

Keywords: Cartan geometry, Bianchi identity, Einstein Cartan Evans (ECE) field theory, Hodge duality, field equations.

9.1 Introduction

It is well known [1] that Cartan's geometry is a self consistent geometry that generalizes Riemann geometry in an elegant manner based on the Cartan structure equations and Bianchi identity. It has been shown recently [2–11] that there is only one Bianchi identity, which must always relate the curvature form to the torsion form. To neglect the latter is arbitrary and inadmissible, yet this is what happens in the standard model theory of general relativity. This flaw has persisted for ninety years, so the only valid theory of general relativity is the well known Einstein Cartan Evans (ECE) theory introduced in 2003 on the basis of Cartan geometry with torsion and curvature. Any self consistent geometry may be used in general relativity, whose fundamental assertion is that natural philosophy is geometry. In ECE theory the philosophy of general relativity is adhered to in the manner of Einstein and Hilbert, but the torsion is re-instated following Cartan. The internal consistency of Cartan geometry depends on the use both of curvature and torsion, and also on the tetrad postulate [1–11]. The latter is the requirement that the complete vector field be invariant under the general coordinate transformation. There is no situation in natural philosophy where this is not true. There may be abstract and abstruse geometries in which the tetrad postulate is not true, but they are not Cartan's geometry as taught for eighty years [1] since first proposed by Cartan circa 1922.

In Section 9.2 the Bianchi identity of Cartan is proven from the commutator of covariant derivatives operating on a four vector in a space-time with curvature and torsion. It is well known [1] that the commutator operating in this manner produces the fundamental definitions of the Riemann and torsion tensors in terms of connections. These definitions are fundamental and are true irrespective of the postulate of metric compatibility [1]. The definitions are therefore true for any type of line element, metric and connection. The Riemann geometry used by Einstein and Hilbert [1] (EH theory) is based on the Christoffel connection, which is symmetric in its lower two indices. This assumption is the basis of EH theory and is based on the arbitrary neglect of the torsion tensor. The Christoffel connection is related to a symmetric metric in EH theory using the postulate of metric compatibility [1]. All line elements and solutions of the EH equation use these arbitrary assumptions. In 1915, when the EH theory was proposed, the existence of the Cartan torsion of 1922 was obviously unknown. In Section 9.2 it is proven that the Bianchi identity of Cartan is a re-expression of the commutator of covariant derivatives acting on the four vector. The Bianchi identity of Cartan is a cyclic sum of fundamental definitions of the curvature tensor. It is therefore an exact identity which is true, however, if and only if the torsion tensor is defined from the same commutator of covariant derivatives acting on the same four vector.

In Section 9.3 the Hodge dual of the Bianchi identity is proven in the same way, by using the Hodge dual of the commutator of covariant derivatives acting on the same four vector. This operation produces the Hodge dual of the Riemann or curvature tensor and the Hodge dual of the torsion tensor. A well defined Hodge dual of the Bianchi identity follows from this proof.

In Section 9.4 it is proven that the use of the Christoffel symbol or connection is incompatible fundamentally with the Bianchi identity of Cartan. This is clear just by considering the torsion tensor, which is the difference of connections as follows:

$$T^{\kappa}_{\mu\nu} = \Gamma^{\kappa}_{\mu\nu} - \Gamma^{\kappa}_{\nu\mu}. \tag{9.1}$$

For the Christoffel connection:

$$\Gamma^{\kappa}_{\mu\nu} = \Gamma^{\kappa}_{\nu\mu} \tag{9.2}$$

and the torsion tensor vanishes. This is incompatible with the fundamental operation of a commutator of covariant derivatives acting on a four vector. In ECE theory the torsion is correctly re-instated, and the general connection used instead of the Christoffel connection. The ECE theory is internally consistent whereas the EH theory is not.

9.2 Proof of the Bianchi Identity

In previous work [1–11] it has been proven that the Bianchi identity of Cartan is the cyclic sum of definitions of the curvature or Riemann tensor. This result is true if and only if the torsion tensor is defined as in Eq. (9.1). In this section the Bianchi identity is proven from the well known equation [1–11]:

$$[D_\mu, D_\nu]V^\rho = R^\rho{}_{\sigma\mu\nu}V^\sigma - T^\lambda_{\mu\nu}D_\lambda V^\rho \tag{9.3}$$

in which the commutator of covariant derivatives acts on a four vector. In Eq. (9.3) no assumption is made concerning metric compatibility or symmetry of the metric or connection, so Eq. (9.3) is a fundamental and general result. In Eq. (9.3) the commutator is an operator defined by:

$$[D_\mu, D_\nu] = D_\mu D_\nu - D_\nu D_\mu \tag{9.4}$$

where the covariant derivative is defined by:

$$D_\mu V^\nu = \partial_\mu V^\nu + \Gamma^\nu_{\mu\lambda}V^\lambda. \tag{9.5}$$

Here V^ν is any four-vector in a space-time with both torsion and curvature. In Eq. (9.3) the curvature tensor is:

$$R^\lambda_{\rho\mu\nu} := \partial_\mu\Gamma^\lambda_{\nu\rho} - \partial_\nu\Gamma^\lambda_{\mu\rho} + \Gamma^\lambda_{\mu\sigma}\Gamma^\sigma_{\nu\rho} - \Gamma^\lambda_{\nu\sigma}\Gamma^\sigma_{\mu\rho} \tag{9.6}$$

and the torsion tensor is:

$$T^\lambda_{\mu\nu} := \Gamma^\lambda_{\mu\nu} - \Gamma^\lambda_{\nu\mu}. \tag{9.7}$$

There is no a priori reason for assuming that the connection must be a Christoffel connection as defined in Eq. (9.2). In almost all standard model general relativity the torsion is arbitrarily neglected by using the Christoffel connection and the symmetric metric:

$$g_{\mu\nu} = g_{\nu\mu}. \tag{9.8}$$

To recast Eq. (9.3) in the form of Cartan's Bianchi identity these assumptions are not used. The only additional equation needed is the tetrad postulate [1–11]:

$$D_\mu q^a_\nu = 0. \tag{9.9}$$

where q^a_ν is the well known Cartan tetrad. The latter is a vector valued one-form of the standard differential geometry introduced by Cartan. The latter's Bianchi identity is:

$$d \wedge T^a + \omega^a{}_b \wedge T^b := R^a{}_b \wedge q^b \tag{9.10}$$

where T^a is the torsion form, a vector valued two-form [1–11], $R^a{}_b$ is the curvature form, a tensor valued two-form, and $\omega^a{}_b$ is the spin connection. The tetrad postulate relates the spin connection and gamma connection as follows [1–11]:

$$\partial_\mu q^a_\sigma + \omega^a_{\mu b} q^b_\sigma = \Gamma^\lambda_{\mu\sigma} q^a_\lambda. \tag{9.11}$$

It is proven as follows that Eqs. (9.3) and (9.10) are the same equation, provided that the tetrad postulate (9.11) is used. First translate Eq. (9.10) into tensor notation using the rules for the wedge product $\wedge$ of differential geometry [1]. This translation from form to tensor notation produces the result:

$$\begin{aligned} &\partial_\mu T^a_{\nu\rho} + \omega^a_{\mu b} T^b_{\nu\rho} + \partial_\rho T^a_{\mu\nu} + \omega^a_{\rho b} T^b_{\mu\nu} + \partial_\nu T^a_{\rho\mu} + \omega^a{}_{\nu b} T^b_{\rho\mu} \\ &\quad := (R^\lambda_{\mu\nu\rho} + R^\lambda_{\rho\mu\nu} + R^\lambda_{\nu\rho\mu}) q^a_\lambda \end{aligned} \tag{9.12}$$

where:

$$T^a_{\nu\rho} = (\Gamma^\lambda_{\nu\rho} - \Gamma^\lambda_{\rho\nu}) q^a_\lambda \quad \text{etc.,} \tag{9.13}$$

$$T^b_{\nu\rho} = (\Gamma^\lambda_{\nu\rho} - \Gamma^\lambda_{\rho\nu}) q^b_\lambda \quad \text{etc.} \tag{9.14}$$

Using the Leibnitz Theorem:

$$\begin{aligned} \partial_\mu T^a_{\nu\rho} = {} & (\partial_\mu \Gamma^\lambda_{\nu\rho} - \partial_\mu \Gamma^\lambda_{\rho\nu}) q^a_\lambda \\ & + (\Gamma^\lambda_{\nu\rho} - \Gamma^\lambda_{\rho\nu}) \partial_\mu q^a_\lambda \quad \text{etc.,} \end{aligned} \tag{9.15}$$

so Eq. (9.12) becomes:

$$\begin{aligned}(\partial_\mu \Gamma^\lambda_{\nu\rho} - \partial_\mu \Gamma^\lambda_{\rho\nu}) q^a_\lambda + (\Gamma^\lambda_{\nu\rho} - \Gamma^\lambda_{\rho\nu})(\partial_\mu q^a_\lambda + \omega^a_{\mu b} q^b_\lambda) + \ldots \\ := (R^\lambda_{\mu\nu\rho} + R^\lambda_{\rho\mu\nu} + R^\lambda_{\nu\rho\mu}) q^a_\lambda.\end{aligned} \tag{9.16}$$

Now re-label dummy (i.e. repeated) indices in the second term on the left hand side:

$$\lambda \to \sigma \tag{9.17}$$

to obtain:

$$\begin{aligned}(\partial_\mu \Gamma^\lambda_{\nu\rho} - \partial_\mu \Gamma^\lambda_{\rho\nu}) q^a_\lambda + (\Gamma^\sigma_{\nu\rho} - \Gamma^\sigma_{\rho\nu})(\partial_\mu q^a_\sigma + \omega^a_{\mu b} q^b_\sigma) + \ldots \\ := (R^\lambda_{\mu\nu\rho} + R^\lambda_{\rho\mu\nu} + R^\lambda_{\nu\rho\mu}) q^a_\lambda\end{aligned} \tag{9.18}$$

Use the tetrad postulate (9.11) to obtain the cyclic sum:

$$\begin{aligned}&\partial_\mu \Gamma^\lambda_{\nu\rho} - \partial_\mu \Gamma^\lambda_{\rho\nu} + \Gamma^\lambda_{\mu\sigma}(\Gamma^\sigma_{\nu\rho} - \Gamma^\sigma_{\rho\nu}) + \partial_\rho \Gamma^\lambda_{\mu\nu} - \partial_\rho \Gamma^\lambda_{\nu\mu} \\ &\quad + \Gamma^\lambda_{\rho\sigma}(\Gamma^\sigma_{\mu\nu} - \Gamma^\sigma_{\nu\mu}) + \partial_\nu \Gamma^\lambda_{\rho\mu} - \partial_\nu \Gamma^\lambda_{\mu\rho} + \Gamma^\lambda_{\nu\sigma}(\Gamma^\sigma_{\rho\mu} - \Gamma^\sigma_{\mu\rho}) \\ &\quad := R^\lambda_{\mu\nu\rho} + R^\lambda_{\rho\mu\nu} + R^\lambda_{\nu\rho\mu}.\end{aligned} \tag{9.19}$$

Re-arrange this cyclic sum as follows:

$$\begin{aligned}&R^\lambda_{\rho\mu\nu} + R^\lambda_{\mu\nu\rho} + R^\lambda_{\nu\rho\mu} \\ &\quad := \partial_\mu \Gamma^\lambda_{\nu\rho} - \partial_\nu \Gamma^\lambda_{\mu\rho} + \Gamma^\lambda_{\mu\sigma}\Gamma^\sigma_{\nu\rho} - \Gamma^\lambda_{\nu\sigma}\Gamma^\sigma_{\mu\rho} \\ &\qquad + \partial_\nu \Gamma^\lambda_{\rho\mu} - \partial_\rho \Gamma^\lambda_{\nu\mu} + \Gamma^\lambda_{\nu\sigma}\Gamma^\sigma_{\rho\mu} - \Gamma^\lambda_{\rho\sigma}\Gamma^\sigma_{\nu\mu} \\ &\qquad + \partial_\rho \Gamma^\lambda_{\mu\nu} - \partial_\mu \Gamma^\lambda_{\rho\nu} + \Gamma^\lambda_{\rho\sigma}\Gamma^\sigma_{\mu\nu} - \Gamma^\lambda_{\mu\sigma}\Gamma^\sigma_{\rho\nu}.\end{aligned} \tag{9.20}$$

It is seen that this is a cyclic sum of three definitions of the curvature tensor:

$$R^\lambda_{\rho\mu\nu} := \partial_\rho \Gamma^\lambda_{\mu\nu} - \partial_\mu \Gamma^\lambda_{\rho\nu} + \Gamma^\lambda_{\rho\sigma}\Gamma^\sigma_{\mu\nu} - \Gamma^\lambda_{\mu\sigma}\Gamma^\sigma_{\rho\nu}, \tag{9.21}$$

$$R^\lambda_{\mu\nu\rho} := \partial_\mu \Gamma^\lambda_{\nu\rho} - \partial_\nu \Gamma^\lambda_{\mu\rho} + \Gamma^\lambda_{\mu\sigma}\Gamma^\sigma_{\nu\rho} - \Gamma^\lambda_{\nu\sigma}\Gamma^\sigma_{\mu\rho}, \tag{9.22}$$

$$R^\lambda_{\nu\rho\mu} := \partial_\nu \Gamma^\lambda_{\rho\mu} - \partial_\rho \Gamma^\lambda_{\nu\mu} + \Gamma^\lambda_{\nu\sigma}\Gamma^\sigma_{\rho\mu} - \Gamma^\lambda_{\rho\sigma}\Gamma^\sigma_{\nu\mu}. \tag{9.23}$$

These definitions come from Eq. (9.3) as does the definition of the torsion tensor needed to obtain the result (9.20), Q.E.D.

It has been proven that Cartan's Bianchi identity (9.10) is the same as Eq. (9.3) given the tetrad postulate (9.11). The identity is exact, because its left hand side is identically the same as its right hand side. One takes three definitions (9.21)–(9.23) and adds them. In general:

$$R^{\lambda}_{\rho\mu\nu} + R^{\lambda}_{\mu\nu\rho} + R^{\lambda}_{\nu\rho\mu} \neq 0 \tag{9.24}$$

because the Bianchi identity is:

$$D_\mu T^a_{\nu\rho} + D_\rho T^a_{\mu\nu} + D_\nu T^a_{\rho\mu} := R^a_{\mu\nu\rho} + R^a_{\rho\mu\nu} + R^a_{\nu\rho\mu} \tag{9.25}$$

and there is no reason for assuming that the torsion is zero, assuming that the connection is symmetric or assuming that the metric is symmetric, not for assuming metric compatibility.

What is usually done in EH theory is to make all these assumptions and to describe the resulting geometry as Riemann geometry. This is arbitrary and unjustifiable. The resulting physics of general relativity is all based on these arbitrary assumptions. The correct Bianchi identity is Eq. (9.25), which can be re-written as:

$$D_\mu \widetilde{T}^{a\,\mu\nu} := \widetilde{R}^{a}{}_{\mu}{}^{\mu\nu} \tag{9.26}$$

where the tilde denotes Hodge duality. In deriving Eq. (9.26) from Eq. (9.25) the following Hodge duals are used [1–11]:

$$\widetilde{T}^{a\,\alpha\beta} = \frac{1}{2}\|g\|^{\frac{1}{2}}\epsilon^{\alpha\beta\mu\nu}T^a_{\mu\nu}, \tag{9.27}$$

$$\widetilde{R}^{a}{}_{b}{}^{\alpha\beta} = \frac{1}{2}\|g\|^{\frac{1}{2}}\epsilon^{\alpha\,\beta\mu\nu}R^{a}{}_{b\,\mu\nu}. \tag{9.28}$$

Here $\|g\|^{\frac{1}{2}}$ is the square root of the positive value of the metric determinant, and $\epsilon^{\alpha\beta\mu\nu}$ is the four dimensional Levi-Civita symbol of Minkowski spacetime. Since two-forms are anti-symmetric by definition [1–11]:

$$T^a_{\mu\nu} = -T^a_{\nu\mu}, \tag{9.29}$$

$$R^a_{b\mu\nu} = -R^a_{b\,\nu\mu}, \tag{9.30}$$

their Hodge duals are also anti-symmetric, and are also two-forms. A particular solution of Eq. (9.26) is the base manifold equation:

$$D_\mu \widetilde{T}^{\kappa\mu\nu} := \widetilde{R}^{\kappa}{}_{\mu}{}^{\mu\nu} \tag{9.31}$$

which is the basis [2–11] of the homogeneous ECE field equation. Note carefully that Eq. (9.26) is less general than Eq. (9.25) because in deriving Eq. (9.26) metric compatibility is used as follows:

$$D_\mu \left(\|g\|^{\frac{1}{2}} \right) = 0. \tag{9.32}$$

However there is no situation in natural philosophy in which metric compatibility is not true, because the metric is defined by:

$$g_{\mu\nu} = q^a_\mu q^b_\nu \eta_{ab}. \tag{9.33}$$

Here η_{ab} is the Minkowski metric [1–11]. The tetrad postulate (9.11) then implies metric compatibility, which is the equation [1–11]:

$$D_\rho g_{\mu\nu} = 0. \tag{9.34}$$

As we have argued, the tetrad postulate is the fundamental requirement that the complete vector field be invariant under general coordinate transformation, and this is always true in natural philosophy.

9.3 Hodge Dual of the Bianchi Identity

This identity of geometry is the basis for the inhomogeneous field equation of ECE theory [2–11]. It is proven by taking the Hodge dual term by term of Eq. (9.3) to give:

$$[D_\mu, D_\nu]_{\mathrm{HD}} V^\rho = \widetilde{R}^\rho{}_{\sigma\mu\nu} V^\sigma - \widetilde{T}^\lambda_{\mu\nu} D_\lambda V^\rho \tag{9.35}$$

where the subscript HD denotes Hodge dual. The Hodge duals of the curvature and torsion tensors are evidently:

$$\widetilde{R}^\lambda{}_{\rho\mu\nu} := \left(\partial_\mu \Gamma^\lambda_{\nu\rho} - \partial_\nu \Gamma^\lambda_{\mu\rho} + \Gamma^\lambda_{\mu\sigma} \Gamma^\sigma_{\nu\rho} - \Gamma^\lambda_{\nu\sigma} \Gamma^\sigma_{\mu\rho} \right)_{\mathrm{HD}}, \tag{9.36}$$

$$\widetilde{T}^\lambda_{\mu\nu} := \left(\Gamma^\lambda_{\mu\nu} - \Gamma^\lambda_{\nu\mu} \right)_{\mathrm{HD}}. \tag{9.37}$$

Therefore, following the methods of Section 9.2, Eq. (9.35) is a re-statement of:

$$d \wedge \widetilde{T}^a + \omega^a{}_b \wedge \widetilde{T}^b := \widetilde{R}^a{}_b \wedge q^b. \tag{9.38}$$

The same connections occur in Eqs. (9.10) and (9.37) and Eqs. (9.3) and (9.35) can be inter-converted by the Hodge dual transformation. Therefore they are duality invariant. The tensor formulation of Eq. (9.37) is:

$$D_\mu \widetilde{T}^a_{\nu\rho} + D_\rho \widetilde{T}^a_{\mu\nu} + D_\nu \widetilde{T}^a_{\rho\mu} := \widetilde{R}^a{}_{\mu\nu\rho} + \widetilde{R}^a{}_{\rho\mu\nu} + \widetilde{R}^a{}_{\nu\rho\mu} \tag{9.39}$$

which is the same as:

$$D_\mu T^{a\mu\nu} := R^a{}_\mu{}^{\mu\nu} \tag{9.40}$$

a special case of which is:

$$D_\mu T^{\kappa\mu\nu} = R^\kappa{}_\mu{}^{\mu\nu} \tag{9.41}$$

This equation is the basis of the inhomogeneous field equation of ECE theory [1–11].

In deriving the Hodge dual (9.35) the $\|g\|^{\frac{1}{2}}$ factor cancels out because it is the same on both sides. Therefore the Hodge duality can be carried out with the $\epsilon^{\mu\nu\rho\sigma}$ tensor of Minkowski space-time. This is the totally anti-symmetric unit tensor in four dimensions. It is important to note that the same connections occur in the Bianchi identity and also in its Hodge dual (9.37), In concise, index-less notation they can be written as:

$$D \wedge T := R \wedge q \tag{9.42}$$

and

$$D \wedge \widetilde{T} := \widetilde{R} \wedge q \tag{9.43}$$

and so are clearly interchangeable under the Hodge dual transforms:

$$T \to \widetilde{T}; \quad R \to \widetilde{R}. \tag{9.44}$$

This is what is meant by duality invariance. The latter forms the basis for topics in physics such as Montonen-Olive duality, topological magnetic monopoles and so forth [12]. The usual Maxwell Heaviside (MH) field equations are not duality invariant because there is no magnetic monopole. The MH equations in form notation are:

$$d \wedge F = 0, \tag{9.45}$$

$$d \wedge \widetilde{F} = \widetilde{j}/\epsilon_0, \tag{9.46}$$

and it is seen that under the Hodge dual transformation of F to $\widetilde{\mathrm{F}}$ they are not duality invariant. In contrast it is shown in the next section that the ECE field equations are duality invariant and have a fundamental symmetry that is missing in the MH theory of special relativity. This symmetry is possible only in a generally covariant unified field theory.

9.4 Incompatibility of the Christoffel Connection

The tensorial formulations of the Bianchi identity and its Hodge dual are duality invariant equations which in the base manifold are as follows:

$$D_\mu \widetilde{T}^{\kappa\mu\nu} := \widetilde{R}^{\kappa}{}_{\mu}{}^{\mu\nu}, \tag{9.47}$$

$$D_\mu T^{\kappa\mu\nu} := \widetilde{R}^{\kappa}{}_{\mu}{}^{\mu\nu}. \tag{9.48}$$

Indices can be raised and lowered on the torsion and curvature tensors in these expressions by use of the metric. For example:

$$T^{\kappa}_{\mu\nu} = g_{\mu\rho} g_{\nu\sigma} T^{\kappa\rho\sigma}. \tag{9.49}$$

By use of computer algebra [2–11] the tensor $R^{\kappa}{}_{\mu}{}^{\mu\nu}$ has been evaluated for various metrics and line elements based on the Christoffel connection (see for example paper 93 of the www.aias.us series). It was found by Maxima that the tensor is non-zero in general for a Christoffel connection. It vanishes only when the line element is constructed from a Ricci flat space-time. Crothers has argued recently on www.aias.us that the use of a Ricci flat space-time is incompatible with the equivalence principle of Einstein. From the point of view of ECE theory such a space-time implies that there is no electromagnetic field because the canonical energy momentum density vanishes. The canonical angular energy momentum density is a rank three tensor as is well known [12] and also vanishes in a Ricci flat space-time. In ECE the electromagnetic field is directly proportional to this rank three tensor density, and this is justified experimentally through the well known fact that the electromagnetic field has angular momentum as observed in the Beth effect. So a Ricci flat space-time is one in which there are no fields and no energy momentum density.

For all line elements that consider a finite energy momentum density the Christoffel connection is always used in Einstein Hilbert field theory. In this case the tensor $R^{\kappa}{}_{\mu}{}^{\mu\nu}$ is non-zero, but for a Christoffel connection the tensor $T^{\kappa\mu\nu}$ is zero. The Christoffel connection is therefore incompatible with the Bianchi identity and its Hodge dual. This is intuitively clear from the fact that one cannot neglect torsion, as argued in sections 9.2 and 9.3. This is an irretrievable flaw in EH theory and progress must be made by solving the Cartan equations without discarding torsion. This is precisely what ECE theory sets out to do and so the well accepted ECE theory is the only self-consistent theory of general relativity and of the generally covariant unified field.

Acknowledgments

The British Government is thanked for a Civil List Pension and the Telesio Galilei Association for generous funding of this work. The staff of AIAS and Telesio Galilei are thanked for many interesting discussions.

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