Solar system orbits from the antisymmetric connection

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Abstract

Keywords:

1 Introduction

2 Self-consistent definition of the metric

3 Derivation of the metric factor $m(r)$

4 Computational analysis of the metric function

Details of cosmological solutions are presented in this section. After showing the general solution $m(r)$ derived in the previous section, the metric function of Kepler orbits (relativistic and non-relativistic, i.e. Newtonian) is derived. These orbits are valid for the solar system. Finally we present the metric of logarithmic spiralling orbits being observed on galaxy scales.

4.1 Properties of the general solution

The general form of $m(r)$ is given in Eq. (39). R is a constant and set to a numerical valud of 1/3 for simplicity. From Fig. 1 it can be seen that this function behaves very similar to the so-called Scharzschild metric Eq. (42) with

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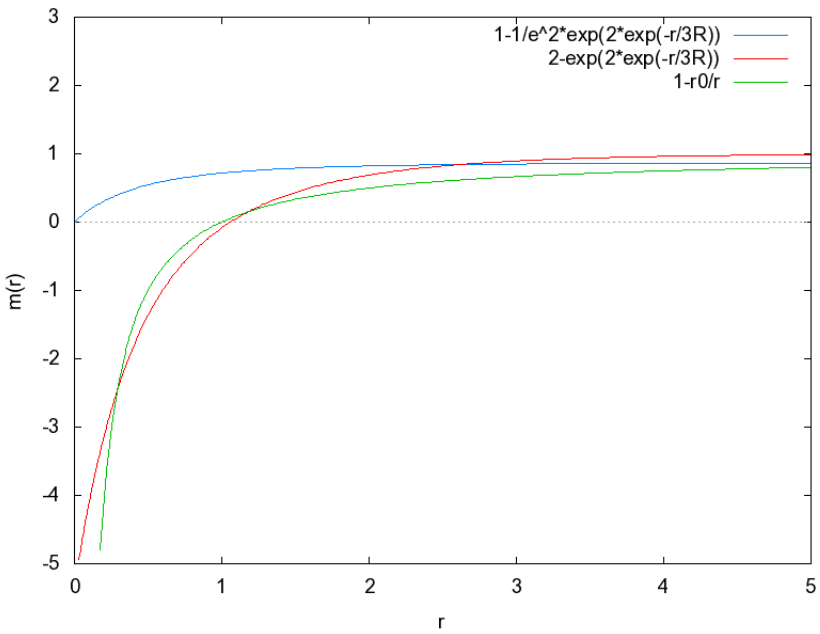

Figure 1: Different forms of $m(r)$.

$r_0 = 1$. In particular there is a zero crossing and a divergent behaviour for $r \to 0$. The zero crossing appears at

$$r_{\text{zero}} = 3\,R \log\left(\frac{2}{\log(2)}\right). \tag{44}$$

The divergent behaviour can be avoided by shifting the r coordinate or using another admissible solution for the metric function, for example

$$m(r) = 1 - \frac{1}{e^2}\exp(2\exp(-\frac{r}{3R})). \tag{45}$$

This function is regular for $r \geq 0$, however the limit of this function is

$$m(r) \to 1 - e^{-2} \tag{46}$$

instead of unity for r going to infinity.

From the solar system it is known that Eq. (42) gives an excellent descrition of gravitation. Therefore we tried to adopt the curve of $m(r)$ to the graph of this equation by least squares fitting. In $m(r)$ there is only one fitting parameter R available, therefore no perfect coincidence can be obtained. The numerical procedure gives

$$R = 0.374\ r_0 \tag{47}$$

as an optimal value. The resulting graph is very similar to that shown in Fig. 1 for $R = 1/3$..

4.2 The relativistic and non-relativistic Kepler Problem

The equation of orbits is found from Eq. (14) for $\mu = \nu = 0$:

$$\partial_0 g_{00} = 0 \tag{48}$$

which is

$$\frac{\partial}{\partial t} m(r,t) = 0. \tag{49}$$

The time dependence of m has not been considered so far. From Eq. (39) it is seen that only the characteristic radius R can be time dependent, therefore

$$m(r,t) = 2 - \exp(2\exp(-\frac{r}{R(t)})). \tag{50}$$

Applying the time derivative in Eq. (49) then leads to the differential equation

$$\frac{1}{R^2(t)} \left(R(t)\frac{dr}{dt} - r\frac{dR(t)}{dt} \right) = 0 \tag{51}$$

or

$$\frac{dr}{dt} = \frac{r}{R(t)} \frac{dR(t)}{dt}. \tag{52}$$

This is an equation for all orbits. The radial coordinate r has to be considered to have a time dependence now which is characteristic for orbital motion. Note that this time dependence does not appear in the metric function (39) a priori.

The orbits of planets in the solar system are described experimentally by a precessing ellipse:

$$r = \frac{\alpha}{1 + \epsilon \cos(y\theta)} \tag{53}$$

with ϵ being the eccentricity, α the semi-major axis and y a parameter describing the precession of the ellipse. In the Newtonian limit we have

$$y \to 1. \tag{54}$$

The precessing ellipse is derved from the so-called Schwarzschild metric

$$ds^2 = (1 - \frac{r_0}{r})c^2 dt^2 - (1 - \frac{r_0}{r})^{-1} dr^2 - r^2 \sin^2(\theta) d\theta^2 \tag{55}$$

which is an approximation to the metric with metric function $m(r,t)$ derived in this paper and passes into the Minkowski metric for

$$\frac{r_0}{r} \to 0. \tag{56}$$

The time derivative of θ in central motion is

$$\frac{d\theta}{dt} = \frac{L}{\mu r^2} \tag{57}$$

where L is the conserved angular momentum and μ the reduced mass. With (53) this is

$$\frac{d\theta}{dt} = \frac{L}{\mu \alpha^2} \left(1 + \epsilon \cos(y\theta)\right)^2 \tag{58}$$

and from this equation and (53) follows by differentiation:

$$\frac{dr}{dt} = \frac{dr}{d\theta}\frac{d\theta}{dt} = \frac{Ly\epsilon}{\mu\alpha^2}\sin(y\theta). \tag{59}$$

Inserting the results into (52) gives a differential equation for $R(t)$:

$$\frac{1}{r}\frac{dr}{dt} = \frac{Ly\epsilon}{\mu\alpha^2}\left(1+\epsilon\cos(y\theta)\right)\sin(y\theta) = \frac{1}{R(t)}\frac{dR(t)}{dt}. \tag{60}$$

Because of

$$\frac{1}{R(t)}\frac{dR(t)}{dt} = \frac{d\log(R(t))}{dt} \tag{61}$$

this equation can be integrated to give

$$R(t) = c\,\frac{Ly\epsilon}{\mu\alpha^2}\int \left(1+\epsilon\cos(y\theta)\right)\sin(y\theta)\,dt. \tag{62}$$

with an integration constant c. The integral cannot be evaluated directly because of the time dependence of θ, but a variable substitution $t \to \theta$ can be performed using Eq. (58). Computer algebra then gives the final result

$$R(\theta) = \frac{c}{\left(1+\epsilon\cos(y\theta)\right)^{1/y}} \tag{63}$$

which with appropriate choice of the constant c can be written as

$$\boxed{R(\theta) = r(\theta)^{1/y}}\,. \tag{64}$$

For Newtonian orbits this further simplifies to the fundamental result

$$\boxed{R(\theta) = r(\theta)}\,. \tag{65}$$

The dependence of $m(r,\theta)$ has been graphed as a suface plot in Fig. 2 for $\epsilon = 0.3$ (all other constants set tu unity). The cyclic weak dependence on the angle θ is visible. The dependence of θ from time can be calculated by integration of Eq. (58). The result obtained from computer algebra is quite complicated:

$$t = \frac{2\,\alpha^2\,\mu}{y\,L}\left(\frac{\operatorname{atan}\left(\frac{(2\,\epsilon-2)\sin(\theta\,y)}{2\sqrt{1-\epsilon^2}\,(\cos(\theta\,y)+1)}\right)}{\sqrt{1-\epsilon^2}\,(\epsilon^2-1)}\right.$$
$$\left. - \frac{\epsilon\sin(\theta\,y)}{(\cos(\theta\,y)+1)\left(\frac{(\epsilon^3-\epsilon^2-\epsilon+1)\sin(\theta\,y)^2}{(\cos(\theta\,y)+1)^2} - \epsilon^3-\epsilon^2+\epsilon+1\right)}\right). \tag{66}$$

The behaviour is illustrated in Fig. 3. It can be seen that for a relatively strong eccentricity of $\epsilon = 0.3$ the dependence remains near to linear as expected for the solar system.

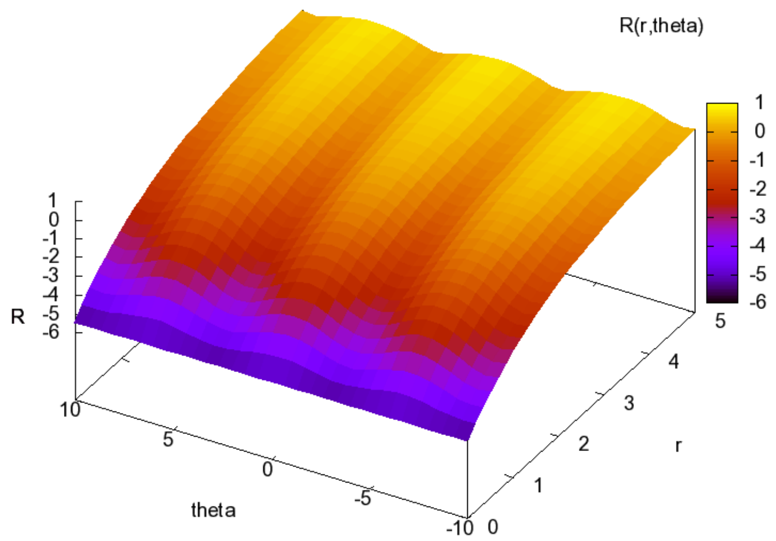

Figure 2: Surface plot of $m(r,t)$ for Keplerian orbits.

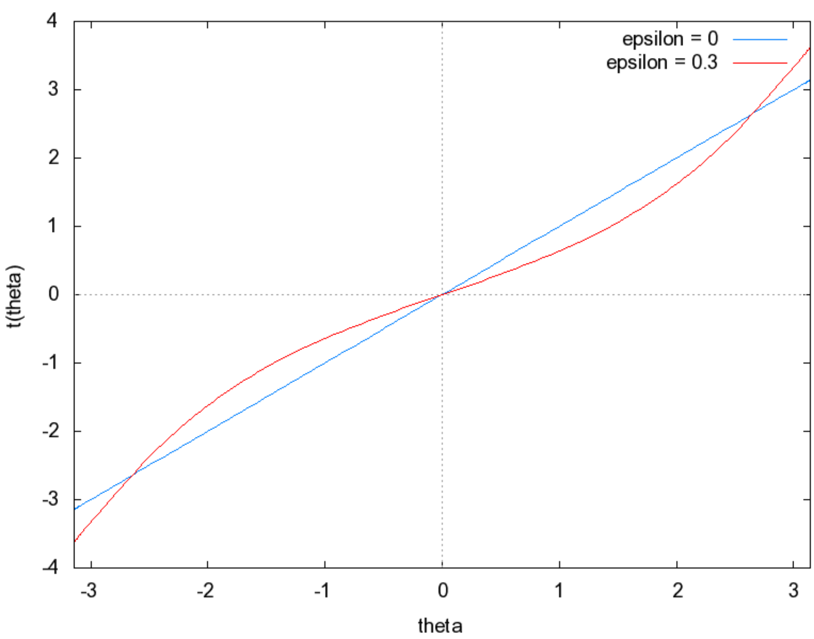

Figure 3: Dependence $t(\theta)$ for Keplerian orbits.

4.3 Logarithmic spiral orbits

The orbit of a logarithmic spiral is given by [13]

$$r = k \exp(\alpha\theta) \tag{67}$$

with k and α being constants. The time dependence of r is

$$r(t) = \left(\frac{2\alpha L}{\mu} t + k^2 C\right)^{1/2} \tag{68}$$

where μ and L are defined as for the Keplerian orbits. From the latter equation follows

$$\frac{d\, r(t)}{dt} = -\frac{\alpha L}{\mu}\left(\frac{2\alpha L}{\mu} t + k^2 C\right)^{-3/2}. \tag{69}$$

With Eq. (52), which is

$$\frac{1}{r}\frac{dr}{dt} = \frac{1}{R(t)}\frac{dR(t)}{dt}, \tag{70}$$

we obtain the the differential equation

$$-\frac{\alpha L}{\mu}\left(\frac{2\alpha L}{\mu} t + k^2 C\right)^{-2} = \frac{1}{R(t)}\frac{dR(t)}{dt}. \tag{71}$$

The solution is

$$R(t) = R_0 \exp\left(-\frac{1}{\mu^2}(\alpha^2 L^2 t^2 + \alpha k^2 \mu C L t)\right) \tag{72}$$

with an integration constant R_0. This is a Gaussian function, depicted in Fig. 4 where all constants have been set to unity again. Inserting $R(t)$ into Eq. (50) gives the netric function for logarithmic orbits. This is also plotted in Fig. 4, showing some kind of inverse Gaussian behaviour. The combined r and t dependence can be observed in the surface plot (Fig. 5). $m(r, t)$ behaves like a Gaussian in time and an $1/r$ function in the radial coordinate. The physically meaningful range begins at $t = 0$ where m has the highest slope, reflecting the fact that m deviates most from free-

space behaviour near to the the center of a spiral. Spiral arms of galaxies should be describable in this way.

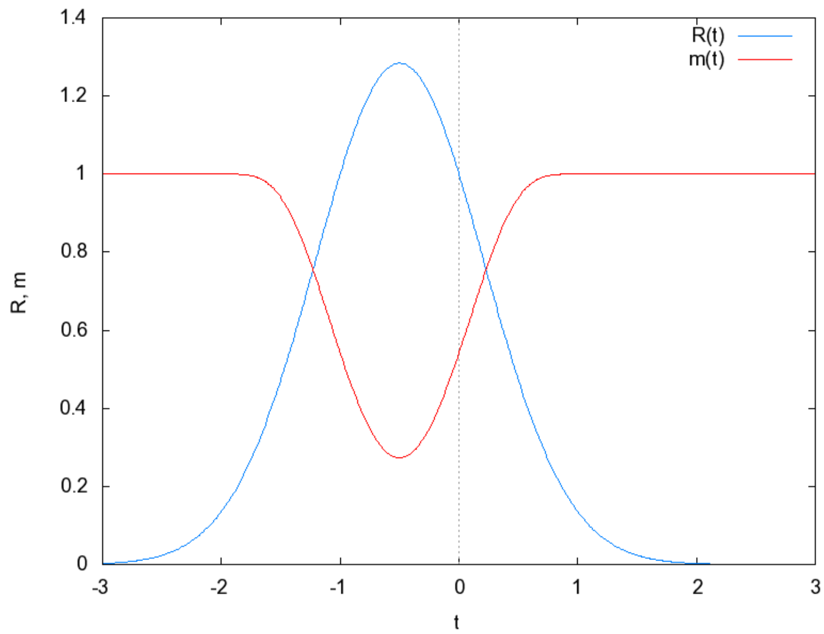

Figure 4: Functions $R(t)$ and $m(r,t)$ (with r=5) for a logarithmic spiral orbit.

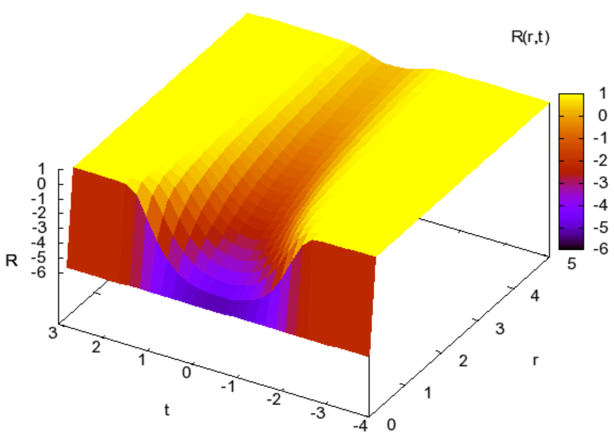

Figure 5: Surface plot of $m(r,t)$ for a logarithmic spiral orbit.