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Development of the Einstein Hilbert Field Equation into the Einstein Cartan Evans (ECE) Field Equation

by

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Abstract

Recently in this series of papers it has been shown that the Einstein Hilbert (EH) field equation is self-inconsistent because the use of the Christoffel connection is inconsistent with the one true Bianchi identity of Cartan geometry. The EH field equation is extended in this paper to include the effect of the Cartan torsion through the angular energy - momentum density tensor. The canonical energy - momentum density tensor of the EH equation is recognized as having an internal structure based on geometry. Therefore the ECE field equation is based on geometry rather than on the lagrangian method leading to the conventional Noether Theorem used in the derivation by Einstein of the EH equation.

Keywords: Einstein Cartan Evans (ECE) field equation, Einstein Hilbert (EH) field equation, Cartan torsion, angular energy momentum density tensor, energy momentum density tensor.

10.1 Introduction

Recently in this series of papers on the Einstein Cartan Evans (ECE) field theory [1–10] it has been shown that there is only one true Bianchi identity of Cartan geometry, and that the use of the Christoffel symbol in general relativity is inconsistent with this one true identity because of the neglect of the Cartan torsion in conventional general relativity [11, 12]. There are many criticisms available [12] of conventional general relativity but this newly discovered [1–10] incompatibility is an irretrievable flaw in the Einstein Hilbert (EH) field equation itself. The ECE theory retains the basic idea of general relativity, that physics or natural philosophy is geometry, but bases the subject of general relativity on the standard and well known Cartan geometry [11] of 1922. The latter is well taught and well known to be rigorously self-consistent [11]. However, this self consistency relies on the recognition of the torsion form of Cartan, as well as the curvature form of Cartan. The one true Bianchi identity is an equation which is a rigorously correct identity, one side of the equation being identically the same as the other [1–11]. In tensor notation the one true Bianchi identity is the cyclic sum of definitions of the Riemann tensor. The Riemann and torsion tensors are defined by the commutator of covariant derivatives acting on the four vector [1–11]. The one true Bianchi identity is a re-expression of this fundamental and well known [1–11] operation. The latter generates the Riemann (or curvature) and torsion tensors simultaneously. One is ineluctably linked to the other, and there is no a priori reason to assert that one or the other must vanish. This is a self-consistent conclusion of Riemann geometry, and Cartan geometry is equivalent to Riemann geometry through the well known and accepted tetrad postulate [1–11]. The latter is the fundamental requirement that the complete vector field be invariant under the general coordinate transformation. This is always true in natural philosophy.

The EH field equation was inferred independently by Einstein and Hilbert in 1915 and was based on the development by Einstein of general relativity from about 1907 onwards using a form of Riemann geometry in which torsion is implicitly zero. The existence of the torsion tensor was unknown in this era, and the Riemann geometry used by Einstein neglects torsion. Einstein inferred the EH equation by making the "second Bianchi identity" torsionless and thus self inconsistent Riemann geometry proportional to the Noether Theorem applied to energy momentum density only, neglecting spin. The covariant derivative of this linear momentum density tensor is zero, but no angular motion is considered. The proportionality constant is k, the Einstein constant. The torsion form was inferred by Cartan in 1922 and it can be translated into the torsion tensor [1–11] using the tetrad postulate. The latter is essential for the compatibility of Cartan and general Riemann geometry. When the torsion tensor is zero the connection must be the Christoffel connection, which is symmetric in its lower two indices. More generally the connection does not have this property, and in general the metric tensor is not symmetric. In conventional general relativity an equation is derived defining the Christoffel connection in terms of the symmetric metric by using the equation of metric compatibility [1–11]. However, the curvature tensor and torsion tensor are found by the operation of the commutator of covariant derivatives on the four vector, as argued already, and this operation does not assume any particular symmetry of the connection or metric. Neither does it assume metric compatibility [1–11]. So conventional or standard model general relativity is based on entirely arbitrary assumptions concerning the symmetry of the metric and connection, and also assumes metric compatibility. Recently it has been found that this inherent arbitrariness leads to a fundamental internal inconsistency in standard model general relativity - the Bianchi identity is fundamentally inconsistent with the Christoffel connection as conventionally defined via the symmetric metric.

In Section 10.2 the geometrical reason for this inconsistency is given. In Section 10.3 a new field equation is inferred geometrically, and this is referred to as the ECE field equation. The latter uses the angular energy momentum density tensor as well as the energy momentum tensor, and shows that the two forms of energy momentum are always inter-convertible. Total energy momentum is conserved only for the whole universe, or for a well defined experimental situation. The curvature tensor can represent both angular or linear motion, and the torsion tensor can represent both angular and linear motion. They are two tensors which are bound together ineluctably by the commutator of covariant derivatives acting on the four vector in a space-time with curvature and torsion. The history of general relativity is such that the torsion tensor was not known in the era leading to the EH equation of 1915. When the torsion form and tensor were inferred by Cartan in 1922 the EH equation remained as it was in 1915. So all claims to the existence of Big Bang, black holes and dark matter are based on an internally incorrect geometry and are therefore unscientific. There are many other criticisms [12] of conventional general relativity which must be addressed if progress is to be made. These criticisms have been made by many leading scientists [12] and ECE is a suggestion for progress.

10.2 Self-Inconsistency of Standard Model Riemann Geometry

The self-inconsistency is most clearly shown by considering the Hodge dual [1–11] of the one true Bianchi identity. Such a consideration leads to a simple tensor equation

$$D_\mu T^{\kappa\mu\nu} = R^{\kappa}{}_{\mu}{}^{\mu\nu} \tag{10.1}$$

which links the covariant derivative of the torsion tensor $T^{\kappa\mu\nu}$ to the Ricci type tensor $R^{\kappa}{}_{\mu}{}^{\mu\nu}$ on the right hand side. Summation over repeated internal indices means that this is a two index tensor. Summation over repeated indices occurs also on the left hand side of Eq. (10.1) so indices are balanced as required in tensor algebra. The Ricci-type tensor on the right hand side of

Eq. (10.1) was evaluated by computer algebra [1–11] using the Christoffel connection as defined in the standard model of general relativity:

$$\Gamma^{\rho}{}_{\mu\alpha} = \frac{1}{2} g^{\rho\lambda}(\partial_\mu g_{\lambda\alpha} + \partial_\alpha g_{\mu\lambda} - \partial_\lambda g_{\alpha\mu}) \tag{10.2}$$

and was found to be non-zero in general (papers 93 onwards of www.aias.us). It vanishes only when the conventional Ricci tensor is assumed to be zero by construction - the so called vacuum or Ricci flat solutions of EH. The Christoffel connection (10.2), as conventionally defined, is symmetric in its lower two indices, because the metric in standard model general relativity is assumed to be symmetric:

$$g_{\mu\nu} = g_{\nu\mu}. \tag{10.3}$$

So the torsion tensor in standard model general relativity is zero:

$$T^{\kappa}_{\mu\nu} = \Gamma^{\kappa}_{\mu\nu} - \Gamma^{\kappa}_{\nu\mu} = 0. \tag{10.4}$$

Therefore this leads to a fundamental internal self-inconsistency in standard model general relativity: the left hand side of Eq. (10.1) is always zero by construction (use of Eq. (10.2)) but the right hand side is not zero in general for the SAME Eq. (10.2).

The origin of this self-inconsistency is the arbitrary neglect of torsion. The one true Bianchi identity can be written most clearly in an indexless notation [1–10]:

$$D \wedge T := R \wedge q \tag{10.5}$$

where $D\wedge$ represents a type of exterior derivative $d\wedge$ supplemented by the spin connection, i.e. a type of covariant exterior derivative [11]. The wedge product is $\wedge$ and T, R and q represent respectively the torsion, curvature and tetrad forms of Cartan geometry [11]. If T is taken arbitrarily to be zero, Eq. (10.5) becomes:

$$R \wedge q = 0 \tag{10.6}$$

which is referred to in the literature as "the first Bianchi identity". In tensor notation Eq. (10.6) is:

$$R_{\kappa\mu\nu\rho} + R_{\kappa\rho\mu\nu} + R_{\kappa\nu\rho\mu} = 0 \tag{10.7}$$

and is true if and only if the metric is symmetric [1–11]. It is not an identity therefore, and was inferred not by Bianchi but by Ricci and Levi-Civita.

For these reasons it is referred to in ECE theory as the Ricci cyclic equation. It is clearly inconsistent with the true Bianchi identity (10.5) due to arbitrary neglect of torsion and this self inconsistency carries through to Eq. (10.1). Similarly the "second Bianchi identity" of standard model general relativity is:

$$D \wedge R = 0 \tag{10.8}$$

and in tensor notation this becomes:

$$D_\rho R^\kappa{}_{\sigma\mu\nu} + D_\mu R^\kappa{}_{\sigma\nu\rho} + D_\nu R^\kappa{}_{\sigma\rho\mu} = 0. \tag{10.9}$$

Again this neglects torsion arbitrarily, and is a special case of [1–10]:

$$D \wedge (D \wedge T) := D \wedge (R \wedge q). \tag{10.10}$$

There is only one true Bianchi identity therefore, Eq. (10.5). The torsionless "second Bianchi identity" Eq. (10.9) can be re-expressed [1–11] as:

$$D^\mu \left(R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \right) = 0 \tag{10.11}$$

where $R_{\mu\nu}$ is the torsionless Ricci tensor, R is the torsionless scalar curvature and $g_{\mu\nu}$ is the torsionless metric. The torsionless Einstein tensor, finally, is defined as:

$$G_{\mu\nu} := R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}. \tag{10.12}$$

The torsionless Noether Theorem is [11] is:

$$D^\mu T_{\mu\nu} = 0 \tag{10.13}$$

and the torsionless EH equation is obtained from:

$$D^\mu G_{\mu\nu} = k D^\mu T_{\mu\nu} = 0 \tag{10.14}$$

as the special case:

$$G_{\mu\nu} = k T_{\mu\nu}. \tag{10.15}$$

This is the well known EH equation which is used as the basis for unscientific assertions such as the existence of black holes (also severely criticized on www.aias.us by Crothers), the existence of an unscientific Big Bang, and the existence of unscientific dark matter. Not only is the basic geometry of EH flawed, but also its methods of solution. Crothers has also criticized the concept of Ricci flat space-times, and has shown that the so called Schwarzschild metric is not due to Schwarzschild. The latter's original solution of 1916 does not contain the mass M, but a parameter alpha unrelated in general to M. Santilli [12] has summarized numerous criticisms of EH theory, criticisms made by well known scientists since inception of EH theory in 1915. Contemporary standard model proponents appear to be unaware of this scholarship, or if they are aware of it, neglect it in the same arbitrary manner in which they neglect Cartan's torsion. So the subject is unscientific and this must be recognized for progress. The following section is a suggestion for progress.

10.3 ECE Field Equation

The ECE equation is based on the proportionality of the Cartan torsion and the three index angular energy momentum density tensor [1–10]:

$$T^{\kappa\mu\nu} = kJ^{\kappa\mu\nu} \tag{10.16}$$

as inferred in previous work. The structure of Eq. (10.1) then suggests that a similar proportionality exists between the Ricci type tensor$R^{\kappa}{}_{\mu}{}^{\mu\nu}$ and an energy momentum density tensor:

$$R^{\kappa}{}_{\mu}{}^{\mu\nu} = kT^{\kappa}{}_{\mu}{}^{\mu\nu}. \tag{10.17}$$

Therefore we arrive at the field equation:

$$D_\mu J^{\kappa\mu\nu} = T^{\kappa}{}_{\mu}{}^{\mu\nu} \tag{10.18}$$

on this geometrical basis. This field equation suggests in turn that there exists a conservation of energy equation:

$$D_\kappa(D_\mu J^{\kappa\mu\nu}) = D_\kappa T^{\kappa}{}_{\mu}{}^{\mu\nu} \tag{10.19}$$

which is a balance between linear and angular energy momentum. In general both sides of Eq. (10.19) are non-zero. Therefore Eq. (10.19) represents a balance of linear and angular energy - momentum densities, both being always non-zero in general. There is no a priori reason for assuming that one or the other should be zero. As a matter of semantics it is possible to assert that "pure translation" is represented by:

$$J^{\kappa\mu\nu} = 0 \tag{10.20}$$

and

$$D_\kappa T^{\kappa}{}_{\mu}{}^{\mu\nu} = 0. \tag{10.21}$$

This is the Noether Theorem used in the EH equation if the following identification is made:

$$\begin{aligned} T^{\kappa\nu} &:= T^{\kappa}{}_{\mu}{}^{\mu\nu} \\ &= T^{\nu}{}_{\mu}{}^{\mu\kappa} \end{aligned} \tag{10.22}$$

This symmetry follows from Eq. (10.17) if the Christoffel connection is used to calculate the Riemann tensor leading to the Ricci type tensor $R^{\kappa}{}_{\mu}{}^{\mu\nu}$. Similarly, it is possible to assert that "pure rotation" is given by:

$$T^{\kappa}{}_{\mu}{}^{\mu\nu} = 0 \tag{10.23}$$

and

$$D_\mu J^{\kappa\mu\nu} = 0. \tag{10.24}$$

This equation is conservation of angular energy momentum density using the covariant derivative. It is the covariant version of an equation given in chapter three of Ryder [13] but it does not appear in Einstein's general relativity [13] because the latter did not consider torsion and did not consider angular energy momentum density. Eq. (10.18) asserts that energy momentum can be interconverted between linear and angular forms, but it is arbitrary to assert a state of pure translation or pure rotation. Similarly, it is arbitrary to assert that torsion and curvature can exist irrespective of each other in geometry. Total energy momentum is conserved as in Eq. (10.19), but this total energy momentum must be carefully defined for each situation under consideration.

Einstein derived the Poisson equation leading to the Newton inverse square law by consideration of a weak field limit of the EH equation. In this derivation the Christoffel symbol is used in the limit of slowly moving fields. However the underlying geometrical entity is the curvature tensor. In the weak field limit the space-time is approximated by a Minkowski space-time in which the curvature becomes infinitesimally small, so the distance between two point masses becomes not curve but a straight line. Using the same ideas in Eq. (10.18), the covariant derivative is replaced in the weak field limit by the ordinary derivative:

$$D_\mu \rightarrow \partial_\mu \tag{10.25}$$

so we obtain:

$$\partial_\mu T^{\kappa\mu\nu} \doteqdot R^{\kappa}{}_{\mu}{}^{\mu\nu}. \tag{10.26}$$

Using $\nu = 0$, Eq. (10.26) reduces to:

$$\boldsymbol{\nabla} \cdot \boldsymbol{g} = c^2 R \tag{10.27}$$

where R has the units of inverse meters squared, i.e. of scalar curvature. Eq. (10.17) gives:

$$R = k\rho_m \tag{10.28}$$

where ρ_m is mass density in kilograms per meters cubed. So:

$$\boldsymbol{\nabla} \cdot \boldsymbol{g} = kc^2 \rho_m \tag{10.29}$$

which can be compared with the Coulomb law of ECE theory [1–11]:

$$\boldsymbol{\nabla} \cdot \boldsymbol{E} = \rho_e/\epsilon_0 \tag{10.30}$$

where ρ_e is charge density in coulombs per meters cubed. It is well known that Eq. (10.30) is a form of the Coulomb inverse square law, so Eq. (10.29) is the Newton inverse square law, derived straightforwardly in the weak field limit Eq. (10.25) from the tensor equation (10.1) that originates in the Hodge dual of the Bianchi identity.

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References

[1] M. W. Evans, "Generally Covariant Unified Field Theory" (Abramis Academic, Suffolk, 2005 onwards), volumes 1–4, ibid., volume five in prep. (www.aias.us, papers 71–93).
[2] L. Felker, "The Evans Equations of Unified Field Theory" (Abramis, Suffolk, 2007).
[3] K. Pendergast, "Crystal Spheres" preprint on www.aias.us, (Abramis, in prep.).
[4] M. W. Evans, et al, over one hundred papers, 2003 to present on www.aias.us, and educational articles on ECE theory.
[5] M. W. Evans (ed.), "Modern Nonlinear Optics", a special topical issue in three parts of I. Prigogine and S. A. Rice (series eds.,), "Modern Nonlinear Optics" (Wiley, New York, 2001, second edition), vols. 119(1) to 119(3); M. W. Evans and S. Kielich (eds.), ibid., first edition (Wiley, New York, 1992, 1993, 1997), vols. 85(1) to 85(3).
[6] M. W. Evans and L. B. Crowell, "Classical and Quantum Electrodynamics and the B(3) Field" (World Scientific, 2001).
[7] M. W. Evans and J.-P. Vigier, "The Enigmatic Photon" (Kluwer, 1994 to 2002 in hardback and softback), in five volumes.
[8] M. W. Evans et al., Omnia Opera section of www.aias.us (1992 to 2003), precursor theories of ECE theory in circa twenty five journals.
[9] M. W. Evans and A. A. Hasanein, "The Photomagneton in Quantum Field Theory" (World Scientific, 1994).
[10] S. P. Carroll, "Space-time and Geometry: an Introduction to General Relativity" (Addison Wesley, New York, 2004).
[11] R. Santilli et al., on www.telesio-galilei.com.
[12] L. H. Ryder, "Quantum Field Theory" (Cambridge Univ. Press, 1996).
[13] A. Einstein, "The Meaning of Relativity" (Princeton Univ. Press, 1921–1954).