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Invariance, Covariance and Duality Properties of the ECE Laws of Dynamics and Electrodynamics

by

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Abstract

The ECE laws of classical dynamics and electrodynamics are based on the Bianchi identity and their duality, covariance and invariance properties in general relativity are also based directly on Cartan geometry. It is shown that the vacuum equations are generally invariant, and that the field matter equations are generally covariant. The structure of the components of the electric and magnetic fields is also determined by the Bianchi identity, and there exist Hodge duals of both the vacuum and field matter equations. In general the equations transform according to the rules of Cartan geometry. The general coordinate transformation becomes the Lorentz transformation in the limit of Minkowski space-time.

Keywords: ECE laws of dynamics and electrodynamics, invariance, covariance, duality.

22.1 Introduction

Recently [1–10] the generally covariant laws of dynamics and electrodynamics have been developed with Cartan geometry [11]. It has been found that they are based on the Bianchi identity of Cartan geometry and the Hodge dual identity. The laws of classical electrodynamics are the same in overall format as the familiar Maxwell Heaviside (MH) field equations of special relativity, but the Einstein Cartan Evans (ECE) laws are written in a space-time with torsion and curvature, and give more information, notably they include the spin connection of space-time. The ECE laws of dynamics are the same in structure as those of electrodynamics, so there are four laws of generally covariant dynamics, akin to the Gauss, Faraday, Coulomb and Ampère-Maxwell laws of classical ECE electrodynamics. In both subject areas there are two orbital laws (Gauss and Coulomb) and two spin laws (Faraday and Ampère Maxwell). The Gauss and Faraday laws refer to the free field or vacuum field and the Coulomb and Ampère Maxwell laws refer to field matter interaction. In Section 22.2, the properties of these laws under coordinate transformation are developed using the methods [1–11] of geometry. It is found that the free field or vacuum laws are invariant under coordinate transformation, and that the field - matter laws are covariant under coordinate transformation. In Section 22.3 the Hodge duality properties of the laws are given in the vacuum and in the presence of field matter interaction.

22.2 Invariance and Covariance

In previous work [1–10] it has been shown that the homogeneous laws of dynamics and electrodynamics are based on the geometrical structure:

$$D_\mu \widetilde{T}^{\kappa\mu\nu} = \widetilde{R}^{\kappa}{}_{\mu}{}^{\mu\nu} \tag{22.1}$$

where $\widetilde{T}^{\kappa\mu\nu}$ is a rank three torsion tensor and where $\widetilde{R}^{\kappa}{}_{\mu}{}^{\mu\nu}$ is a rank four curvature tensor. The D_μ denotes covariant derivative in a space-time with torsion and curvature [11]. The inhomogeneous laws are based on the Hodge dual of Eq. (22.1):

$$D_\mu T^{\kappa\mu\nu} = R^{\kappa}{}_{\mu}{}^{\mu\nu}. \tag{22.2}$$

These equations can be rearranged to give:

$$\partial_\mu \widetilde{T}^{\kappa\mu\nu} = \widetilde{j}^{\kappa\nu} \tag{22.3}$$

and

$$\partial_\mu T^{\kappa\mu\nu} = j^{\kappa\nu} \tag{22.4}$$

where the right hand side terms include the spin connection in their structure. It has been shown that Eqs. (22.3) and (22.4) give the equations of dynamics in generally covariant unified field theory [1–10]. If a primordial voltage is defined by:

$$\phi = cA^{(0)} \tag{22.5}$$

where c is the vacuum speed of light, the equations of classical electrodynamics are given by the fundamental hypothesis:

$$F^{\kappa\mu\nu} = A^{(0)} T^{\kappa\mu\nu} \tag{22.6}$$

which defines the electromagnetic field tensor in terms of the torsion tensor. The ECE equations of electrodynamics are therefore:

$$\partial_\mu \widetilde{F}^{\kappa\mu\nu} = A^{(0)} j^{\kappa\nu}, \tag{22.7}$$

$$\partial_\mu F^{\kappa\mu\nu} = A^{(0)} \widetilde{j}^{\kappa\nu\prime}. \tag{22.8}$$

Experimentally it is found that:

$$\partial_\mu \widetilde{F}^{\kappa\mu\nu} = 0, \tag{22.9}$$

$$\partial_\mu F^{\kappa\mu\nu} = A^{(0)} j^{\kappa\nu}, \tag{22.10}$$

because a magnetic monopole (part of $j^{\kappa\nu}$) has never been observed. There have been some claims to magnetic monopole observation, but these claims appear not to be reproducible. For all practical purposes therefore the ECE laws of classical electrodynamics are:

$$\partial_\mu \widetilde{F}^{\kappa\mu\nu} = 0, \tag{22.11}$$

$$\partial_\mu F^{\kappa\mu\nu} = J^{\kappa\nu}/\epsilon_0. \tag{22.12}$$

For comparison, the MH laws are well known to be:

$$\partial_\mu \widetilde{F}^{\mu\nu} = 0, \tag{22.13}$$

$$\partial_\mu F^{\mu\nu} = J^{\nu}/\epsilon_0. \tag{22.14}$$

The main differences between the ECE and MH laws include the fact that the latter are restricted to Minkowski space-time, whereas the former are written in a space-time with torsion and curvature present and are part of a generally covariant unified field. In ECE theory the electromagnetic field is the correct rank three tensor related [1–10] to canonical angular momentum/energy density as required in general relativity. In MH theory the electromagnetic field is a rank two tensor which is an integral [12–14] over a rank three density.

The concept of vacuum electromagnetic field is used routinely in classical electrodynamics and is defined as the field propagating infinitely far from its source. This concept is a mathematical limit, not a physical reality, because without a source there is no field. However, if we transfer this concept of the received view to ECE theory the vacuum field is defined by:

$$\partial_\mu \widetilde{F}^{\kappa\mu\nu} = 0 \tag{22.15}$$

$$\partial_\mu F^{\kappa\mu\nu} = 0 \tag{22.16}$$

and in vector notation the ECE vacuum equations of electrodynamics are:

$$\boldsymbol{\nabla} \cdot \boldsymbol{B} = 0. \tag{22.17}$$

$$\boldsymbol{\nabla} \times \boldsymbol{E} + \frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{0}, \tag{22.18}$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{E} = 0, \tag{22.19}$$

$$\boldsymbol{\nabla} \times \boldsymbol{B} - \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t} = \boldsymbol{0}, \tag{22.20}$$

in which $\boldsymbol{B}$ is the magnetic flux density in tesla and $\boldsymbol{E}$ is the electric field strength in volts per meter. Eqs. (22.17) and (22.19) are laws of orbital torsion [1–10] in which the field components are defined by:

$$\boldsymbol{B} = B^{001}\boldsymbol{i} + B^{002}\boldsymbol{j} + B^{003}\boldsymbol{k} \tag{22.21}$$

and

$$\boldsymbol{E} = E^{010}\boldsymbol{i} + E^{020}\boldsymbol{j} + E^{030}\boldsymbol{k} \tag{22.22}$$

respectively. Eq. (22.18) is a law of spin torsion in which:

$$\boldsymbol{E} = E^{332}\boldsymbol{i} + E^{113}\boldsymbol{j} + E^{221}\boldsymbol{k} \tag{22.23}$$

and:

$$\boldsymbol{B} = B^{101}\boldsymbol{i} + B^{202}\boldsymbol{j} + B^{303}\boldsymbol{k} \tag{22.24}$$

and Eq. (22.20) is a law of spin torsion in which:

$$\boldsymbol{E} = E^{110}\boldsymbol{i} + E^{220}\boldsymbol{j} + E^{330}\boldsymbol{k} \tag{22.25}$$

and

$$\boldsymbol{B} = B^{332}\boldsymbol{i} + B^{113}\boldsymbol{j} + B^{221}\boldsymbol{k}. \tag{22.26}$$

The right hand sides of Eqs. (22.15) and (22.16) are null rank two tensors. If the latter is denoted by:

$$X^{\mu\nu} = 0 \tag{22.27}$$

it transforms as another null tensor:

$$X'^{\mu\nu} = \frac{\partial x^{\mu'}}{\partial x^{\mu}} \frac{\partial x^{\nu'}}{\partial x^{\nu}} X^{\mu\nu} = 0 \tag{22.28}$$

where x^{μ} is the coordinate four-vector [1–11]. Therefore this property is necessary and sufficient to show that the ECE vacuum equations are generally invariant. This means that the equations are the same under arbitrary coordinate transform from a frame K to K′. In contrast the vacuum MH equations are invariant only under the Lorentz transform from frame K to K′. In other words the ECE equations are those of a generally covariant unified field, and the MH equations are those of Lorentz covariant and un-unified field. Note carefully that the electromagnetic field tensor itself transforms as a rank three tensor as follows [1–11]:

$$T^{\kappa\mu\nu'} = \left(\frac{\partial x^{\kappa'}}{\partial x^{\kappa}}\right) \left(\frac{\partial x^{\mu'}}{\partial x^{\nu}}\right) \left(\frac{\partial x^{\nu'}}{\partial x^{\nu}}\right) T^{\kappa\mu\nu} \tag{22.29}$$

and is not invariant. It is well known that the MH field tensor transform as a rank two tensor using two Lorentz transform matrices. They are transformed from frame K to a frame K′ moving at a constant velocity v with respect to K. When:

$$v/c \ll 1 \tag{22.30}$$

this type of transform produces:

$$\boldsymbol{E}' = \boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B} \tag{22.31}$$

and

$$\boldsymbol{B}' = \boldsymbol{B} - \frac{1}{c^2} \boldsymbol{v} \times \boldsymbol{E}, \tag{22.32}$$

and the Lorentz force law is:

$$\boldsymbol{F} = e(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}) \tag{22.33}$$

where e is a charge. So the transform of frames has a physical effect. Similarly in ECE theory the general transform (22.29) will have a physical effect. Therefore coordinate transform of the vacuum ECE field equations leaves them invariant, but the fields themselves are changed.

In order to develop the concept of coordinate transform the general rule for any tensor is [11]:

$$T^{\mu'_1 \cdots \mu'_\kappa}_{\nu'_1 \cdots \nu'_\ell} = \left(\frac{\partial x^{\mu'_1}}{\partial x^{\mu_1}} \cdots \frac{\partial x^{\mu'_\kappa}}{\partial x^{\mu_\kappa}} \right) \left(\frac{\partial x^{\nu_1}}{\partial x^{\nu'_1}} \cdots \frac{\partial x^{\nu}_{\ell}}{\partial x^{\nu}_{\ell'}} \right) T^{\mu_1 \cdots \mu_\kappa}_{\nu_1 \cdots \nu_\ell} \tag{22.34}$$

For a mixed index tensor in Cartan geometry [1–10]:

$$T^{a'\mu'}_{b'\nu'} = \Lambda^{a'}_{a} \frac{\partial x^{\mu'}}{\partial x^{\mu}} \Lambda^{b}_{b'} \frac{\partial x^{\nu}}{\partial x^{\nu'}} T^{a\mu}_{b\nu} \tag{22.35}$$

where $\Lambda^{a'}_{a}$ denotes Lorentz transform of the Minkowski tangent space-time at point P to the base manifold. The partial derivative transforms as:

$$\partial_{\mu'} = \frac{\partial x^{\mu}}{\partial x^{\mu'}} \partial_{\mu}. \tag{22.36}$$

The covariant derivative of a vector transforms as:

$$(D_\mu V^\nu)' = \frac{\partial x^{\mu}}{\partial x^{\mu'}} \frac{\partial x^{\nu'}}{\partial x^{\nu}} D_\mu V^\nu \tag{22.37}$$

because by definition D_μ is covariant, whereas ∂_μ acting on a tensor produces extra terms. For example the homogeneous MH equation transforms using the Leibniz theorem as:

$$(\partial_\mu \widetilde{F}^{\mu\nu})' = \Lambda^{\nu'}_{\nu} \partial_\mu \widetilde{F}^{\mu\nu} + \Lambda^{\mu}_{\mu'} \widetilde{F}^{\mu\nu} \partial_\mu (\Lambda^{\mu'}_{\mu} \Lambda^{\nu'}_{\nu}) \tag{22.38}$$

and because of the second term on the right hand side the homogeneous MH equation would appear not to transform covariantly. However, using Eq. (22.28) it is known that:

$$(\partial_\mu \widetilde{F}^{\mu\nu})' = \partial_\mu \widetilde{F}^{\mu\nu} = 0 \tag{22.39}$$

so the homogeneous MH equation is frame invariant.

Similarly the homogeneous ECE equation transforms as:

$$(\partial_\mu \widetilde{F}^{\kappa\mu\nu})' = \frac{\partial x^{\nu'}}{\partial x^{\nu}} \frac{\partial x^{\kappa'}}{\partial x^{\kappa}} \partial_\mu \widetilde{F}^{\kappa\mu\nu} + \widetilde{F}^{\kappa\mu\nu} \partial_\mu \left(\frac{\partial x^{\mu'}}{\partial x^{\mu}} \frac{\partial x^{\nu'}}{\partial x^{\nu}} \frac{\partial x^{\kappa'}}{\partial x^{\kappa}} \right) \tag{22.40}$$

and from Eq. (22.15):

$$(\partial_\mu \widetilde{F}^{\kappa\mu\nu})' = \partial_\mu \widetilde{F}^{\kappa\mu\nu} = 0. \tag{22.41}$$

For example, consider a rotation of the ECE Gauss law about the Z axis and without loss of generality assume that:

$$\kappa' = \kappa = 0 \tag{22.42}$$

so

$$\frac{\partial x^{\kappa'}}{\partial x^{\kappa}} = 1. \tag{22.43}$$

For rotation about the Z axis:

$$\frac{\partial x^{\nu'}}{\partial x^{\nu}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \tag{22.44}$$

Therefore:

$$\begin{bmatrix} V^{0'} \\ V^{1'} \\ V^{2'} \\ V^{3'} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V^{0} \\ V^{1} \\ V^{2} \\ V^{3} \end{bmatrix} \tag{22.45}$$

which is equivalent to the rotation of a four-vector:

$$\begin{aligned} V^{0'} &= V^0 \\ V^{1'} &= V^0 \cos\theta + V^1 \sin\theta \\ V^{2'} &= V^1 \sin\theta + V^2 \cos\theta \\ V^{3'} &= V^3. \end{aligned} \tag{22.46}$$

The time-like part of this vector is:

$$V^0 = \boldsymbol{\nabla} \cdot \boldsymbol{B}, \tag{22.47}$$

and the space-like part is:

$$\boldsymbol{V} = \boldsymbol{\nabla} \times \boldsymbol{E} + \frac{\partial \boldsymbol{B}}{\partial t}, \tag{22.48}$$

where the components are:

$$V^1 = V_X = \left(\boldsymbol{\nabla} \times \boldsymbol{E} + \frac{\partial \boldsymbol{B}}{\partial t} \right)_X \tag{22.49}$$

and so on. The structure of this vector is derived from the vector equivalent of Eq. (22.15):

$$\partial_\mu \widetilde{F}^{0\mu\nu} = 0 := V^\nu \tag{22.50a}$$
$$\boldsymbol{\nabla} \cdot \boldsymbol{B} = 0$$
$$\boldsymbol{\nabla} \times \boldsymbol{E} + \frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{0} \tag{22.50b}$$

and the vector is defined by:

$$V^\nu = \left(c\boldsymbol{\nabla} \cdot \boldsymbol{B}, \quad \boldsymbol{\nabla} \times \boldsymbol{E} + \frac{\partial \boldsymbol{B}}{\partial t} \right) = 0. \tag{22.51}$$

The rotation of the column vector about the Z axis is therefore:

$$V^{\nu\prime} = \Lambda^{\nu'}{}_{\nu} V^\nu \tag{22.52}$$

which gives the four equations:

$$(\boldsymbol{\nabla} \cdot \boldsymbol{B})' = \boldsymbol{\nabla} \cdot \boldsymbol{B} = 0 \tag{22.53}$$
$$\left(\boldsymbol{\nabla} \times \boldsymbol{E} + \frac{\partial \boldsymbol{B}}{\partial t} \right)'_X = \left(\boldsymbol{\nabla} \times \boldsymbol{E} + \frac{\partial B}{\partial t} \right)_X = 0 \tag{22.54}$$

The rotation of the ECE Gauss law about the Z axis is given by Eq. (22.46a), from which:

$$(\boldsymbol{\nabla}' \cdot \boldsymbol{B}' = 0) = (\boldsymbol{\nabla} \cdot \boldsymbol{B} = 0). \tag{22.55}$$

This result means that the ECE Gauss law is invariant under Z axis rotation, Q.E.D. This is an example of the fact that the ECE Gauss law is invariant under any type of transform from frame K to K′.

Similarly, Eq. (22.46b) gives the transform under Z axis rotation of the X component of the ECE Faraday law:

$$\left(\boldsymbol{\nabla} \times \boldsymbol{E} + \frac{\partial \boldsymbol{B}}{\partial t} \right)'_X = c\boldsymbol{\nabla} \cdot \boldsymbol{B} \cos\theta + \left(\boldsymbol{\nabla} \times \boldsymbol{E} + \frac{\partial \boldsymbol{B}}{\partial t} \right)_X \sin\theta = 0. \tag{22.56}$$

Using the K frame results:

$$c\boldsymbol{\nabla} \cdot \boldsymbol{B} = \left(\boldsymbol{\nabla} \times \boldsymbol{E} + \frac{\partial \boldsymbol{B}}{\partial t} \right)_X = 0 \tag{22.57}$$

it is found that:

$$\left(\boldsymbol{\nabla} \times \boldsymbol{E} + \frac{\partial \boldsymbol{B}}{\partial t} \right)'_X = 0 \tag{22.58}$$

and Z axis rotation gives us the original equation again, QED. Similarly for the Y and Z components of the ECE Faraday law. The same result is true for the MH Gauss and Faraday laws, which are also invariant under Z axis rotation. As argued, the basic reason for the invariance is that a null tensor in frame K is a null tensor in frame K′ (Eq. (22.28)).

Adopting differential form notation [1–11] the free space or vacuum ECE equations of classical electrodynamics are defined by:

$$d \wedge \widetilde{F}^a = 0 \tag{22.59}$$

and

$$d \wedge F^a = 0. \tag{22.60}$$

Therefore in the vacuum:

$$\widetilde{R}^a{}_b \wedge q^b = \omega^a{}_b \wedge \widetilde{T}^b \tag{22.61}$$

and

$$R^a{}_b \wedge q^b = \omega^a{}_b \wedge T^b. \tag{22.62}$$

As shown in precious ECE papers this vacuum geometry can be interpreted to mean that the spin connection form $\omega^a{}_b$ is the tensor dual of the tetrad form:

$$\omega^a{}_b = -\frac{\kappa}{2} \epsilon^a{}_{bc} q^c \tag{22.63}$$

and that the curvature form is the tensor dual of the torsion form:

$$R^a{}_b = -\frac{\kappa}{2} \epsilon^a{}_{bc} T^c. \tag{22.64}$$

Here $\frac{k}{2}$ is a proportionality coefficient with the units of wave-number. Such dualities define the vacuum electromagnetic field in ECE theory, and also the vacuum dynamical field. They are analogous to, and generalize, the well known duality in Euclidean space between an axial vector V_k and a anti-symmetric tensor V_{ij}:

$$V_{ij} = \frac{1}{2}\epsilon_{ijk}V_k \tag{22.65}$$

where ϵ_{ijk} is the rank three totally anti-symmetric unit tensor. For example the rotation generator is, within a factor $-i$, an anti-symmetric unit tensor dual to an axial vector. Therefore Eq (22.58) and its Hodge dual (22.59) define the motion for example of a vacuum plane wave, both in ECE electrodynamical radiation and in ECE gravitational radiation. For a plane wave propagating in the Z axis, the motion is a rotation about the Z axis superimposed on translation. The electric and magnetic components of the plane wave are:

$$\boldsymbol{E} = \frac{E^{(0)}}{\sqrt{2}}(\boldsymbol{i} - i\boldsymbol{j})\ \exp(i(\omega t - \kappa Z)) \tag{22.66}$$

and

$$\boldsymbol{B} = \frac{B^{(0)}}{\sqrt{2}}(i\boldsymbol{i} + \boldsymbol{j})\ \exp(i(\omega t - \kappa Z)) \tag{22.67}$$

where ω is the angular frequency of the wave at instant t, and κ is its wave-number at point Z. The mathematical law (22.58) indicates that $\boldsymbol{E}$ and $\boldsymbol{B}$ of the plane wave will transform covariantly in general. However the vacuum plane wave is already propagating at c, so the addition of v to c in special relativity produces c again by the velocity addition law of special relativity [1–12]. This law is derived from the Lorentz transform as is well known. So vacuum plane waves propagating at c are invariant under the Lorentz transform. They are already propagating at c and cannot travel faster than c. In ECE theory the constancy of c is also a fundamental hypothesis and plane waves such as (22.65) and (22.66) are also solutions in ECE theory. Note carefully that plane waves are mathematical idealizations, not physical entities.

In vector notation the inhomogeneous field equation (22.10) can be written as the two vector equations that constitute the Coulomb law:

$$\boldsymbol{\nabla} \cdot \boldsymbol{D} = \rho \tag{22.68}$$

and the Ampère Maxwell law:

$$\boldsymbol{\nabla} \times \boldsymbol{H} - \frac{\partial \boldsymbol{D}}{\partial t} = \boldsymbol{J} \tag{22.69}$$

where $\boldsymbol{D}$ is the electric displacement and where $\boldsymbol{H}$ is the magnetic field strength. Here $\boldsymbol{J}$ is the current density and ρ is the charge density. The electric displacement in the Coulomb law is defined by an orbital torsion as follows:

$$\boldsymbol{D} = D^{010}\boldsymbol{i} + D^{020}\boldsymbol{j} + D^{030}\boldsymbol{k} \tag{22.70}$$

The magnetic field strength in the Ampère Maxwell law is defined in terms of spin torsion as:

$$\boldsymbol{H} = H^{332}\boldsymbol{i} + H^{113}\boldsymbol{j} + H^{221}\boldsymbol{k}. \tag{22.71}$$

and the electric displacement in the Ampère Maxwell law is defined in terms of spin torsion by:

$$\boldsymbol{D} = D^{110}\boldsymbol{i} + D^{220}\boldsymbol{j} + D^{330}\boldsymbol{k}. \tag{22.72}$$

Using the methods given above for the vacuum laws, it is found that under a Z axis rotation the ECE Coulomb and Ampère Maxwell laws are generally covariant. Under the arbitrary transformation from frame K to K′ the laws in frame K′ become:

$$(\boldsymbol{\nabla} \cdot \boldsymbol{D} = \rho) \rightarrow (\boldsymbol{\nabla} \cdot \boldsymbol{D} = \rho)' \tag{22.73}$$

and

$$\left(\boldsymbol{\nabla} \times \boldsymbol{H} - \frac{\partial \boldsymbol{D}}{\partial t} = \boldsymbol{J}\right) \rightarrow \left(\boldsymbol{\nabla} \times \boldsymbol{H} - \frac{\partial \boldsymbol{D}}{\partial t} = \boldsymbol{J}\right)'. \tag{22.74}$$

They retain their vector format but

$$\boldsymbol{D} \rightarrow \boldsymbol{D}', \quad \rho \rightarrow \rho', \quad \boldsymbol{H} \rightarrow \boldsymbol{H}', \quad \boldsymbol{J} \rightarrow \boldsymbol{J}' \tag{22.75}$$

and:

$$\boldsymbol{\nabla} \rightarrow \boldsymbol{\nabla}', \quad \frac{\partial}{\partial t} \rightarrow \frac{\partial}{\partial t'}, \tag{22.76}$$

So the general transformation in this case produces new physical effects, the essential reason being that the charge current density is changed.

As argued, the vacuum field equations of both ECE and the standard model are invariant under respectively the general coordinate transformation and the Lorentz transformation, while the vacuum fields $\boldsymbol{E}$ and $\boldsymbol{B}$ themselves change. One consequence of this property is that the standard model is unable to describe the Faraday disk generator - the well known Faraday paradox. In the K frame the Faraday law of induction in the standard model is:

$$\boldsymbol{\nabla} \times \boldsymbol{E} + \frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{0} \tag{22.77}$$

and in order for induction to take place of a field $\boldsymbol{E}$ by a field $\boldsymbol{B}$ the experimental condition needed is:

$$\frac{\partial \boldsymbol{B}}{\partial t} \neq \boldsymbol{0}. \tag{22.78}$$

In the Faraday disk generator this condition is not fulfilled. The generator consists of a disk of uncharged metal placed on a magnet. The condition for induction is that the disk rotates relative to the observing apparatus in frame K. Whether or not the magnet is static or spinning about its own Z axis:

$$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{0} \tag{22.79}$$

and no induction of $\boldsymbol{E}$ occurs according to Eq. (22.76). This paradox is not resolved by Lorentz transformation, under which Eq. (22.76) stays the same and under which the $\boldsymbol{E}$ and $\boldsymbol{B}$ fields change as in Eqs. (22.31) and (22.32). After Lorentz transformation therefore:

$$\boldsymbol{\nabla} \times (\boldsymbol{E} + \boldsymbol{V} \times \boldsymbol{B}) + \frac{\partial}{\partial t}\left(\boldsymbol{B} - \frac{1}{c^2}\boldsymbol{V} \times \boldsymbol{E}\right) = \boldsymbol{0}. \tag{22.80}$$

Using Eq. (22.76):

$$\boldsymbol{\nabla} \times (\boldsymbol{v} \times \boldsymbol{B}) - \frac{1}{c^2}\frac{\partial}{\partial t}(\boldsymbol{v} \times \boldsymbol{E}) = \boldsymbol{0}. \tag{22.81}$$

Now use the vector identities [15]:

$$\boldsymbol{\nabla} \times (\boldsymbol{a} \times \boldsymbol{b}) = \boldsymbol{a}\boldsymbol{\nabla} \cdot \boldsymbol{b} - \boldsymbol{b}\boldsymbol{\nabla} \cdot \boldsymbol{a} + (\boldsymbol{b} \cdot \boldsymbol{\nabla})\boldsymbol{a} - (\boldsymbol{a} \cdot \boldsymbol{\nabla})\boldsymbol{b} = \boldsymbol{0} \tag{22.82}$$

and

$$\boldsymbol{a} \times (\boldsymbol{\nabla} \times \boldsymbol{a}) = \frac{1}{2}\boldsymbol{\nabla} a^2 - (\boldsymbol{a} \cdot \boldsymbol{\nabla})\boldsymbol{a} \tag{22.83}$$

to find that:

$$\boldsymbol{\nabla} \times (\boldsymbol{v} \times \boldsymbol{B}) = \boldsymbol{v}\boldsymbol{\nabla} \cdot \boldsymbol{B} - \boldsymbol{B}\boldsymbol{\nabla} \cdot \boldsymbol{v} + (\boldsymbol{B} \cdot \boldsymbol{\nabla})\boldsymbol{v} - (\boldsymbol{v} \cdot \boldsymbol{\nabla})\boldsymbol{B}. \tag{22.84}$$

For a constant v:

$$\boldsymbol{\nabla} \times (\boldsymbol{v} \times \boldsymbol{B}) = -(\boldsymbol{v} \cdot \boldsymbol{\nabla})\boldsymbol{B}. \tag{22.85}$$

From Eq. (22.82):

$$\boldsymbol{v} \times (\boldsymbol{\nabla} \times \boldsymbol{B}) = \frac{1}{2}\boldsymbol{\nabla}(\boldsymbol{v} \cdot \boldsymbol{B}) - (\boldsymbol{v} \cdot \boldsymbol{\nabla})\boldsymbol{B} = -(\boldsymbol{v} \cdot \boldsymbol{\nabla})\boldsymbol{B} \tag{22.86}$$

for constant v. Therefore:

$$\boldsymbol{\nabla} \times (\boldsymbol{v} \times \boldsymbol{B}) = \boldsymbol{v} \times (\boldsymbol{\nabla} \times \boldsymbol{B}) \tag{22.87}$$

and for constant v:

$$\frac{\partial}{\partial t}(\boldsymbol{v} \times \boldsymbol{E}) = \boldsymbol{v} \times \frac{\partial \boldsymbol{E}}{\partial t}. \tag{22.88}$$

So Eq. (22.79) is:

$$\boldsymbol{v} \times \left(\boldsymbol{\nabla} \times \boldsymbol{B} - \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t}\right) = \boldsymbol{0} \tag{22.89}$$

which is the vacuum Ampère Maxwell law of the standard model cross multiplied by $\boldsymbol{v}$. Therefore the Lorentz transformation has produced the Hodge dual of the Faraday law, which is the vacuum Ampère Maxwell law:

$$\boldsymbol{\nabla} \times \boldsymbol{B} - \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t} = \boldsymbol{0}. \tag{22.90}$$

Since:

$$\boldsymbol{\nabla} \times \boldsymbol{B} = \boldsymbol{0} \tag{22.91}$$

experimentally the $\boldsymbol{B}$ field cannot again induce an $\boldsymbol{E}$ field in the standard model, and the Faraday paradox remains in the standard model. There have been some claims in the literature that Lorentz induction explains the Faraday disk, but these are based on the transform of $\boldsymbol{E}$, neglecting the transform of $\boldsymbol{B}$. When both transforms are accounted for correctly it is seen that

$$\left(\boldsymbol{\nabla} \times \boldsymbol{E} + \frac{\partial \boldsymbol{B}}{\partial t}\right)' = 0 \tag{22.92}$$

and there is no induction of $\boldsymbol{E}$ by $\boldsymbol{B}$ in any frame of reference, contrary to observation. This result is shown in another way by noting that the linear velocity at the rim of a disk rotating at an angular frequency Ω is the real part of:

$$\boldsymbol{v} = \frac{v^{(0)}}{\sqrt{2}}(\boldsymbol{i} - i\boldsymbol{j}) \ \exp(i\Omega t) \tag{22.93}$$

which is:

$$\text{Real } (\boldsymbol{v}) = \frac{v^{(0)}}{\sqrt{2}}(\boldsymbol{j}\cos(\Omega t) + \boldsymbol{j}\sin(\Omega t)). \tag{22.94}$$

The Lorentz transform of the electric field:

$$\boldsymbol{E}' = \boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B} \tag{22.95}$$

produces:

$$\boldsymbol{v} \times \boldsymbol{B} = v^{(0)}\frac{B^{(0)}}{\sqrt{2}}(\boldsymbol{i}\sin(\Omega t) - \boldsymbol{j}\cos(\Omega t)) \tag{22.96}$$

where:

$$\boldsymbol{B} = B^{(0)}\boldsymbol{k}, \tag{22.97}$$

which is the product of $v^{(0)}$ with a rotating magnetic field:

$$\boldsymbol{B} = -\frac{B^{(0)}}{\sqrt{2}}(i\boldsymbol{i} + \boldsymbol{j}) \ \exp(i\Omega t) \tag{22.98}$$

which has the real part of:

$$\text{Real } (\boldsymbol{B}) = \frac{B^{(0)}}{\sqrt{2}}(\boldsymbol{i}\sin(\Omega t) - \boldsymbol{j}\cos(\Omega t)). \tag{22.99}$$

The Lorentz transform does not produce a rotating electric field as claimed for example by Feynman [16]. It produces a rotating magnetic field. Also, when rotation is present, the inertial Lorentz transform is not applicable, and the standard model cannot explain the Faraday paradox. For this ECE theory is needed [1–10].

In contrast to the standard model the ECE explanation of the Faraday disk generator is based directly on the fundamental hypothesis:

$$A^a_\mu = A^{(0)}q^a_\mu \tag{22.100}$$

that a vector potential is generated by the Cartan tetrad. The disk rotates at an angular frequency Ω and generates the potential [1–10]:

$$\boldsymbol{A}^{(2)} = \boldsymbol{A}^{(2)*} = \frac{A^{(0)}}{\sqrt{2}}(\boldsymbol{i} - i\boldsymbol{j})e^{i\Omega t} \tag{22.101}$$

where the C negative A is the magnitude of the vector potential of the magnet. From the ECE equations linking the field and potential, the following rotating electric field is generated:

$$\boldsymbol{E}^{(2)} = \boldsymbol{E}^{(1)*} = -\left(\frac{\partial}{\partial t} + i\Omega\right)\boldsymbol{A}^{(2)} \tag{22.102}$$

where the spin connection in units of inverse meters is:

$$\omega = \frac{\Omega}{c}. \tag{22.103}$$

The real part of this electric field rotates around the rim of the Faraday disk and is detected by apparatus in the observer frame K. As observed experimentally, $\boldsymbol{E}^{(1)}$ is proportional to Ω multiplied by $A^{(0)}$, the magnitude of the vector potential of the magnetic flux density $\boldsymbol{B}$. As noted, the standard model has no explanation for the Faraday disk generator.

Similarly the standard model has no explanation for the Sagnac effect, which is a phase shift induced by rotation. As argued, the free space Maxwell Heaviside (MH) equations are invariant under frame rotation, so they cannot explain the Sagnac effect because their solutions are the same in any frame of reference. This has been a well known cause of difficulty since the effect was discovered in 1913, and in the standard physics there continues to be no satisfactory explanation for the effect. In ECE theory [1–10] the effect is explained in the same way as the Faraday disk. A vector potential rotates around the platform of the Sagnac interferometer (ring laser gyroscope). Rotation to the left is described by:

$$\boldsymbol{A}_L^{(2)} = \frac{A^{(0)}}{\sqrt{2}}(\boldsymbol{i} - i\boldsymbol{j})\ \exp\ (i\omega_1 t) \tag{22.104}$$

and to the right by:

$$\boldsymbol{A}_R^{(2)} = \frac{A^{(0)}}{\sqrt{2}}(\boldsymbol{i} + i\boldsymbol{j})\ \exp\ (i\omega_1 t). \tag{22.105}$$

When the platform is at rest the time delay is:

$$\Delta t = 2\pi \left(\frac{1}{\omega_1} - \frac{1}{\omega_1} \right) = 0. \tag{22.106}$$

Eqs. (22.103) and (22.104) are tetrad equations of spinning space-time, a concept that does not exist in standard electrodynamics. In ECE theory the electromagnetic field is the frame of reference itself, so a beam of light traveling in a circular path is equivalent to a rotating tetrad multiplied by $A^{(0)}$, giving Eq. (22.103). When the platform is spun left at an angular frequency Ω:

$$\omega_1 \rightarrow \omega_1 + \Omega \tag{22.107}$$

for the left rotating beam and:

$$\omega_1 \rightarrow \omega_1 - \Omega \tag{22.108}$$

for the right rotating beam. This is because the rotating platform causes an additional or subtractive frame rotation, i.e. of space-time itself. There is a time delay for light going around a right or left spinning platform:

$$\Delta t = 2\pi \left(\frac{1}{\omega_1 - \Omega} - \frac{1}{\omega_1 + \Omega} \right) \tag{22.109}$$

which is the well known Sagnac effect.

Experimentally it is found that:

$$\Delta t = \frac{4}{c^2} Ar\Omega \tag{22.110}$$

where Ar is the area of the platform. For a circular platform:

$$Ar = \pi r^2 \tag{22.111}$$

and experimentally:

$$\Omega \ll \omega_1 \tag{22.112}$$

so

$$\omega_1 = \frac{c}{r} := \kappa_1 c. \tag{22.113}$$

The Sagnac effect is therefore an effect of spinning space-time, and Eq. (22.113) defines the frequency ω_1 and wave-number k_1 for a platform radius r.

22.3 Hodge Duality

In this section Hodge dual transformations are defined for free and interacting fields in ECE theory. In index-less shorthand notation [1–10] the Bianchi identity is:

$$D \wedge T := R \wedge q \tag{22.114}$$

and its Hodge dual is:

$$D \wedge \widetilde{T} := \widetilde{R} \wedge q. \tag{22.115}$$

These equations are written as:

$$d \wedge T = j = R \wedge q - \omega \wedge T \tag{22.116}$$

$$d \wedge \widetilde{T} = \widetilde{j} = \widetilde{R} \wedge q - \omega \wedge \widetilde{T}. \tag{22.117}$$

Free fields are defined by the geometry:

$$j = \widetilde{j} = 0 \tag{22.118}$$

i.e.

$$R \wedge q = \omega \wedge T, \tag{22.119}$$

$$\widetilde{R} \wedge q = \omega \wedge \widetilde{T}. \tag{22.120}$$

The free field is defined as the field infinitely distant from its source, a mathematical limit defined by:

$$j \to 0, \tag{22.121}$$

$$\widetilde{j} \to 0. \tag{22.122}$$

Therefore the free field geometry is:

$$d \wedge T \to 0, \tag{22.123}$$

$$d \wedge \widetilde{T} \to 0. \tag{22.124}$$

a) Free Electromagnetic Field
Use the ECE hypothesis:

$$F = A^{(0)} T \tag{22.125}$$

to find the ECE equations of the free electromagnetic field:

$$d \wedge F = 0, \tag{22.126}$$

$$d \wedge \widetilde{F} = 0. \tag{22.127}$$

These equations imply:

$$R \wedge A = \omega \wedge F, \tag{22.128}$$

$$\widetilde{R} \wedge A = \omega \wedge \widetilde{F}. \tag{22.129}$$

For the free field, propagating in vacuo at c, the spin connection is of the same order as the torsion, tetrad and curvature. Therefore this is not Minkowski space-time because the free field is due to space-time torsion. In a Minkowski space-time, the torsion, curvature and spin connection all vanish.

Translating into vector notation, Eqs. (22.125) and (22.126) become the familiar:

$$\boldsymbol{\nabla} \cdot \boldsymbol{B} = 0, \tag{22.130}$$

$$\boldsymbol{\nabla} \times \boldsymbol{E} + \frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{0}, \tag{22.131}$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{E} = 0, \tag{22.132}$$

$$\boldsymbol{\nabla} \times \boldsymbol{B} - \frac{1}{c^2} \frac{\partial \boldsymbol{E}}{\partial t} = \boldsymbol{0}. \tag{22.133}$$

For example, the Coulomb law for the free field is Eq. (22.131), with the solution:

$$\boldsymbol{E} \rightarrow \boldsymbol{0}. \tag{22.134}$$

This result means that in electro-statics, the static electric field tends to zero if the distance between two charges approaches infinity. This result can be seen from the Coulomb law:

$$\boldsymbol{F} = \frac{e_1 e_2}{4\pi\epsilon_0 r^3} \boldsymbol{r} \tag{22.135}$$

where:

$$\boldsymbol{E} = \frac{e_2}{4\pi\epsilon_0 r^3}\ \boldsymbol{r}. \tag{22.136}$$

Here e_1 is a charge interacting with e_2, ϵ_0 is the S.I. vacuum permittivity, $\boldsymbol{r}$ is the radial coordinate, and $\boldsymbol{F}$ is the Coulomb force of repulsion. In electro-dynamics there are plane wave solutions to Eqs. (22.129) to (22.132), plane waves which propagate through the vacuum infinitely distant from the source. This is a mathematical concept. The plane waves are:

$$\boldsymbol{E} = \frac{E^{(0)}}{\sqrt{2}}(\boldsymbol{i} - i\boldsymbol{j}) \ \exp(i(\omega t - \boldsymbol{\kappa} \cdot \boldsymbol{r})) \tag{22.137}$$

and

$$\boldsymbol{B} = \frac{B^{(0)}}{\sqrt{2}}(i\boldsymbol{i} + \boldsymbol{j}) \ \exp(i(\omega t - \boldsymbol{\kappa} \cdot \boldsymbol{r})) \tag{22.138}$$

and there are other types of dynamical solution such as spherical waves based on spherical harmonics. The plane waves have angular frequency ω at an instant t, and wave-number $\boldsymbol{k}$ at position $\boldsymbol{r}$.

b) Free Gravitational Fields

The free gravitational fields are defined in ECE theory by equations which have the same structure as (22.129) to (22.132) [1–10]:

$$\boldsymbol{\nabla} \cdot \boldsymbol{h} = 0, \tag{22.139}$$

$$\boldsymbol{\nabla} \times \boldsymbol{g} + \frac{1}{c}\frac{\partial \boldsymbol{h}}{\partial t} = \boldsymbol{0}, \tag{22.140}$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{g} = 0, \tag{22.141}$$

$$\boldsymbol{\nabla} \times \boldsymbol{h} - \frac{1}{c}\frac{\partial \boldsymbol{g}}{\partial t} = \boldsymbol{0}, \tag{22.142}$$

where $\boldsymbol{g}$ is the acceleration due to gravity (an orbital component of torsion). Eq. (22.140) can be interpreted in the same way as Eq. (22.131) for the static electric field. The Newton inverse square law is a well defined limit of ECE theory and is a (negative valued) force of attraction between two masses m and M:

$$\boldsymbol{F} = -\frac{mMG}{r^3}\boldsymbol{r} \tag{22.143}$$

in which:

$$\boldsymbol{g} = -\frac{mG}{r^3}\boldsymbol{r} \tag{22.144}$$

from the weak equivalence principle. Therefore for infinitely distant masses the acceleration due to gravity approaches zero, which is the physical interpretation of Eq. (22.140). Eqs. (22.138) to (22.141) show that besides this well known law, there are three other laws of classical gravitation whose structure is the same as the laws of classical electrodynamics. This is a major result of the EEC unified field theory. The classical vacuum is defined as the absence of mass and charge, so:

$$\boldsymbol{E} \to \boldsymbol{0}, \ \boldsymbol{g} \to \boldsymbol{0} \tag{22.145}$$

in the vacuum.

The radiated gravitational plane waves are evidently:

$$\boldsymbol{g} = \frac{g^{(0)}}{\sqrt{2}}(i\boldsymbol{i} - \boldsymbol{j}) \ \exp(i(\omega t - \boldsymbol{\kappa} \cdot \boldsymbol{r})) \tag{22.146}$$

and

$$\boldsymbol{h} = \frac{h^{(0)}}{\sqrt{2}}(i\boldsymbol{i} + \boldsymbol{j}) \ \exp(i(\omega t - \boldsymbol{\kappa} \cdot \boldsymbol{r})) \tag{22.147}$$

and are about twenty one orders of magnitude weaker in the laboratory than the radiated plane waves of classical electromagnetism. Note carefully that gravitational radiation cannot be deduced from the Einstein field equation, because of the latter's neglect of torsion. As shown in papers 93 and following on www.aias.us, the neglect of torsion leads to an incorrect geometry, so nothing can be deduced form the Einstein field equation. The latter is regarded in ECE theory as obsolete.

When fields interact with matter the basic geometry is:

$$d \wedge T = (R \wedge q - \omega \wedge T)_{\text{int}} \tag{22.148}$$

and its Hodge dual:

$$d \wedge \widetilde{T} = (\widetilde{R} \wedge q - \omega \wedge \widetilde{T})_{\text{int}}. \tag{22.149}$$

a) In electrodynamics in the laboratory, the magnetic monopole is unmeasurably small experimentally, so

$$d \wedge F \to 0, \tag{22.150}$$

$$d \wedge \widetilde{F} = \widetilde{j}/\epsilon_0. \tag{22.151}$$

Eqs. (22.149) and (22.150) translate into the free field homogeneous equations:

$$\boldsymbol{\nabla} \cdot \boldsymbol{B} = 0, \tag{22.152}$$

$$\boldsymbol{\nabla} \times \boldsymbol{E} + \frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{0}, \tag{22.153}$$

and the field matter inhomogeneous equations:

$$\boldsymbol{\nabla} \cdot \boldsymbol{D} = \rho, \tag{22.154}$$

$$\boldsymbol{\nabla} \times \boldsymbol{H} - \frac{\partial \boldsymbol{D}}{\partial t} = \boldsymbol{J}, \tag{22.155}$$

where $\boldsymbol{E}$ is the electric field strength, $\boldsymbol{B}$ is the magnetic flux density, $\boldsymbol{D}$ is the electric displacement, ρ is the electric charge density, $\boldsymbol{H}$ is the magnetic field strength, and $\boldsymbol{J}$ is the electric current density. The displacement is defined as:

$$\boldsymbol{D} = \epsilon_0 \boldsymbol{E} + \boldsymbol{P} \tag{22.156}$$

where $\boldsymbol{P}$ is the electric polarization, and the magnetic field strength is defined by:

$$\boldsymbol{B} = \mu_0 (\boldsymbol{H} + \boldsymbol{M}) \tag{22.157}$$

where $\boldsymbol{M}$ is the magnetization and μ_0 is the magnetic vacuum permeability [1–10].

In general, an asymmetric connection must be used to find j in Eq. (22.150). The Hodge dual of the interaction current:

$$\widetilde{j} := (\widetilde{R} \wedge q - \omega \wedge \widetilde{T})_{\text{int}} \tag{22.158}$$

is another interaction current:

$$j := (R \wedge q - \omega \wedge T)_{\text{int}}. \tag{22.159}$$

Therefore the Hodge dual of the pair of vector equations:

$$\boldsymbol{\nabla} \cdot \boldsymbol{D} = \rho, \tag{22.160}$$

$$\boldsymbol{\nabla} \times \boldsymbol{H} + \frac{\partial \boldsymbol{D}}{\partial t} = \boldsymbol{J} \tag{22.161}$$

is

$$\boldsymbol{\nabla} \cdot \boldsymbol{H} = -\boldsymbol{\nabla} \cdot \boldsymbol{M} := \widetilde{\rho}, \tag{22.162}$$

$$\boldsymbol{\nabla} \times \boldsymbol{D} + \frac{1}{c^2} \frac{\partial \boldsymbol{H}}{\partial t} = \boldsymbol{\nabla} \times \boldsymbol{P} - \frac{1}{c^2} \frac{\partial \boldsymbol{M}}{\partial t} := \boldsymbol{J} \tag{22.163}$$

where we have used Eqs. (22.151) and (22.152). The Hodge dual of these latter equations are Eqs. (22.131) and (22.132).

b) The interaction of the gravitational field with matter is given by:

$$d \wedge \widetilde{T} = \widetilde{j}_{\text{int}} = (\widetilde{R} \wedge q - \omega \wedge \widetilde{T})_{\text{int}} \tag{22.164}$$

so that Eq. (141) for example, becomes:

$$\boldsymbol{\nabla} \cdot \boldsymbol{g} = 4\pi G \rho_m \tag{22.165}$$

where $\boldsymbol{G}$ is the Newton constant and ρ_m is the mass density [1–10]. This is the gravitational analogue of the Coulomb law (22.159). In ECE theory there is also a gravitational analogue of Eq. (22.160), a law that should be investigated experimentally. The gravitational law (22.139) has already been observed by Tajmar, de Matos et al. [16].

The homogeneous gravitational current:

$$j = (R \wedge q - \omega \wedge T)_{\text{free}} \tag{22.166}$$

must be interpreted carefully as being due to the interaction of gravitational and electromagnetic fields infinitely far from their sources. This type of interaction in classical electrodynamics would mean that the Gauss law of magnetism and Faraday law of induction become minutely different due to the existence of a magnetic charge density (monopole) and magnetic current density. The magnetic charge density or monopole, and the magnetic current density are both due to the geometry of Eq. (22.1). This is a concept of general relativity and not of special relativity. The ECE magnetic monopole is not a Dirac monopole, and not a topological monopole of gauge theory in special relativity. The ECE monopole is due to the geometry of space-time.

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