18

The Origin of Orbits in Spherically Symmetric Space-Time

by

Myron W. Evans,
Alpha Institute for Advanced Study, Civil List Scientist.
(emyrone@aol.com and www.aias.us)

and

Horst Eckardt,
Alpha Institute for Advanced Study/S.G.
(www.telesio-galilei.com)

Abstract

It is shown that the origin of all known orbital trajectories is the spherical symmetry of space-time, and not the Einstein field equation. A consideration of simple integral equations leads to the mathematical structure of the line element needed to describe all known orbits. The orbital equation is deduced as usual from the line element, which results directly from the spherical symmetry of space-time. All known orbits are described therefore by the spherical symmetry of space-time, irrespective of any field equation. The Einstein field equation is shown to be geometrically incorrect, and replaced by the ECE field equation of dynamics, based directly on the Bianchi identity.

Keywords: Origin of orbits, spherical symmetry of space-time, ECE equations of dynamics.

18.1 Introduction

Contemporary high precision satellite data are available to re-assess the validity of the Einstein field equation. It is known [1–12] that this equation is based directly on a type of Riemann geometry that is intrinsically self-inconsistent.

This type of Riemann geometry is based in turn on the well known Christoffel connection, which is symmetric in its lower two indices. Such a connection means that the torsion tensor is zero. However, this is inconsistent with the fact that the torsion tensor is produced by the commutator of covariant derivatives [11] acting on the vector in any dimension and irrespective of any assumed connection. The torsion tensor is thereby an ineluctable counterpart of the curvature tensor and it cannot be assumed that the torsion tensor vanishes. It was pointed out by Alley [13], in the 2006 Wheeeler Fest at Princeton, that as early as 1918, Bauer and Schroedinger demonstrated independently that there are severe internal inconsistencies in the approach to relativity taken by Einstein and by Hilbert in 1915. Einstein agreed with Schroedinger that his field equation is self-inconsistent [13], but this fact has been forgotten by history. Bauer pointed out that the Minkowski metric, which is constructed in the absence of gravitation, nevertheless produces a non zero type of stress energy momentum tensor for gravitation, if Einstein's method is applied. Schroedinger pointed out that the Einstein field equation does not self-consistently produce this type of tensor at all. Yilmaz [13] has attempted to remedy this self-inconsistency, as has Wheeler [13]. However, the attempts by Yilmaz and Wheeler still assume a zero torsion tensor. In the work of Rapoport, Santilli and others [14], the original approach by Einstein has been greatly extended. Rapoport [14] has successfully applied the torsion tensor to several areas of physics, including Brownian motion, stochastic vacuum physics, fractal physics and thermodynamics.

From 2003 onwards a generally covariant unified field theory [1–10] has been developed based on standard Cartan geometry [11], namely the two Cartan structure equations, the Bianchi/Cartan identity, and the tetrad postulate. The first and second structure equations respectively define the torsion as the covariant exterior derivative of the tetrad, and define the curvature in terms of the spin connection. The Bianchi identity has been proven [1–10] to be invariant under the Hodge duality transformation in four dimensions. In general both the Bianchi identity and its Hodge dual must be considered. They are two parts of the Bianchi identity and it is not admissible to consider one without the other. As pointed out by Santilli [14] there are several logical flaws in the Einstein field equation and in its geometry. Another one of these was discovered in 2007 in paper 93 of the ECE series on www.aias.us. Computer algebra was applied to the complete, duality invariant, Bianchi identity. It was found by direct computation that exact solutions of the Einstein field equation do not in general obey the complete Bianchi identity. The reason is the neglect of torsion, i.e. the use of the Christoffel connection is not consistent with the Bianchi identity. The so called first and second Bianchi identities that appear in the gravitational literature are incomplete, because they neglect torsion. The so-called first Bianchi identity was in fact inferred by Ricci and Levi-Civita, and is not in fact an identity. It is an equation that is true if an only if the metric and connection are symmetric, i.e. if and only if the torsion vanishes. The so called second Bianchi identity is in fact not an identity, it is an equation which is again true if and only if the torsion vanishes. The Einstein approach to relativity is based directly on this torsion-less second Bianchi identity. Paper 93 of www.aias.us shows that the Einstein approach is not just incomplete, but incorrect. Such a conclusion has been reached repeatedly, this is the ninetieth anniversary of the criticisms by Bauer and Schroedinger, criticisms with which Einstein agreed [13] and which are reviewed in a book by Santilli on www.telesio-galilei.com and www.aias.us.

The contemporary assertion that there is a "standard model" of physics therefore collapses entirely, because standard cosmologies are based on the incorrect Einstein field equation. The ECE field equations of dynamics, electrodynamics and cosmology [1–10] on the other hand are based on the self-consistent use of torsion in the duality invariant Bianchi identity of Cartan geometry. Clearly, ECE theory can be extended with the use of other geometries, such as those advocated by Rapoport [14], Santilli [14], Schadeck and Connes [14].

In Section 18.2 it is demonstrated straightforwardly that all known orbits are due to the spherical symmetry of space-time, irrespective of any assumed field equation or connection. The general line element for a spherically symmetric space-time is well known to have a general mathematical structure, and in Section 18.2 it is demonstrated in a very simple way that a particular solution this general structure is responsible for all known orbits to known experimental precision.

In Section 18.3 the line element derived in Section 18.2 is developed with the ECE equations of dynamics to produce a self consistent result. Computer algebra is used in both Sections 18.2 and 18.3 to eliminate the labor of calculation and to eliminate all possibility of purely calculational human error.

18.2 Line Element for a Spherically Symmetric Space-Time

Consider the simple equation:

$$r = \int dr \tag{18.1}$$

where r is the radial vector. This equation is developed in a spherically symmetric space-time which can be represented with coordinate systems such as the spherical polar (r, θ, ϕ) or Cartesian (X, Y, Z). In Eq. (18.1) the constant of integration, denoted μ, has been assumed to be zero:

$$\mu = 0. \tag{18.2}$$

The following line element [11] is a representation of a spherically symmetric space-time:

$$ds^2 = -mc^2dt^2 + ndr^2 + r^2d\Omega^2 \tag{18.3}$$

where $d\Omega^2$ is the infinitesimal volume element in spherical polar coordinates. Eq. (18.3) is not the most general line element possible mathematically for a spherically symmetric space-time, but as the experimental satellite data show, will be sufficient to describe the experiments.

Assume that the function m is defined by:

$$mr = \int dr \tag{18.4}$$

and that the function n is defined by:

$$\frac{r}{n} = \int dr \tag{18.5}$$

and assume that the constant of integration μ is not zero, and is the same in Eqs. (18.4) and (18.5). It follows that:

$$m = \frac{1}{n} = 1 + \frac{\mu}{r} \tag{18.6}$$

and that a spherically symmetric metric can be described by the structure:

$$ds^2 = -\left(1 + \frac{\mu}{r}\right)c^2dt^2 + \left(1 + \frac{\mu}{r}\right)^{-1} + r^2d\Omega^2. \tag{18.7}$$

It is known from experimental satellite data that in almost all known orbits:

$$\mu = -\frac{2MG}{c^2} \tag{18.8}$$

where M is the mass of the object about which orbits an object of mass m. Here G is Newton's constant and c is the vacuum speed of light.

Therefore nearly all known orbits are manifestations of the spherically symmetry of space-time, because the orbital equation is obtained from the line element as is well known [1–12]. The only known deviations from Eq. (18.8) are the orbits of binary pulsars, in which the orbits slowly spiral inwards by an extremely tiny amount per revolution. It was shown in paper 108 of the

ECE series (www.aias.us) that binary pulsar orbits are described in all detail by:

$$\mu = -\left(\frac{2MG}{c^2} + \frac{a}{r}\right) \tag{18.9}$$

where a is a constant perturbation of very tiny magnitude. There are well known systematic anomalies recorded by the Cassini and Pioneer spacecraft independently, these may indicate a deviation from Eq. (18.8), but may be due to mundane artifacts such as leaking gas in the propellant system. If these anomalies are indeed systematic within the uncertainty, as seems likely, they are described by Eq. (18.9), again with a spherically symmetric space-time.

Therefore equations of type (18.1), (18.4) and (18.5) are the basic reason for all known orbits. It may be that this type of simple integral structure has a fundamental significance in topology.

Using computer algebra (Maxima, paper 93 of www.aias.us) a line element of type (18.8) was evaluated with an assumed Christoffel connection. It was found that this assumption produces a correct solution of the Bianchi identity:

$$D \wedge T^a = R^a{}_b \wedge q^b \tag{18.10}$$

and also of the Hodge dual identity

$$D \wedge \widetilde{T}^a = \widetilde{R}^a{}_b \wedge q^b. \tag{18.11}$$

Here T is shorthand (index - less) notation for the Cartan torsion, $D\wedge$ denotes the covariant exterior derivative and R denotes the Cartan curvature. The tilde denotes Hodge dual. The reason for this is that if the Christoffel connection is used with the spherically symmetric line element of type (18.8), then it follows that:

$$T^a = 0, \ \widetilde{T}^a = 0, \ R^a{}_b \wedge q^b = 0, \ \widetilde{R}^a{}_b \wedge q^b = 0. \tag{18.12}$$

This result may be considered to be the fundamental reason why orbits are as observed in the vast majority of cases. The reason is that orbits are central phenomena, for which a symmetric connection suffices. If rotation is considered, a symmetric connection no longer suffices to describe the dynamics. Note carefully that the result (18.7) does not rely on any field equation. The explanation for orbits cannot be the incorrect Einstein field equation. The result (18.12) is fortuitously true for vacuum (Ricci flat) solutions of the Einstein field equation, but such vacuum solutions cannot have any physical significance.. As soon as a finite energy momentum density tensor is introduced, i.e. as soon as we consider:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} := G_{\mu\nu} = kT_{\mu\nu} \tag{18.13}$$

all exact solutions of the Einstein field equation VIOLATE the Bianchi identity [1–10], and so the Einstein field equation is clearly seen to be geometrically incorrect. Crothers has shown on www.aias.us and elsewhere that the Ricci flat assumption violates the Einstein principle of equivalence. The reason why central orbits are as observed is the spherical symmetry of space-time, not the vanishing of the Ricci tensor, and above all, not the Einstein field equation (18.13).

If the binary pulsar perturbation a/r is added, i.e. if we consider:

$$ds^2 = -\left(1 - \frac{2MG}{c^2 r} - \frac{a}{r^2}\right) c^2 dt^2 + \left(1 - \frac{2MG}{c^2 r} - \frac{a}{r^2}\right)^{-1} dr^2 + r^2 d\Omega^2 \tag{18.14}$$

computer algebra can be used to work out all the Christoffel connections from Eq. (18.14), assuming metric compatibility [11]. Knowing the Christoffel connections, the computer algebra is used to work out all the Riemann tensor elements from (18.14), the Ricci tensor elements, and the Ricci scalar. Finally the code was used to investigate whether Eq. (18.14) obeys metric compatibility and also both Eq. (18.10) and Eq. (18.11) under the assumption of a Christoffel connection. It was found that Eq. (18.14), used with an assumed Christoffel connection, obeys metric compatibility and Eq. (18.10), but does not obey Eq. (18.11). It does not produce a zero Ricci tensor in general, but produces a zero Ricci scalar:

$$R = g^{\mu\nu} R_{\mu\nu} = 0. \tag{18.15}$$

Therefore if a Christoffel connection is assumed, internal inconsistencies appear in the approach taken by Einstein to relativity, in that the complete, duality invariant, Bianchi identity is not obeyed, and in that:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = kT_{\mu\nu} \neq 0 \tag{18.16}$$

but, self-inconsistently:

$$R = -kT = 0. \tag{18.17}$$

It is known from paper 108 of www.aias.us that Eq. (18.14) produces all known features of the orbits of binary pulsars, so it is concluded that the Einstein field equation cannot describe such orbits. In the following section the ECE field equation is shown to provide a self-consistent explanation of all known orbits. This is achieved directly from the Bianchi identity and its Hodge dual, which give rise to the homogeneous and inhomogeneous ECE field equations of dynamics, cosmology and electrodynamics, and all fundamental interactions of classical fields.

18.3 General Equation for the Metric in ECE Theory

It has been shown in paper 108 of the ECE series (www.aias.us) that the general expression for the potential energy from a line element is:

$$V = \frac{m}{2} g_{00} \left(c^2 + \frac{L^2}{r^2} \right) \tag{18.18}$$

where m is the mass of an object orbiting an object of mass M, g_{00} is the time-like metric, L is the reduced angular momentum, the constant of motion:

$$L = r^2 \frac{\partial \phi}{\partial \lambda} \tag{18.19}$$

and λ a differential parameter. The gravitational potential Φ is obtained from the potential energy by:

$$\Phi = \frac{V}{m} \tag{18.20}$$

and in ECE theory [1–10] the acceleration due to gravity is related to the gravitational potential by:

$$\boldsymbol{g} = -(\boldsymbol{\nabla} + \boldsymbol{\omega})\Phi \tag{18.21}$$

where $\boldsymbol{\omega}$ is the spin connection vector. The Bianchi identity leads to the equation:

$$\boldsymbol{\nabla} \cdot \boldsymbol{g} = c^2 (R - \omega T) \tag{18.22}$$

which is the gravitational equivalent of the Coulomb law. Therefore:

$$\boldsymbol{\nabla} \cdot (\boldsymbol{\nabla}\Phi + \boldsymbol{\omega}\Phi) = c^2 (\omega T - R) \tag{18.23}$$

which is a differential equation for g_{00} in terms of ω, T, R and L in general. This equation makes no assumption other than those of Cartan geometry [1–10] (paper 105 of www.aias.us). In spherical polar coordinates:

$$\boldsymbol{\nabla} \cdot (\boldsymbol{\nabla}\Phi) = \frac{\partial^2 \Phi}{\partial r^2} + \frac{2}{r}\frac{\partial \Phi}{\partial r}, \tag{18.24}$$

$$\boldsymbol{\nabla} \cdot (\boldsymbol{\omega}\Phi) = \boldsymbol{\omega} \cdot \boldsymbol{\nabla}\Phi + (\boldsymbol{\nabla} \cdot \boldsymbol{\omega})\Phi \tag{18.25}$$

so:

$$\frac{\partial^2 \Phi}{\partial r^2} + \left(\frac{2}{r} + \omega\right)\frac{\partial \Phi}{\partial r} + \frac{\partial \omega}{\partial r}\Phi = c^2(\omega T - R). \tag{18.26}$$

This equation has resonance solutions under well defined conditions.

From Eqs. (18.20) and (18.26):

$$\begin{aligned} &\frac{1}{2}\left(c^2 + \frac{L^2}{r^2}\right)\frac{\partial^2 g_{00}}{\partial r^2} + \left(\frac{1}{2}\left(\frac{2}{r} + \omega\right)\left(c^2 + \frac{L^2}{r^2}\right) - \frac{L^2}{r^3}\right)\frac{\partial g_{00}}{\partial r} \\ &+ \left(L^2\left(\frac{6}{r^4} - \left(\frac{2}{r} + \omega\right)\frac{1}{r^3}\right) + \frac{1}{2}\left(c^2 + \frac{L^2}{r^2}\right)\frac{\partial \omega}{\partial r}\right) g_{00} \\ &= c^2(\omega T - R) \end{aligned} \tag{18.27}$$

This equation may be integrated numerically to give g_{00}, or otherwise solved by computer algebra in given approximations. Precise satellite data show that:

$$g_{00} = 1 + \frac{\mu}{r} \tag{18.28}$$

where

$$\mu = -\frac{2MG}{c^2} \tag{18.29}$$

except for binary pulsars and the apparent Pioneer/Cassini anomalies. In the limit:

$$r \to \infty, \quad g_{00} \to 1 \tag{18.30}$$

so

$$\frac{\partial^2 g_{00}}{\partial r^2} \to 0, \quad \frac{\partial g_{00}}{\partial r} \to 0 \tag{18.31}$$

In this limit:

$$\omega \to 0, \quad \frac{\partial \omega}{\partial r} \to 0 \tag{18.32}$$

so:

$$\omega T - R \to \frac{4L^2}{c^2 R^4}\left(1 + \frac{\mu}{r}\right) \tag{18.33}$$

A particular solution of Eq. (18.33) is:

$$\omega T = \frac{4L^2}{c^2 r^4}\left(\frac{\mu}{r}\right) \tag{18.34}$$

so:

$$\boldsymbol{\nabla} \cdot \boldsymbol{g} = c^2 R\left(1 - \frac{\mu}{r}\right). \tag{18.35}$$

Empirically, except for binary pulsars:

$$\boldsymbol{\nabla} \cdot \boldsymbol{g} = c^2 k \rho_m \left(1 + \frac{2MG}{c^2 r}\right) \tag{18.36}$$

where ρ_m is the mass density (paper 105 of www.aias.us). The structure of ECE theory means that the Coulomb law is corrected in this weak field limit by:

$$\boldsymbol{\nabla} \cdot \boldsymbol{E} = \frac{\rho_e}{\epsilon_0}\left(1 + \frac{2MG}{c^2 r}\right) \tag{18.37}$$

where ρ_e is the charge density. In this weak field limit:

$$\boldsymbol{E} \to -\boldsymbol{\nabla}\phi \tag{18.38}$$

and the Poisson equation is corrected by mass as follows:

$$\nabla^2 \phi = -\frac{\rho_e}{\epsilon_0}\left(1 + \frac{2MG}{c^2 r}\right). \tag{18.39}$$

Therefore the scalar potential of electrodynamics is corrected by:

$$\phi = \frac{e}{4\pi\epsilon_0 r}\left(1 + \frac{MG}{rc^2}\right). \tag{18.40}$$

In the laboratory and in quantum chemistry this correction of the Coulomb law is entirely negligible, except when spin connection resonance occurs (for example papers 63 and 94 of www.aias.us). When the mass M becomes very large however, the correction becomes significant and may become experimentally observable.

In this analysis the torsion is self-consistently incorporated. The analysis starts from the Hodge dual of the Bianchi identity, which is:

$$D \wedge \widetilde{T}^a := \widetilde{R}^a{}_b \wedge q^b. \tag{18.41}$$

In tensor notation this becomes:

$$D_\mu T^{a\mu\nu} := R^a{}_\mu{}^{\mu\nu} \tag{18.42}$$

a particular solution of which is:

$$D_\mu T^{\kappa\mu\nu} = R^\kappa{}_\mu{}^{\mu\nu} \tag{18.43}$$

Eq. (18.43) may be written as:

$$\partial_\mu T^{\kappa\mu\nu} = R^\kappa{}_\mu{}^{\mu\nu} - \omega^\kappa{}_{\mu\lambda} T^{\lambda\mu\nu} \tag{18.44}$$

and this can be expressed as two vector equations, the inhomogeneous field equations of ECE theory. One of these may be written as:

$$\boldsymbol{\nabla} \cdot \boldsymbol{T} = R - \omega T \tag{18.45}$$

and using the hypothesis in paper 105 of www.aias.us:

$$\boldsymbol{g} = c^2 \boldsymbol{T} \tag{18.46}$$

Eq. (18.22) is obtained. Here [1–10]:

$$\boldsymbol{T} := T^{010}\boldsymbol{i} + T^{020}\boldsymbol{j} + T^{030}\boldsymbol{k}, \tag{18.47}$$

$$R := R^0{}_1{}^{10} + R^0{}_2{}^{20} + R^0{}_3{}^{30}, \tag{18.48}$$

$$\omega T := (\omega^0{}_{1\lambda} T^{\lambda 10} + \omega^0{}_{2\lambda} T^{\lambda 20} + \omega^0{}_{3\lambda} T^{\lambda 30}) \tag{18.49}$$

The relation between **g** and Φ in Eq. (18.21) is derived from the first Cartan structure equation [1–10]. Therefore the equations of dynamics and cosmology, and of classical electrodynamics, are all based rigorously on Cartan geometry with torsion.

In his two original papers of 1916, Schwarzschild derived:

$$g_{00} = 1 + \frac{\mu}{r} \tag{18.50}$$

without consideration of torsion. Eq. (18.50) as derived by Schwarzschild is a purely geometrical solution (as pointed out by Crothers on www.aias.us). It is a solution of

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 0 \tag{18.51}$$

upon assuming that the connection is symmetric. As such it has no physical significance, because Eq. (18.51) eliminates all mass density from consideration, meaning that the object M cannot interact with the object m. Crothers has also shown that Eq. (18.51) violates the Einstein equivalence principle, and has also demonstrated basic errors in the theory of "big bang" and of "black holes". There are plentiful experimental data [1–10] that refute big bang comprehensively. In this paper it has been shown that the satellite data now available can be explained by considering only the spherical symmetry of space-time.

Acknowledgments

The British Government is thanked for a Civil List Pension to MWE and the staff of AIAS/SG for many interesting discussions.

References

[1] M. W. Evans, "Generally Covariant Unified Field Theory" (Abramis Academic, 2005 onwards), vols. 1–4, vol. 5 in prep. (preprints on www.aias.us).

[2] L .Felker, "The Evans Equations of Unified Field Theory" (Abramis, 2007).

[3] K. Pendergast, "Crystal Spheres" (preprint on www.aias.us).

[4] M. W. Evans, Acta Physica Polonica, **38B**, 2211 (2007).

[5] M. W. Evans and H. Eckardt, Physica B, **400**, 175 (2007).

[6] M. W. Evans, Physica B, **403**, 517 (2008).

[7] M. W. Evans (ed.), Adv. Chem. Phys., **119** (2001), ibid., M. W. Evans and S. Kielcih (eds.), Adv. Chem. Phys., **85** (1992, 1993, 1997).

[8] M. W. Evans and L. Crowell, "Classical and Quantum Electrodynamics and the B(3) Field" (World Scientific, 2001).

[9] M. W. Evans and J.-P. Vigier, "The Enigmatic Photon" (Kluwer, 1994 to 2002, hardback and softback), in five volumes.

[10] M. W. Evans, Omnia Opera section of www.aias.us (1992 onwards).

[11] S. P. Carroll, "Space-time and Geometry - an Introduction to General Relativity" (Addisaon-Wesley, New York, 2004, online notes of 1997).

[12] R. Wald, "General Relativity" (Univ. Chicago Press, 1986).

[13] C. Alley, tycho.usno.navy.mil/IAU31/alley.ppt (2006).

[14] AIAS/SG Discussions of May 2008.