ESSAY 26: QUANTUM HAMILTON EQUATIONS

In the previous essay the Heisenberg uncertainty principle was criticised because of its introduction into Baconian natural philosophy of the unscientific concept of indeterminacy, which means that a quantity can be absolutely unknowable. Central to the Heisenberg uncertainty principle is the commutator [A, B] of two hermitian operators A and B. Hermitian operators have real eigenvalues corresponding to observables and the Copenhagen philosophy of indeterminacy relies on the product of the root mean square deviations from the mean of operators A and B. This is asserted to be greater than or equal to half the expectation value of C, denoted <C>, where [A, B] = iC. Simply by investigating higher order commutators such as [pp, xx] it was found in UFT 175 that C can be zero or non-zero from the same Schroedinger equation. This means in general that A and B can be Aunknowable@ or Aknowable@, according to Copenhagen, from the same equation. So by consensus to date, the Copenhagen interpretation has been abandoned, in other words there has been no objection internationally to UFT 175.

As an unexpected by product of this research, a research initially intended to show that the Heisenberg uncertainty principle is a tautology, new equations of motion of quantum mechanics have been discovered. These have been named AThe Quantum Hamilton Equations@, and will be written up in UFT 176. They were derived by considering simple tautologies: d<x> / dx = 1 and d<p> / dp = 1. In the classical limit these tautologies become Poisson brackets equal to unity, and this is the root of the so called Heisenberg uncertainty principle. It is well known that [A, B] divided by i h bar becomes the Poisson bracket (A, B) as h bar goes to zero. For [x, p], the corresponding Poisson bracket (x, p) is always unity. This is a tautology, i.e. a statement of the obvious, and a statement of the fact that there is no deep philosophical significance in unity or the Heisenberg uncertainty principle. When [xx, pp] is considered for example, the corresponding Poisson bracket (x squared, p squared) is 4xp, which is not constant. It was found in UFT 175 that <[xx, pp]> is zero for all harmonic oscillator wavefunctions and non zero for H atom wavefunctions. So the familiar constancy of [x, p] = i h bar disappears and for higher order commutators It is not possible to assert that the relevant product of root mean square deviations form the mean is a constant. It can be zero or non-zero. If zero, the operators are precisely knowable, and the basis of indeterminacy disappears.

Although this is an important philosophical result, clearing up years of needless debate caused by Copenhagen, the emergence of new equations of motion of quantum mechanics is even more so. They emerge from the classical tautologies: dq / dq = 1 = (q, p) and dp / dp = 1 = (p, q), where p and q are the canonical variables of the Hamilton equations of motion: dp / dt = - dH / dq and dq / dt = dH / dp where H is the classical hamiltonian. Upon quantization the following two operator equations emerge from the correspondence between the Poisson bracket and commutator: i h bar dq /dq = [q, p] and i h bar d p / dp = [p, q]. Finally the two quantum Hamilton equations are obtained by realizing that these tautologies can be generalized as follows: i h bar dH / dq = [H, p] and i h bar dH / dp = [H, q]. These two equations are the Hamiltonian formulation of the subject of quantum dynamics and are new equations of quantum mechanics and quantum field theory. The older and well known Newtonian formulation of quantum dynamics is i h bar dp / dt = [p, H].

For a time independent wave function a particularly simple and useful result is obtained from the quantum Hamilton equations: partial (H psi) / partial q = 0, where H is the classical hamiltonian H = T + V where T is the kinetic energy and V is the potential energy. Knowing the hamiltonian gives the wavefunction psi for a given canonical variable such as x in the one dimensional Cartesian representation. Therefore it may be possible to find an exact wavefunction for the helium atom for example. This is possible neither from the Schroedinger equation nor from matrix mechanics. The emergence of a new fundamental equation of quantum mechanics after more than a hundred years of development of the subject is due to ECE theory and the systematic development of a unified field theory of physics. In this case the investigation proceeded from the tetrad postulate of Cartan=s geometry to the fermion equation, and thence to the new quantum Hamilton equations of immediate and widespread utility.

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