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The Coulomb and Ampère Maxwell Laws in the Friedmann Lemaître Robertson Walker Metric

by

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Abstract

The Coulomb and Ampère Maxwell laws are derived in the Friedmann Lemaître Robertson Walker metric and it is shown that these laws of classical electrodynamics depend on the evolution of the universe. This is the result of a generally covariant unified field theory in which all sectors are rigorously objective. The charge density and current density of these laws depend on the age of the universe, a new result in physics. The experimental implications of this result are discussed in terms of the change in the optical properties of light deflected by gravitation.

Keywords: Coulomb law, Ampère Maxwell law, Friedmann Lemaître Robertson Walker metric, generally covariant unified field theory.

2.1 Introduction

Recently a generally covariant unified field theory has been developed on the basics of general relativity, that physics is geometry, and that objectivity in physics is measured by geometry. The latter in Einstein Cartan Evans (ECE) field theory is represented by standard Cartan geometry [1–10] in which space-time torsion is present in general as well as space-time curvature. Within this geometrical structure the laws of classical electrodynamics have been unified with the laws of gravitation [11] for several representative metrics.

In Section 2.2 the implications of this result are developed for the most used metric in cosmology, the Friedmann Lemaître Robertson Walker (FLRW) metric [12] that is an exact solution of the Einstein Hilbert field equation. Many such exact solutions are now known [13] but few of these are likely to give scientifically acceptable charge and current densities of the Coulomb and Ampère Maxwell laws. This procedure has been made possible by the development [1–10] of ECE theory (www.aias.us and www.atomicprecision.com). The charge and current densities of the FLRW metric are given in Section 2 for the usual cases developed in cosmology, a flat, closed and open universe [12]. In Section 2.3 the results are analyzed, and the FLRW metric criticized on the geometrical ground given by Crothers [11]. Some elementary errors [12] in units used in cosmology are corrected. The main result is that these laws of classical electrodynamics depend on the evolution and age of the universe. They are not immutable laws as in the Maxwell Heaviside field theory of special relativity. As Crothers has argued [11] the singularity in FLRW at $t = 0$ does not signify Big Bang, it is due to a geometrical misconception carried through uncritically in the standard model literature. The correct treatment of metrics of this type has been given by Eddington [14]. This paper is intended to develop the laws of electrodynamics in the FLRW metric as usually used in cosmology, in a more rigorous treatment in forthcoming work, the rigorously correct Crothers metric [11] will be used.

2.2 The Laws of Electrodynamics in the FLRW Metric

The FLRW metric is [12]:

$$ds^2 = -c^2 dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right) \tag{2.1}$$

and is an exact solution of the Einstein Hilbert (EH) field equation. Here a is dimensionless, k is in inverse square meters and r is in meters squared. The usual habit in cosmology is to regard k as "dimensionless". If this is done then $k = -1$ is an open universe, $k = 0$ is a flat universe and $k = 1$ is a closed universe. We replace this habit as follows in rigorous S.I. units:

$$k = 0, \quad \pm 1 \quad m^{-2}. \tag{2.2}$$

Matter and energy in this metric may be modeled by a perfect fluid [12], whose canonical energy momentum density is:

$$T_{\mu\nu} = (\boldsymbol{p} + \rho)U_\mu U_\nu + \rho g_{\mu\nu}. \tag{2.3}$$

Here ρ and p are respectively the energy density and pressure as measured in the rest frame, and U^μ is the four velocity of the fluid. If both the fluid and metric are isotropic then their frames coincide and they are known as "co-moving" [12]. The fluid is at rest in co-moving coordinates. In this case:

$$U^\mu = (1, 0, 0, 0) \tag{2.4}$$

and

$$T_{\mu\nu} = \begin{bmatrix} \rho & 0 & 0 & 0 \\ 0 & & g_{(ij)}\boldsymbol{p} & \\ 0 & & & \\ 0 & & & \end{bmatrix}. \tag{2.5}$$

Lowering an index it is found that [12]:

$$T^\mu{}_\nu = \text{diag}\,(-\rho, \boldsymbol{p}, \boldsymbol{p}, \boldsymbol{p}) \tag{2.6}$$

whose trace is:

$$T = T^\mu{}_\mu = -\rho + 3p. \tag{2.7}$$

An equation of state [12] must now be defined. The simplest choice is:

$$p = w\rho \tag{2.8}$$

where w is a constant independent of time. The law of conservation of energy is then given by [12]:

$$\frac{\dot{\rho}}{\rho} = -3(1 + w)\frac{\dot{a}}{a} \tag{2.9}$$

i.e.

$$\rho \propto a^{-3(1+w)}. \tag{2.10}$$

Dust and radiation are examples of cosmological fluids. The former is defined as collision-less and non-relativistic with no pressure (w is zero). Examples are stars and galaxies. The energy density of dust is dominated by the rest energy, which is proportional to the number density. In this paper we will restrict consideration to dust defined in this way. The EH equation is written [12] in the form:

$$R_{\mu\nu} = 8\pi G\left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\right) \tag{2.11}$$

where $R_{\mu\nu}$ is the Ricci tensor, $\boldsymbol{G}$ is Newton's constant, $g_{\mu\nu}$ is the symmetric metric and $\boldsymbol{T}$ the trace of $T_{\mu\nu}$. For:

$$\mu = 0, \nu = 0 \tag{2.12}$$

Eq. (2.11) gives, for the FLRW metric (2.1):

$$\frac{-3\ddot{a}}{a} = 4\pi G\left(\rho + 3\boldsymbol{p}\right) \tag{2.13}$$

where:

$$\dot{a} = \frac{da}{dt}. \tag{2.14}$$

For

$$\mu = i, \quad \nu = j \tag{2.15}$$

we obtain:

$$\frac{\ddot{a}}{a} + 2\left(\frac{\dot{a}}{a}\right)^2 + \frac{2k}{a^2} = 4\pi G(\rho - p). \tag{2.16}$$

Re-arrangement of terms gives the Friedmann equations [12]:

$$\frac{\ddot{a}}{a} = -\frac{4}{3}\pi G(\rho + 3\boldsymbol{p}) \tag{2.17}$$

and

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}. \tag{2.18}$$

In ref. [11] it was found that the charge density of the Coulomb law given by this metric is:

$$J^0 = -3\phi\frac{\ddot{a}}{a} \tag{2.19}$$

in Vm^{-2} (S.I. units). Here ϕ is the primordial scalar potential in volts (J/C) of the ECE theory. Using the first Friedmann equation (2.17):

$$J^0 = 4\pi\phi G(\rho + 3p) \tag{2.20}$$

For dust:

$$p = 0 \tag{2.21}$$

and we obtain:

$$J^0 = 4\pi\phi G\rho. \tag{2.22}$$

The Coulomb law in this case is therefore:

$$\boldsymbol{\nabla} \cdot \mathbf{E} = \rho_e/\epsilon_0 \tag{2.23}$$

where the electric charge density ρ_e is defined by:

$$\rho_e = 4\pi\epsilon_0\phi G\rho \tag{2.24}$$

The S.I. units of this equation are as follows:

$$\begin{aligned} &\rho_e = Cm^{-3}; \epsilon_0 = J^{-1}C^2m^{-1}; \phi = \text{volt} = JC^{-1}; \\ &G = mkgm^{-1}; \rho = kgm\, m^{-3}; J^0 = \text{volt}\, m^{-2}. \end{aligned} \tag{2.25}$$

The precise self consistency of ECE theory is demonstrated by the fact that Eq. (2.24) is the same exactly in structure as Eq. (5.102) vol. 1 of ref. (1). Eq (2.24) shows that charge density cannot exist without mass density, and the implications of this result were discussed in chapter five vol. 1 of ref. (1). For example the electron's charge is always found with the electron's mass. This finding has now been explained from first principles using the required generally covariant unified field theory [1–11]. In Maxwell Heaviside (MH) theory there is no explanation for it, because MH is special relativity whose flat or Minkowski metric is:

$$ds^2 = -c^2dt^2 + dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2). \tag{2.26}$$

In ECE theory [11] the Minkowski metric produces zero charge density in the Coulomb law because curvature and torsion are zero in the Minkowski metric.

The radial and angular components of the current density of the FLRW metric [11] were found to be as follows:

$$J_r = \frac{A^{(0)}}{\mu_0}\left(\frac{1-kr^2}{a^2}\right)\left(\frac{2}{a^2}(k+\dot{a}^2)+\frac{\ddot{a}}{a}\right) \tag{2.27}$$

and

$$J_\theta = J_\phi \sin^2\theta = \frac{A^{(0)}}{\mu_0 r^2 a^2}\left(\frac{2}{a^2}(k+\dot{a}^2)+\frac{\ddot{a}}{a}\right), \tag{2.28}$$

where $A^{(0)}$ is the magnitude of the primordial vector potential of ECE theory and μ_0 is the vacuum permeability. The S.I. units of the ratio of these two factors are as follows:

$$\frac{A^{(0)}}{\mu_0} = \frac{Js\,C^{-1}m^{-1}}{Js^2\,C^{-2}m^{-1}} = Cs^{-1} = \text{ampere}, \tag{2.29}$$

so (A/μ_0) has the units of Ampères (C/s). The results for current density depend on the parameter k, i.e. on whether the universe is closed, flat or open. The results for charge density however are independent of k.
a) For a flat universe:

$$k = 0 \tag{2.30}$$

and

$$J_r = r^2 J_\theta = r^2 \sin^2\theta J_\phi = \frac{A^{(0)}}{\mu_0}\left(\frac{4\pi G\rho}{a^2}\right). \tag{2.31}$$

b) For a closed universe:

$$k = 1m^{-2} \tag{2.32}$$

and

$$J_r = \frac{A^{(0)}}{\mu_0}\left(4\pi G\rho\left(\frac{1-kr^2}{a^2}\right)\right) \tag{2.33}$$

with:

$$r^2 J_\theta = r^2 \sin^2\theta J_\phi = \frac{A^{(0)}}{\mu_0}\left(\frac{4\pi G\rho}{a^2}\right) \tag{2.34}$$

c) For an open universe:

$$k = -1m^{-2} \tag{2.35}$$

and:

$$J_r = \frac{A^{(0)}}{\mu_0}\left(4\pi G\rho\left(\frac{1-kr^2}{a^2}\right)\right) \tag{2.36}$$

with:

$$r^2 J_\theta = r^2 \sin^2\theta J_\phi = \frac{A^{(0)}}{\mu_0}\left(\frac{4\pi G\rho}{a^2}\right). \tag{2.37}$$

In each case the factor a^2 appears in the denominator. This is defined for a flat universe by:

$$a \propto t^{2/3} \tag{2.38}$$

for a closed universe by:

$$a \propto \left(\frac{1-\cos\phi}{\phi-\sin\phi}\right) t \tag{2.39}$$

and for an open universe by:

$$a \propto \left(\frac{1-\cosh\phi}{\phi-\sinh\phi}\right) t. \tag{2.40}$$

Unfortunately the S.I. units of a^2 given in ref. (12) are incorrect, so we adopt a proportionality sign for a. In ref. (12) a constant C is defined by:

$$C := \frac{8\pi G}{3}\rho a^3 \tag{2.41}$$

which must have the units of inverse square meters because a is unitless. However, in ref. (12) the following equations are used respectively for a flat, closed and open universe:

$$\begin{aligned} a &= \frac{C}{2}(1-\cos\phi), t = \frac{C}{2}(\phi-\sin\phi), k = 1, \\ a &= \left(\frac{9C}{4}\right)^{1/3} t^{2/3}, k = 0, \\ a &= \frac{C}{2}(\cosh\phi - 1), t = \frac{C}{2}(\sinh\phi - \phi), k = -1. \end{aligned} \tag{2.42}$$

It is seen that the units in these equations are not correct. This is a problem of usage in standard model cosmology. The units in the results of this paper are rigorously correct S.I. units of charge density (C/m^3) and current density($C/s/m^2$).

2.3 Discussion

The main result is that the Coulomb and Ampère Maxwell laws of classical electrodynamics depend on the evolution of the universe from an initial:

$$t = 0. \tag{2.43}$$

This is usually interpreted as Big Bang [12]. At $t = 0$ the charge density of Eq. (2.24) is finite but the current densities are infinite. If the volume at this initial instant is zero, then the mass density and charge density of Eq. (2.24) become infinite also. However, Crothers [11] has shown that there is an irretrievable error in this theory because of confusion in geometry. The Universe may only go to a finite radius at the initial instant. We accept this criticism by Crothers [11] and in future work will replace the FLRW metric by the rigorously correct Crothers metric evaluated in ref. (11). The purpose of this paper is to show that in a generally covariant unified field theory the laws of electrodynamics evolve, they are not immutable as in the Maxwell Heaviside field theory. The FLRW metric can only be used as a guide to this overall result. As shown in ref. (11), the correct treatment of the metric, following Eddington [14] leads to a universe that is infinite in extent. The standard model FLRW metric leads to the flawed idea of Big Bang. In ref. (11), several other metrics were used to evaluate the charge density of the Coulomb law and the current density of the Ampère Maxwell laws. The FLRW metric is simply the one most used in cosmology [12].

In a generally covariant unified field theory the charge density of the Coulomb law and the current density of the Ampère Maxwell law must be well defined and well behaved. Few metrics of the many now known [13] will satisfy this requirement. For example the charge and current densities must be free of mathematical singularities. In the FLRW metric a singularity occurs at the initial point in time, so at that point the metric is not well behaved in general relativity. As argued already the source of this is incorrect [11] evaluation of geometry. However, for finite t the charge density of FLRW appears to be well behaved and finite. Unfortunately it is not possible to fully evaluate the behavior of the current densities for finite t because of elementary errors in units in ref. (12). It may be concluded that the current densities of the Ampère Maxwell law from the standard model FLRW metric (which is flawed [11] geometrically) are different for a flat, closed and open universe, and provided that t is not zero, they have no mathematical singularities.

In order to test these conclusions experimentally the properties of light deflected by gravitation can be analyzed for changes in electrodynamic and kinematic properties, for example changes in polarization [1–10] which are known to occur from objects such as a white dwarf. The reason for this method is that the current density of the Ampère Maxwell law is defined in ECE by curvature [11], so the existence of the current density changes the vacuum Ampère Maxwell law from:

$$\boldsymbol{\nabla} \times \boldsymbol{B} = \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t} \tag{2.44}$$

to

$$\boldsymbol{\nabla} \times \boldsymbol{B} = \frac{1}{c^2}\frac{\partial \boldsymbol{E}}{\partial t} + \mu_0 \boldsymbol{J}. \tag{2.45}$$

Similarly the vacuum Coulomb law is changed from

$$\nabla \cdot \boldsymbol{E} = 0 \tag{2.46}$$

to

$$\nabla \cdot \boldsymbol{E} = \rho_e/\epsilon_0 \tag{2.47}$$

In other words charge and current density exist in ECE where there appears to be no mass present, for example at the edges of a very heavy object such as a white dwarf. The existence of $\boldsymbol{J}$ in eq. (2.45) changes the polarization from for example circular to elliptical [1–10].

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