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Derivation of the Gravitational Red Shift from the Theorem of Orbits

by

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Abstract

The experimentally observable gravitational red shift is derived by rotating the line element derived from the Theorem of Orbits. The latter is a simple special case of the Frobenius Theorem for a spherically symmetric space-time. All known orbits are described by the geometry of the Theorem of Orbits, and the gravitational red shift is shown to be the precession or phase shift caused by rotating the line element of the Theorem of Orbits.

Keywords: ECE Theory, gravitational red shift, Theorem of Orbits, line element rotation.

21.1 Introduction

Recently in the ECE series of papers [1–10] it has been shown that all known orbits can be described directly by the spherical symmetry of space-time with torsion and curvature without having to use any field equation a priori. The Theorem of Orbits (paper 111) has been derived from the well known [11] Frobenius Theorem applied to a spherically symmetric space-time. From the Theorem of Orbits the line element is derived, giving the orbital equation. Therefore the field of force (which becomes the Newtonian field of force in the appropriate limit) is derived directly from spherical space-time symmetry. This procedure is summarized in Section 21.2, and in Section 21.3 the well known gravitational red shift is given a new meaning by deriving it from rotation of the line element of Section 21.2. It is found that the gravitational red shift is a precession or phase shift - essentially a property purely of spherical space-time and not of any field equation. In the standard model the gravitational red shift is thought to be a wavelength change and incorrectly derived from a space-time that has no torsion.

21.2 Line Element and Orbital Equation from Theorem of Orbits

The Theorem of Orbits is a simple example of the Frobenius Theorem [11] which defines the most general line element. The Theorem of Orbits is:

$$nr = \frac{r}{m} = \int dr = r + \mu \tag{21.1}$$

where n and m are functions of r, the radial coordinate in spherical polar coordinates. The constant of integration is in general non-zero, and goes to zero in a Minkowski space-time. If the Frobenius Theorem is applied [11] to a spherically symmetric space-time the line element is:

$$ds^2 = -n(r)c^2dt^2 + m(r)dr^2 + r^2 d\Omega^2. \tag{21.2}$$

From the Theorem of Orbits it is found that:

$$n = 1 + \frac{\mu}{r}, \tag{21.3}$$

$$m = \left(1 + \frac{\mu}{r}\right)^{-1}, \tag{21.4}$$

so that the line element becomes:

$$ds^2 = -\left(1 + \frac{\mu}{r}\right)c^2dt^2 + \left(1 + \frac{\mu}{r}\right)^{-1} dr^2 + r^2 d\Omega^2 \tag{21.5}$$

in spherical polar co-ordinates.

The orbital equation is obtained by considering the special case of orbits in a plane, so the line element (21.5) reduces to:

$$ds^2 = -\left(1 + \frac{\mu}{r}\right)c^2dt^2 + \left(1 + \frac{\mu}{r}\right)^{-1} dr^2 + r^2 d\phi^2. \tag{21.6}$$

Define [1–11] the constant of motion:

$$\begin{aligned} -\epsilon &= -\left(\frac{ds}{d\lambda}\right)^2 = -c^2\left(\frac{d\tau}{d\lambda}\right)^2 \\ &= -\left(1+\frac{\mu}{r}\right)c^2\left(\frac{dt}{d\lambda}\right)^2 + \left(1+\frac{\mu}{r}\right)^{-1}\left(\frac{dr}{d\lambda}\right)^2 + r^2\left(\frac{d\phi}{d\lambda}\right)^2 \end{aligned} \quad (21.7)$$

where $d\tau$ is the infinitesimal element of proper time. Now make the choice:

$$\lambda = \tau \quad (21.8)$$

to find:

$$-c^2 = -\left(1+\frac{\mu}{r}\right)c^2\left(\frac{dt}{d\tau}\right)^2 + \left(1+\frac{\mu}{r}\right)^{-1}\left(\frac{dr}{d\tau}\right)^2 + r^2\left(\frac{d\phi}{d\tau}\right)^2. \quad (21.9)$$

To convert to S.I. units multiply throughout by $\frac{1}{2}m$, where m is to be determined:

$$\begin{aligned} &\frac{1}{2}mr^2\left(\frac{d\phi}{d\tau}\right)^2 - \frac{1}{2}m\left(1+\frac{\mu}{r}\right)c^2\left(\frac{dt}{d\tau}\right)^2 + \frac{1}{2}m\left(1+\frac{\mu}{r}\right)^{-1}\left(\frac{dr}{d\tau}\right)^2 \\ &\qquad = -\frac{1}{2}mc^2. \end{aligned} \quad (21.10)$$

Multiply through by $\left(1+\frac{\mu}{r}\right)$: to find that:

$$\begin{aligned} &\frac{1}{2}mr^2\left(\frac{d\phi}{d\tau}\right)^2\left(1+\frac{\mu}{r}\right) - \frac{1}{2}m\left(1+\frac{\mu}{r}\right)^2c^2\left(\frac{dt}{d\tau}\right)^2 + \frac{1}{2}m\left(\frac{dr}{d\tau}\right)^2 \\ &\qquad = -\frac{1}{2}mc^2\left(1+\frac{\mu}{r}\right). \end{aligned} \quad (21.11)$$

This is the orbital equation:

$$\frac{1}{2}m\left(\frac{dr}{d\tau}\right)^2 + V = E. \quad (21.12)$$

The total energy in S.I. units is:

$$E = \frac{1}{2}mc^2\left(1+\frac{\mu}{r}\right)^2\left(\frac{dt}{d\tau}\right)^2. \quad (21.13)$$

The potential energy in S.I. units is:

$$V = \frac{1}{2}m\left(1+\frac{\mu}{r}\right)\left(c^2+\frac{L^2}{r^2}\right) \tag{21.14}$$

where:

$$L = r^2\frac{d\phi}{d\tau} \tag{21.15}$$

is a constant of motion having the units of angular momentum per unit mass. The factor $\frac{1}{2}$ is introduced [11] to write the equation in standard dynamical form. The potential energy is therefore:

$$V = \frac{1}{2}mc^2 + \frac{1}{2}mc^2\frac{\mu}{r} + \frac{1}{2}m\frac{L^2}{r^2} + \frac{1}{2}\frac{mL^2\mu}{r^3} \tag{21.16}$$

and is made up of four terms which are identified below. For all orbits excluding binary pulsars and the Cassini/Pioneer anomaly it is found by experimental observation that:

$$\mu = -\frac{2mG}{c^2}. \tag{21.17}$$

Therefore the potential energy becomes:

$$V = \frac{1}{2}mc^2 - m\frac{mG}{r} + \frac{1}{2}\frac{mL^2}{r^2} - \frac{L^2mMG}{c^2r^3}. \tag{21.18}$$

Therefore it becomes possible to identify the four terms as follows.

1) A constant term proportional to rest energy, $\frac{1}{2}mc^2$.
2) The Newtonian potential of attraction, $-mMG/r$.
3) The centripetal repulsion, $mL^2/(2r^2)$.
4) The relativistic correction to the Newtonian attraction, $-L^2mMG/(c^2r^3)$.

Therefore the factor m is the mass of an object attracted by an object of mass M. The Theorem of Orbits (21.1) is the "geometrical control" over the way m and M interact. The introduction of m, M and G introduces physics into pure geometry.

The Newtonian limit is defined by:

$$r \to \infty \tag{21.19}$$

when the familiar Newtonian terms (21.2) and (21.3) dominate. The Newtonain force of attraction is:

$$F = -\frac{\partial V}{\partial r} = -\frac{mMG}{r^2} \tag{21.20}$$

which is the inverse square law of Newton. From Eqs (21.18) and (21.20) the total force between m and M is:

$$F = -\frac{mMG}{r^2} + \frac{mL^2}{r^3} - \frac{3L^2 mMG}{c^2 r^4}. \tag{21.21}$$

This force law describes the vast majority of known orbits with great accuracy. It describes perihelion advance, deflection of light by gravity, frame dragging, Shapiro time delay and all the phenomena incorrectly attributed in the standard model to the now obsolete [1–10] Einstein field equation. As argued, these phenomena are due purely to the spherical symmetry of space-time. The masses m and M are introduced following experimental observation. The Newtonian force is Eq. (21.20). Newton did not realize the existence of the centripetal force, and of course did not realize the existence of the relativistic correction.

In binary pulsars (paper 108) the orbits decrease by a few millimeters per revolution. This effect is described by the addition of a very small perturbation as follows:

$$\mu = -\left(\frac{2mG}{c^2} + \frac{a}{r}\right) \tag{21.22}$$

which generates an additional attraction potential:

$$\Delta V = -\frac{1}{2} ma \left(\frac{c^2}{r^2} + \frac{L^2}{c^2 r^4}\right) \tag{21.23}$$

and an additional force of attraction:

$$\Delta F = -ma \left(\frac{c^2}{r^3} + \frac{2L^2}{c^2 r^5}\right) \tag{21.24}$$

which causes the two objects of a binary pulsar to spiral in towards each other from a relativistic orbit whose perihelion advance is a few degrees per revolution. This is a very small effect, but reproducible and repeatable. The same type of phenomenon is found in the solar system in the well known Pioneer/Cassini anomalies. Both spacecraft see a tiny additional force of attraction not present in Eq. (21.21). The complete force law for binary pulsars and the Pioneer Cassini orbits is therefore:

$$F = -\frac{mMG}{r^2} + \frac{m}{r^3}(L^2 - ac^2) - \frac{3LmMG}{c^2r^4} - \frac{2amL^2}{c^2r^5}, \tag{21.25}$$

the Newtonian force in this case being only one of five terms.

21.3 The Gravitational Red Shift

Consider the line element (21.5) in cylindrical polar co-ordinates:

$$-ds^2 = \left(1 + \frac{\mu}{r}\right) c^2dt^2 - \left(1 + \frac{\mu}{r}\right)^{-1} dr^2 - r^2d\phi^2 - dZ^2. \tag{21.26}$$

Now rotate it (see paper 110) at an angular velocity ω as follows:

$$\phi' = \phi + \omega t. \tag{21.27}$$

The rotated line element is therefore:

$$-ds'^2 = \left(1 + \frac{\mu}{r}\right) c^2dt^2 - \left(1 + \frac{\mu}{r}\right)^{-1} dr^2 - r^2d\phi'^2 - dZ^2 \tag{21.28}$$

where:

$$d\phi' = d\phi + \omega dt \tag{21.29}$$

and

$$d\phi'^2 = d\phi^2 + 2\omega d\phi dt + \omega^2dt^2. \tag{21.30}$$

It is found that:

$$\begin{aligned} -ds'^2 = &\left(1 + \frac{\mu}{r} - \frac{v^2}{c^2}\right)(c^2dt^2 - 2r^2\Omega d\phi dt) \\ &- \left(1 + \frac{\mu}{r}\right)^{-1} dr^2 - r^2d\phi^2 - dZ^2 \end{aligned} \tag{21.31}$$

where the orbital linear velocity of rotation is defined by

$$v = r\omega. \tag{21.32}$$

Identify the relativistic angular velocity (compare paper 110 on www.aias.us) as:

$$\Omega = \omega \left(1 + \frac{\mu}{r} - \frac{v^2}{c^2}\right)^{-1} \tag{21.33}$$

and the infinitesimal of proper time by:

$$d\tau = \left(1 + \frac{\mu}{r} - \frac{v^2}{c^2}\right)^{\frac{1}{2}} dt. \tag{21.34}$$

The change of phase, or precession, upon rotating by 2π radians is:

$$\alpha = \Omega d\tau - \omega dt = 2\pi \left(\left(1 + \frac{\mu}{r} - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} - 1\right). \tag{21.35}$$

The limit

$$r \to \infty \tag{21.36}$$

defines the Thomas precession (paper 110):

$$\alpha(\text{Thomas}) = 2\pi \left(\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} - 1\right) \tag{21.37}$$

and the limit:

$$v \to 0 \tag{21.38}$$

defines the gravitational red shift:

$$\alpha(\text{grav}) = 2\pi \left(\left(1 + \frac{\mu}{r}\right)^{-\frac{1}{2}} - 1\right). \tag{21.39}$$

For almost all orbits, as argued in Section 21.2, it is found by experimental observation that:

$$\mu = -\frac{2mG}{c^2} \tag{21.40}$$

so the gravitational red shift is:

$$\alpha(\text{grav}) = 2\pi\left(\left(1-\frac{2mG}{c^2r}\right)^{-\frac{1}{2}}-1\right) \sim \frac{2\pi mG}{c^2r}. \tag{21.41}$$

as observed experimentally as is well known. The Thomas precession is well observed experimentally in atomic and molecular spectra in spin orbit coupling.

It is concluded that both the gravitational red shift and the Thomas precession are due purely to the spherical symmetry of space-time. The standard model's Einstein field equation is known to be geometrically incorrect because of its neglect of torsion, so the standard explanation of the gravitational red shift cannot be correct. Similarly, the standard model's cosmological red shift (which is different from the gravitational red shift) is an artifact based on the Roberston Walker metric, which in paper 93 on www.aias.us was shown to violate basic geometry (the Hodge dual of the Bianchi identity).

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