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A New Theory of Light Deflection Due to Gravitation

by

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Abstract

A new theory of light deflection due to gravitation is proposed based on Einstein Cartan Evans (ECE) unified field theory. It is shown that the Schwarzschild parameter of 1916 can be expressed as:

$$\alpha = -T/R$$

where T/R is the ratio of torsion to curvature, c is the vacuum speed of light and G is Newton's constant. In the Einstein Hilbert theory alpha is identified with the mass M of the object that deflects light. However this identification of alpha with M was not made by Schwarzschild and conflicts with the concept of Ricci flat vacuum used in the EH equation. In the new theory the deflection of light is governed by the ratio R/T and the theory suggests a simple explanation for the Pioneer anomaly, in which a small and anomalous gravitational attraction towards the sun has been found within experimental error.

Keywords: Light deflection due to gravitation, ECE theory, Pioneer anomaly.

12.1 Introduction

Recently a generally covariant unified field theory (GCUFT) has been developed [1–10] and accepted as providing one means of unifying physics with geometry, the aim of relativity theory. During the course of development of the theory it was found that the geometry of the Einstein Hilbert field equation is incompatible with the Hodge dual of the Bianchi identity. This incompatibility was discovered through the use of computer algebra, which showed that the Ricci type tensor $R^{\kappa}{}_{\mu}{}^{\mu\nu}$ is non-zero for all space-times that are not Ricci flat space-times. This result is incompatible with the fact that the inhomogeneous field equation of ECE theory is

$$D_{\mu}T^{\kappa\mu\nu} = R^{\kappa}{}_{\mu}{}^{\mu\nu} \tag{12.1}$$

On the left hand side occurs the covariant derivative of the torsion tensor $T^{\kappa\mu\nu}$, and for all Christoffel symbols this must be zero [11]. On the right hand side occurs the tensor $R^{\kappa}{}_{\mu}{}^{\mu\nu}$ which is non-zero in general as argued.

Therefore the Einstein Hilbert equation cannot be used to correctly describe the phenomenon of light deflection by an object of mass M. In order to achieve this aim this paper sets out to develop an expression for the parameter α used by Schwarzschild in 1916 [1–10] in two exact solutions of the EH field equation. It is known from this work by Schwarzschild that a line element of the type:

$$ds^2 = -(1-\alpha)c^2dt^2 + (1-\alpha)^{-1}dr^2 + r^2d\Omega^2 \tag{12.2}$$

with

$$\alpha = \frac{2GM}{c^2r} \tag{12.3}$$

describes the light deflection due to gravitation to an accuracy of 1: 100,000 [1–10]. Here G is the Newton constant, M is the mass of the gravitating object, c is the speed of light and r is the distance between the center of mass of the gravitating object and a point mass m called the particle. In the case of light being deflected by M the particle m is the photon. In the EH theory however, the Schwarzschild solution (12.2) is obtained with a Christoffel connection and by using a Ricci flat assumption. The Christoffel connection is incompatible with the Hodge dual of the Bianchi identity 1-10 and Crothers [1–10] has shown that the Ricci flat assumption is incompatible with the equivalence principle. Also, the Ricci flat assumption means a vanishing Ricci tensor, so that there is no energy momentum density present, no source present, and therefore no M. This is why the Schwarzschild solution is known as a vacuum solution. Computer algebra shows that the tensor $R^{\kappa}{}_{\mu}{}^{\mu\nu}$ is zero only for a vacuum solution, otherwise it is non-zero, but the torsion tensor is always zero. This is self-inconsistent and is the result of assuming a Christoffel connection. In the presence of torsion, the latter cannot be used to produce line elements, and conversely, a line element such as (12.2) cannot be used to compute ${R^\kappa}_\mu{}^{\mu\nu}$ through the use of a Christoffel symbol and a symmetric metric. These are all procedures of the EH equation, which arbitrarily and incorrectly neglects torsion. In the correct procedure adopted in this paper, the tensor and the torsion tensor are always non-zero, there is always a source for gravitation, and both the metric and connection are asymmetric. In this paper no assumption is made concerning their symmetry.

There are several reasons therefore for discarding the EH explanation of light bending due to gravitation, primarily because it neglects torsion, and also neglects the fact that a finite source M requires a finite energy momentum density. In general, the Christoffel connection must be asymmetric for finite torsion, and both the tensors $T^{\kappa\mu\nu}$ and ${R^\kappa}_\mu{}^{\mu\nu}$ must be non-zero. These fundamental geometrical requirements of Cartan geometry are reviewed in Section 12.2, leading to a definition of **g** in terms of elements of the torsion tensor. In Section 12.3 a new explanation of the experimental result for light bending is given, and it is shown that the Schwarzschild parameter is:

$$\alpha = -\frac{T}{R} \tag{12.4}$$

where T/R is a ratio of a well defined scalar torsion to a well defined scalar curvature. The experimental result of NASA Cassini is therefore produced when:

$$\alpha = -\frac{T}{R} = \frac{2GM}{c^2} \cdot \frac{1}{r} \tag{12.5}$$

but the small Pioneer anomaly [11, 12] requires alpha to be different from this. This is straightforwardly explained in ECE theory when the ratio T/R is slightly different from that needed to give M, the mass of the object that deflects light. There is no explanation for the Pioneer anomaly in the EH theory, as is well known. The calculation leading to (12.4) does not make any assumptions concerning the metric or connection, and self consistently accounts for the existence of torsion as well as curvature in the general four dimensional space-time. The result (12.4) is also compatible with both the Bianchi identity and the Hodge dual of the Bianchi identity.

12.2 The Fundamental Geometry

It has been shown in previous work that the Bianchi identity as developed by Cartan is, in an index-less notation [1–10]:

$$D \wedge T := R \wedge q \tag{12.6}$$

where $D\wedge$ denotes the covariant derivative, T denotes the torsion form, R denotes the curvature form, $\wedge$ denotes the wedge product and q denotes the tetrad form. In the base manifold, Eq. (12.6) is equivalent to the tensorial expression:

$$D_\mu \widetilde{T}^{\kappa\mu\nu} = \widetilde{R}^{\kappa}{}_{\mu}{}^{\mu\nu} \tag{12.7}$$

where D_μ is the covariant derivative, $\widetilde{T}^{\kappa\mu\nu}$ is the Hodge dual of the torsion tensor, and where $\widetilde{R}^{\kappa}{}_{\mu}{}^{\mu\nu}$ is the Hodge dual of the curvature tensor $R^{\kappa}{}_{\mu\alpha\beta}$. It has been shown [1–10] that there also exists the Hodge dual of Eq. (12.6), denoted:

$$D \wedge \widetilde{T} := \widetilde{R} \wedge q \tag{12.8}$$

and that the tensorial expression of Eq. (12.8) is:

$$D_\mu T^{\kappa\mu\nu} = R^{\kappa}{}_{\mu}{}^{\mu\nu}. \tag{12.9}$$

Eqs. (12.6) to (12.9) are the equations of gravitation in the presence of torsion. In general, neither the metric nor the connection in these equations can be symmetric, because a symmetric metric produces:

$$T^{\kappa\mu\nu} = 0, \;\; R^{\kappa}{}_{\mu}{}^{\mu\nu} \neq 0 \tag{12.10}$$

which is incompatible with Eq. (12.10). The geometry of the Einstein Hilbert (EH) equation produces Eq. (12.10), and so a new approach to the subject of gravitation is needed, one that is fully compatible with the Bianchi identity (12.6) and with its Hodge dual (12.8).

In the EH theory:

$$\widetilde{R}^{\kappa}{}_{\mu}{}^{\mu\nu} = 0 \tag{12.11}$$

because:

$$R^{\kappa}{}_{\mu\nu\rho} + R^{\kappa}{}_{\rho\mu\nu} + R^{\kappa}{}_{\nu\rho\mu} = 0. \tag{12.12}$$

This is the same as the so-called [11] "first Bianchi identity":

$$R \wedge q = q \wedge R = 0. \tag{12.13}$$

Eqs. (12.11) and (12.12) are true if and only if:

$$g_{\mu\nu} = g_{\nu\mu} \tag{12.14}$$

and

$$\Gamma^{\kappa}_{\mu\nu} = \Gamma^{\kappa}_{\nu\mu} \tag{12.15}$$

which is the Christoffel connection. Under the geometrically arbitrary assumptions (12.14) and (12.15) the torsion tensor vanishes:

$$T^{\kappa}_{\mu\nu} = \Gamma^{\kappa}_{\mu\nu} - \Gamma^{\kappa}_{\nu\mu} = 0. \tag{12.16}$$

It has also been shown that there is only one true Bianchi identity (12.6) in Cartan geometry. The so called "second Bianchi identity" of EH theory is a special case of Eq. (12.6) when there is no torsion. This was shown [1–10] by taking the covariant exterior derivative on both sides of Eq. (12.6) to give:

$$D \wedge (D \wedge T) := D \wedge (R \wedge q). \tag{12.17}$$

This reduces to

$$D \wedge R = 0 \tag{12.18}$$

when the torsion is arbitrarily neglected. Reinstating the indices in Eq. (12.18) gives:

$$D \wedge R^{\kappa}{}_{\mu\nu\rho} = 0 \tag{12.19}$$

i.e:

$$D_\sigma R^{\kappa}{}_{\mu\nu\rho} + D_\rho R^{\kappa}{}_{\mu\sigma\nu} + D_\nu R^{\kappa}{}_{\mu\rho\sigma} = 0 \tag{12.20}$$

which is almost always referred to in the EH literature as "the second Bianchi identity", whereas the true identity is Eq. (12.6) and its Hodge dual, Eq. (12.8).

The usual EH approach to light deflection in the solar system is therefore geometrically incorrect. It also suffers from the assumption that the Ricci tensor is zero by construction [1–10], and from the assumption that the Schwarzschild parameter α of 1916 is assumed arbitrarily and incorrectly [1–10] to be determined by the mass M of the attracting object (the solar mass for example). So the EH method is self-inconsistent fundamentally. Furthermore, Crothers [1–10] has shown that the method of solution used in EH field theory is also fundamentally flawed, notably the proper radius and radius of curvature of a line element is confused [1–10]. This confusion means that an initial singularity cannot be assumed in the EH theory, and cannot be deduced geometrically. Therefore theories that depend on an initial singularity such as Big Bang, cannot be correct geometrically. They have apparently developed uncritically for ninety years. There are several other well known criticisms and limitations of the EH theory on a website such as www.telesio-galilei.com.

The geometrically correct equations of Cartan geometry are (12.6) and (12.8). In principle these must be solved without any a priori assumption about the symmetry of the metric and connection. Therefore this solution in general must be a numerical one, using a supercomputer. In an analytical approximation however the weak field limit may be considered. The latter can be defined as the limiting approach to Minkowski or flat space-time. In this limit it is assumed that the spin connection goes to zero, so Eq. (12.9) is approximated by:

$$\partial_\mu T^{\kappa\mu\nu} \doteq R^{\kappa}{}_{\mu}{}^{\mu\nu}. \tag{12.21}$$

In vector notation [1–10], Eq. (12.21) is:

$$\boldsymbol{\nabla} \cdot \boldsymbol{T}_1 = R_1, \tag{12.22}$$

$$\boldsymbol{\nabla} \times \boldsymbol{T}_2 - \frac{1}{c}\frac{\partial \boldsymbol{T}_3}{\partial t} = \boldsymbol{R}_2, \tag{12.23}$$

were the subscripts denote particular types of torsion and curvature defined by:

$$\boldsymbol{T}_1 = T^{010}\boldsymbol{i} + T^{020}\boldsymbol{j} + T^{030}\boldsymbol{k}, \tag{12.24}$$

$$R_1 = R^{0}{}_{1}{}^{10} + R^{0}{}_{2}{}^{20} + R^{0}{}_{3}{}^{30}, \tag{12.25}$$

$$\boldsymbol{T}_2 = T^{332}\boldsymbol{i} + T^{113}\boldsymbol{j} + T^{221}\boldsymbol{k}, \tag{12.26}$$

$$\boldsymbol{T}_3 = T^{110}\boldsymbol{i} + T^{220}\boldsymbol{j} + T^{330}\boldsymbol{k}, \tag{12.27}$$

and where $\mathbf{R}_2$ is defined by Eqs. (12.26) and (12.27). The Newton inverse square law is obtained in this weak field limit as:

$$\boldsymbol{\nabla} \cdot \boldsymbol{g} = c^2 R. \tag{12.28}$$

The structures of Eqs. (12.22) and (12.28) are the same so:

$$\boldsymbol{g} = c^2 \boldsymbol{T}_1, R = R_1. \tag{12.29}$$

The acceleration due to gravity in the weak field limit is therefore:

$$\boldsymbol{g} = c^2 (T^{010}\boldsymbol{i} + T^{020}\boldsymbol{j} + T^{030}\boldsymbol{k}) \tag{12.30}$$

and the mass density is defined by [1–11]:

$$\rho_m = \frac{1}{k}(R^0{}_1{}^{10} + R^0{}_2{}^{20} + R^0{}_3{}^{30}) \tag{12.31}$$

where k is Einstein's constant.

In a Ricci flat space-time, and using a symmetric metric and connection, it is found from Eq. (12.31) using computer algebra [1–10] that:

$$\rho_m(\text{Ricci flat}) = 0. \tag{12.32}$$

In such a space-time there is therefore no acceleration due to gravity because:

$$\boldsymbol{g} = \boldsymbol{0}. \tag{12.33}$$

The elements T^{010}, T^{020}, and T^{030} are proportional to elements of the angular energy momentum density tensor. In other words if there is no source, i.e. no angular energy momentum density, there is no mass density ($\rho_m = 0$) and no field ($\mathbf{g} = \mathbf{0}$). Therefore the Ricci flat approach to solutions of the EH equation is self-inconsistent. Light bending due to gravitation is governed by Eq. (12.28), and by six components $T^{010}, T^{020}, T^{030}, R^0{}_1{}^{10}, R^0{}_2{}^{20}$ and $R^0{}_3{}^{30}$ in general. If $\mathbf{g}$ is restricted to one axis, e.g. $\mathbf{k}$, then the problem reduces to two components, T^{030} and $R^0{}_3{}^{30}$, i.e.:

$$\boldsymbol{g} = c^2 T^{030}\boldsymbol{k},\ \ \rho_m = \frac{1}{k}R^0{}_3{}^{30}. \tag{12.34}$$

12.3 The General Geodesic Method

The basic method used in EH theory to calculate the light deflection due to gravitation is the null geodesic method based on the following condition on the general line element. Cartesian coordinates are used in the following:

$$\begin{aligned} ds^2 &= -g_{00}c^2dt^2 + g_{11}dX^2 + g_{22}dY^2 + g_{33}dZ^2 \\ &= 0. \end{aligned} \tag{12.35}$$

This method and the geodesic equation remain valid in ECE theory provided no assumptions are made concerning the symmetry of the metric and connection. The general geodesic equation is [11]:

$$\epsilon = -g_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda} = \text{constant} \tag{12.36}$$

where $g_{\mu\nu}$ is the general asymmetric metric. In spherical polar coordinates Eq. (12.36) is

$$-g_{00}c^2\left(\frac{dt}{d\lambda}\right)^2+g_{11}\left(\frac{dr}{d\lambda}\right)^2+g_{22}r^2\left(\frac{d\phi}{d\lambda}\right)^2=-\epsilon \tag{12.37}$$

if the metric matrix is diagonal. The conserved quantities may be defined in general [11] by:

$$E=g_{00}c\frac{dt}{d\lambda},\ \ L=g_{22}r^2\frac{d\phi}{d\lambda} \tag{12.38}$$

so that Eq. (12.37) becomes:

$$-\frac{E^2}{2}+\frac{1}{2}g_{00}g_{11}\left(\frac{dr}{d\lambda}\right)^2+\frac{g_{00}}{2}\left(g_{22}r^2\left(\frac{d\phi}{d\lambda}\right)^2+\epsilon\right)=0. \tag{12.39}$$

Define the potential in reduced units by:

$$V:=\frac{1}{2}g_{00}\left(g_{22}r^2\left(\frac{d\phi}{d\lambda}\right)^2+\epsilon\right). \tag{12.40}$$

where

$$\lambda=c\tau \tag{12.41}$$

and where τ is the proper time. Define the potential energy in units of joules by:

$$V:=\frac{1}{2}mc^2g_{00}\left(g_{22}r^2\left(\frac{d\phi}{d\lambda}\right)^2+\epsilon\right). \tag{12.42}$$

To consider light deflected by mass the null geodesic is used, so:

$$\epsilon=0 \tag{12.43}$$

and in this case:

$$V=\frac{1}{2}mg_{00}g_{22}r^2\left(\frac{d\phi}{d\tau}\right)^2. \tag{12.44}$$

Without loss of generality it may be assumed that:

$$g_{22} = 1 \tag{12.45}$$

so that the potential energy in joules is:

$$V = \frac{1}{2} m g_{00} r^2 \left(\frac{d\phi}{d\tau} \right)^2 . \tag{12.46}$$

This expression must now be related to the potential Φ of the ECE theory, and to the curvature, spin connection and torsion. From this, an explanation of may g_{00} be found from first principles. The ECE theory gives in general:

$$\boldsymbol{\nabla} \cdot \boldsymbol{g} = c^2 (R - \omega T) \tag{12.47}$$

where R, omega and T are defined by:

$$R := R^{0}{}_{1}{}^{10} + R^{0}{}_{2}{}^{20} + R^{0}{}_{3}{}^{30}, \tag{12.48}$$

$$\omega T := (\omega^{0}{}_{1\lambda} T^{\lambda 10} + \omega^{0}{}_{2\lambda} T^{\lambda 20} + \omega^{0}{}_{3\lambda} T^{\lambda 30}). \tag{12.49}$$

Here the Einstein and Newton constants in S. I. units are:

$$k = 1.86595 \times 10^{-26} N s^2 kg^{-2}, \tag{12.50}$$

$$G = 6.6726 \times 10^{-11} N m^2 kg^{-2}. \tag{12.51}$$

Firstly the Newtonian limit may be defined by:

$$R \gg \omega T \tag{12.52}$$

i.e.

$$\omega \to 0. \tag{12.53}$$

In this limit:

$$\boldsymbol{g} = -\boldsymbol{\nabla} \Phi = \frac{G m_2}{r^2} \boldsymbol{r} \tag{12.54}$$

and the force is:

$$\boldsymbol{F} = m_1 \boldsymbol{g} = -\left(\frac{G m_1 m_2}{r^2} \right) \boldsymbol{r}. \tag{12.55}$$

Therefore:

$$\nabla \cdot \boldsymbol{g} = 2G\frac{m_2}{r^3} \tag{12.56}$$

and

$$\rho_m = \frac{m_2}{4\pi r^3} := \frac{m_2}{3V_0}. \tag{12.57}$$

Therefore the mass m_2 is defined by a curvature:

$$R = \frac{km_2}{3V_0} = R^0{}_1{}^{10} + R^0{}_2{}^{20} + R^0{}_3{}^{30}. \tag{12.58}$$

If the spin connection is fully considered, the acceleration due to gravity is defined in a manner analogous to the electric field strength in the Coulomb law of ECE theory [1–10]

$$\boldsymbol{g} = -(\nabla + \boldsymbol{\omega})\Phi. \tag{12.59}$$

Here

$$V = m_2\Phi. \tag{12.60}$$

The basic equations of the system are therefore:

$$V = m_2 g_{00} r^2 (d\phi/d\tau)^2, \tag{12.61}$$

$$\boldsymbol{g} = -(\nabla + \boldsymbol{\omega})\Phi, \tag{12.62}$$

$$\nabla \cdot \boldsymbol{g} = c^2(R - \omega T), \tag{12.63}$$

$$V = m_2\Phi, \tag{12.64}$$

and the mathematical problem is to solve this system of equations to give an expression for g_{00} in terms of R, ω, and T. The expression for g_{00} may then be used with the structure of Schwarzschild's solution to define his α parameter in terms of R and T.

From Eq. (12.63):

$$g = c^2 \int (R - \omega T)\, dr. \tag{12.65}$$

From Eq. (12.62):

$$g = -\frac{\partial \Phi}{\partial r} - \omega \Phi \tag{12.66}$$

where

$$\Phi = g_{00} r^2 \left(\frac{d\phi}{d\tau} \right)^2 . \tag{12.67}$$

From Eqs. (12.66) and (12.67):

$$g = -g_{00} r \left(\frac{d\phi}{d\tau} \right)^2 (2 + \omega r). \tag{12.68}$$

If it is assumed as in paper 63 of www.aias.us that the radial component of the spin connection vector is:

$$\omega = -\frac{1}{r} \tag{12.69}$$

then:

$$g = -g_{00} r \left(\frac{d\phi}{d\tau} \right)^2 = c^2 \int \left(R + \frac{T}{r} \right) dr. \tag{12.70}$$

Differentiating both sides of this equation gives:

$$-g_{00} \left(\frac{d\phi}{d\tau} \right)^2 = c^2 \left(R + \frac{T}{r} \right) \tag{12.71}$$

and

$$g_{00} = c^2 \left(R + \frac{T}{r} \right) / \left(\frac{d\phi}{d\tau} \right)^2 . \tag{12.72}$$

This is the required expression for g_{00} in terms of R and T.

The experimental result of NASA Cassini is that the light deflection by the sun is twice the Newtonian value within 1 : 100,000. Such a result is given by [1–11]:

$$g_{00} = -\left(1 - \frac{2GM}{c^2 r} \right) \tag{12.73}$$

where M is the mass of the sun. It is seen that the structures of Eqs. (12.72) and (12.73) are the same. Comparing the equations:

$$R = \frac{1}{c^2}\left(\frac{d\phi}{d\tau}\right)^2 \tag{12.74}$$

and

$$T = -\frac{2GM}{c^4}\left(\frac{d\phi}{d\tau}\right)^2 \tag{12.75}$$

i.e.

$$T = -\frac{2GM}{c^2}R. \tag{12.76}$$

In the original solution of EH for a vacuum, given by Schwarzschild in 1916, a parameter α was used, defined by:

$$\alpha = \frac{2GM}{rc^2}(\text{Cassini result}) \tag{12.77}$$

so

$$\alpha = -\frac{T}{R}. \tag{12.78}$$

Therefore the structure of α has been deduced from the true Bianchi identity without making any assumptions concerning either the metric or the spin connection and also without assuming a Ricci flat condition. The EH result coincides with:

$$-\frac{T}{R} = \frac{2GM}{rc^2} \tag{12.79}$$

but as the Pioneer anomaly [12] shows, this may not be the case in general, there is a small anomalous acceleration of $(8.74 \pm 1.33) \times 10^{-10}\text{ms}^{-2}$ towards the sun which cannot be explained in EH because M cannot be adjusted. However it is explained in ECE as a small deviation from the result (12.79), i.e. a small increment in T/R.

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