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The elementary static magnetic field of the photon

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It is argued that the photon generates on a fundamental level a static magnetic field, a flux density operator $\hat{B}_{\Pi} = B_0\hat{J}/\hbar$ where $\hat{J}$ is the photon's angular momentum operator and B_0 the scalar magnetic flux density amplitude of a beam of N photons. A key experimental test of the theory is proposed, whereby a circularly polarised laser beam is reflected at right angles from a beam of electrons, and the frequency shift Δf (hertz) $= (\hat{m} \cdot \hat{B}_{\Pi})/h$ measured in the reflected laser beam as a function of incident laser intensity I_0. Here $\hat{m}$ is the magnetic dipole moment operator of the electron.

1. Introduction

It is well known that the photon has an intrinsic, unremovable spin, which can be expressed as its quantised angular momentum operator $\hat{J}$ [1–4]. This is the essential explanation in quantum field theory for the existence of classical left and right circular polarisation in electromagnetic plane waves. In this paper it is argued that the photon also generates an intrinsic and unremovable *static magnetic field* (flux density in tesla) which can be described through the operator equation

$$\hat{B}_{\Pi} = B_0 \frac{\hat{J}}{\hbar} \tag{1}$$

where B_0 is the scalar magnetic flux density amplitude of a beam of N photons (for example a circularly polarised laser beam). The expectation value of the component of the operator $\hat{B}_{\Pi}$ in the propagation axis of the laser is $B_0 M_J$, where M_J is the azimuthal quantum number associated with the operator $\hat{J}$. Classically, this expectation value is $\pm \boldsymbol{B}_{\Pi} \cdot \boldsymbol{k}$, where $\boldsymbol{k}$ is a unit axial vector in the propagation axis Z of the laboratory frame of reference (X, Y, Z).

The derivation of the fundamental operator (1) is given in section 2, followed in section 3 by a suggestion for a key experiment to test the theory, an experiment which consists of reflecting at right angles a circularly polarised laser beam from a beam of electrons, and of measuring the frequency shift in the reflected laser due to the interaction

$$\Delta H_{\Pi} = -\hat{m} \cdot \hat{B}_{\Pi} \tag{2}$$

between $\hat{B}_{\Pi}$ and the electron's magnetic dipole moment operator $\hat{m}$. Section 3 develops some consequences in spectroscopy of the existence of $\hat{B}_{\Pi}$, specifically a quantum field theory of the optical Zeeman effect, splitting due to $\hat{B}_{\Pi}$ in spectra at visible frequencies, and optical NMR and ESR [5], in which $\hat{B}_{\Pi}$ shifts and splits conventional resonance features in liquids and condensed matter. Finally, a discussion is given of some other immediately interesting consequences of the existence of $\hat{B}_{\Pi}$, for example an optical Stern–Gerlach effect.

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2. Derivation of the operator equation for $\hat{B}_\Pi$

It is seen immediately that the operator equation (1) can be derived on the basis of symmetry and dimensions alone, and in this section the rigorous quantum field theoretical derivation is given using the recent results of Tanaś and Kielich [6] and of the present author [7–10]. Before embarking on this it is instructive to note the role of fundamental symmetries, namely the motion reversal operator $\hat{T}$, and the parity inversion operator $\hat{P}$. The operators $\hat{B}_\Pi$ and $\hat{J}$ have the same $\hat{P}$ and $\hat{T}$ symmetries, respectively positive and negative, so one is proportional to the other through a $\hat{T}$ and $\hat{P}$ positive scalar quantity. Furthermore, the unit of the operator $\hat{J}$ in quantum-mechanics is the reduced Planck constant $\hbar$, and therefore the scalar proportionality constant must be a scalar magnetic flux density amplitude, B_0, in tesla. In a laser beam of N photons the constant is the *laser's* scalar flux density amplitude in tesla. It is clear that when $N = 1$ (one photon), B_0 remains finite, and it follows that the single photon generates a quantum of magnetostatic flux density, described in quantum field theory by $\hat{B}_\Pi$ of eq. (1).

To derive this result rigorously it is convenient to consider first the classical equivalent of $\hat{B}_\Pi$, which is a vector quantity in the propagation axis of the laser:

$$\boldsymbol{B}_\Pi = B_0 \boldsymbol{k} \tag{3}$$

where $\boldsymbol{k}$ is a unit *axial* vector. The classical $\boldsymbol{B}_\Pi$ is proportional [7–10] to the well-known conjugate product [11–15]

$$\boldsymbol{\Pi}^{(\mathrm{A})} = \boldsymbol{E} \times \boldsymbol{E}^* , \tag{4}$$

a vector cross product of the *electric field strength* $\boldsymbol{E}$ of a circularly polarised laser with its complex conjugate $\boldsymbol{E}^*$:

$$\begin{aligned} \boldsymbol{E} &= E_0(\boldsymbol{i} + \mathrm{i}\boldsymbol{j}) \exp(\mathrm{i}\phi) , \\ \boldsymbol{E}^* &= E_0(\boldsymbol{i} - \mathrm{i}\boldsymbol{j}) \exp(-\mathrm{i}\phi) . \end{aligned} \tag{5}$$

Here, as usual, E_0 is the scalar electric field strength amplitude in volts per metre of the laser, $\boldsymbol{i}$ and $\boldsymbol{j}$ are unit polar vectors in X and Y of the laboratory frame (X, Y, Z), mutually orthogonal to the propagation axis Z, and ϕ is the phase. From eqs. (5),

$$\boldsymbol{\Pi}^{(\mathrm{A})} = -2E_0^2 \boldsymbol{k}\mathrm{i} \tag{6}$$

where $\boldsymbol{k}$ is a unit *axial* vector in Z. The product $\boldsymbol{\Pi}^{(\mathrm{A})}$ is an axial vector which is also negative to T and positive to P. Equation (6) can be rewritten using the fundamental in vacuo relation

$$E_0 = cB_0 \tag{7}$$

as

$$\boldsymbol{\Pi}^{(\mathrm{A})} = -2E_0 c(B_0 \boldsymbol{k})\mathrm{i} \equiv -2E_0 c \boldsymbol{B}_\Pi \mathrm{i} , \tag{8}$$

with the definition

$$\boldsymbol{B}_\Pi \equiv B_0 \boldsymbol{k} , \tag{9}$$

where c is the (scalar) speed of light.

Clearly, $\boldsymbol{\Pi}^{(\mathrm{A})}$ and $\boldsymbol{B}_\Pi$ must have the same $\hat{T}$ and $\hat{P}$ symmetries, and so does the unit axial vector $\boldsymbol{k}$. The classical quantity $\boldsymbol{B}_\Pi$ has been defined [7–10] as equivalent to the magnetostatic flux density in vacuo of a circularly polarised plane wave. It has no dependence on the phase ϕ of the wave, and therefore none on the angular frequency ω and propagation vector $\boldsymbol{\kappa}$. Using the relation between intensity (watts per square metre) and electric field strength

$$I_0 = \tfrac{1}{2}\varepsilon_0 c E_0^2 , \tag{10}$$

where ε_0 is the permittivity in vacuo [1]:

$$\varepsilon_0 = 8.854 \times 10^{-12}\ \mathrm{C^2 J^{-1} m^{-1}} , \tag{11}$$

we arrive at

$$\boldsymbol{B}_\Pi = \mathrm{i}\left(\frac{\varepsilon_0}{8I_0 c}\right)^{1/2} (\boldsymbol{E} \times \boldsymbol{E}^*) \tag{12}$$

which shows clearly that $\boldsymbol{B}_\Pi$ is directly proportional to the conjugate product $\boldsymbol{E} \times \boldsymbol{E}^*$, and that $\boldsymbol{B}_\Pi$ is a real quantity.

The classical field $\boldsymbol{B}_\Pi$ is proportional to the vector, or antisymmetric part of the intensity tensor $I_{ij} = \frac{1}{2}\varepsilon_0 c E_i E_j^*$ of the circularly polarised electromagnetic plane wave in free space, and vanishes if there is no degree of circular polarity. By expressing the antisymmetric part of the tensor $E_iE_j^*$ as the vector product $\boldsymbol{E}\times\boldsymbol{E}^*$ it becomes clear that $\boldsymbol{B}_\Pi$ is relative of the well known Poynting vector $\boldsymbol{N}=\boldsymbol{E}\times\boldsymbol{B}^*/\mu_0$, where μ_0 is the free-space magnetic permeability and $\boldsymbol{B}$ is the usual oscillating magnetic part [7–10] of the electromagnetic plane wave. Note that $\boldsymbol{B}_\Pi$ and $\boldsymbol{B}$ are fundamentally different quantities and should not be confused. The vector $\boldsymbol{B}$ is complex, vanishes when time-averaged, because it is phase dependent, and has no component in the propagation axis Z of the wave. The vector $\boldsymbol{B}_\Pi$ is real, and is directed exclusively in the Z axis. $\boldsymbol{B}$ is a plane wave solution of Maxwell's equations and $\boldsymbol{B}_\Pi$ is not a direct solution, being constructed from a cross product of direct plane wave solutions of Maxwell's equations.

Note also that the polar vector $\boldsymbol{N}$ is $\hat{P}$ and $\hat{T}$ negative, whereas the axial vector $\boldsymbol{B}_\Pi$ is $\hat{T}$ negative, $\hat{P}$ positive. The well-known $\boldsymbol{N}$ is interpreted physically as a flux of energy density, the novel $\boldsymbol{B}_\Pi$ as a flux of magnetic density. Furthermore, it is well known that $\boldsymbol{N}$ is proportional to the photon linear momentum and as we shall see, $\boldsymbol{B}_\Pi$ is proportional to the photon angular momentum. In the same way that the ratio of energy to linear momentum for an electromagnetic plane wave is the same as that for a particle (the photon) travelling at the speed of light, the ratio of magnetic flux density $\boldsymbol{B}_\Pi$ to angular momentum of the plane wave is the same as that for the photon travelling at c.

The conjugate product $\frac{1}{2}\varepsilon_0 c\boldsymbol{E}\times\boldsymbol{E}^*$ being the antisymmetric part of the intensity tensor $I_{ij}=\frac{1}{2}\varepsilon_0 cE_iE_j^*$ of circularly polarised light, is nonzero in free space, i.e. propagates in a vacuum in direct analogy with $\boldsymbol{N}$, which can be defined in terms of the scalar part of I_{ij} through the well known $\boldsymbol{N}=I_0\boldsymbol{n}$, where $\boldsymbol{n}$ is a unit propagation vector in free space, related to the wave vector $\boldsymbol{k}$, by $\boldsymbol{k}=\omega\boldsymbol{n}/c$. Therefore, although Maxwell's equations allow no direct phase-independent solutions in free space, vector products of allowed solutions, such as $\boldsymbol{N}$ and $\boldsymbol{B}_\Pi$, are physically meaningful phase-independent quantities whose time averages are non-zero. It follows that both $\boldsymbol{B}_\Pi$ and $\boldsymbol{E}\times\boldsymbol{E}^*$ can interact with matter to produce observable effects, in direct analogy with $\boldsymbol{N}$. The scalar magnitude of $\boldsymbol{N}$, i.e. the scalar intensity of light, I_0, is clearly a quantity which travels through a vacuum and interacts with matter to produce, for example, heat. Similarly, $\boldsymbol{E}\times\boldsymbol{E}^*$ travels through a vacuum and interacts with the antisymmetric part of the electric polarisability of an atom or molecule to produce an interaction energy which is measurable spectroscopically. The vector $\boldsymbol{B}_\Pi$ at the same time interacts with an electronic or nuclear magnetic dipole moment to produce a different interaction energy and a different spectroscopic signature. These effects have been demonstrated with the recently observed phenomenon of optical NMR spectroscopy (vide infra) in which circularly polarized light shifts NMR resonances at frequencies far from optical resonance. Note that $\boldsymbol{E}\times\boldsymbol{E}^*$ and $\boldsymbol{B}_\Pi$ both play a role in this phenomenon: the former is second order in $\boldsymbol{B}_\Pi$ and the latter is first order in $\boldsymbol{B}_\Pi$. This, together with other mechanisms, leads to a richly structured ONMR spectrum which is site specific, i.e. each resonance is shifted to a different extent by the circularly polarised light, leading to a useful new fingerprint of the sample. It is significant that no ONMR shifts are observed in linear polarisation, in which both $\boldsymbol{E}\times\boldsymbol{E}^*$ and $\boldsymbol{B}_\Pi$ vanish. However, it is clear that $\boldsymbol{B}_\Pi$ (and also $\boldsymbol{E}\times\boldsymbol{E}^*$) is a property of light, and is not interpretable as an ordinary uniform, magnetostatic field, such as that produced by an ordinary magnet. In this paper, it is shown that $\boldsymbol{B}_\Pi$ can be defined for the individual photon in terms of the angular momentum boson operator $\hat{J}$ of that photon, which is shown to be proportional to the photon's individual flux density quantum $\hat{B}_\Pi$, also a boson operator. Therefore $\hat{B}_\Pi$ is defined directly in terms of the photon's spin as it propagates linearly at the speed of light c. The classical vector $\boldsymbol{B}_\Pi$ is shown to be an expectation value over the operator $\hat{B}_\Pi$ in eigenstates $|\alpha\rangle$, coherent eigenstates of the quantised electromagnetic field. This conclusion is based directly on the work of Tanaś and Kielich [6], who have defined the four Stokes parameters of an elliptically polarised electromagnetic plane wave as expectation values over the four Stokes operators of the quantised field in eigenstates $|\alpha\rangle$.

It is well known, furthermore, that the classical third Stokes parameter is the real scalar

$$S_3 = -\mathrm{i}(E_X E_Y^* - E_Y E_X^*) \tag{13}$$

so that

$$\boldsymbol{B}_\Pi = -\left(\frac{\varepsilon_0}{8I_0C}\right)^{1/2} S_3 \boldsymbol{k} . \tag{14}$$

Equation (14) defines $\boldsymbol{B}_\Pi$ in eq. (8). Equation (10) defines $\boldsymbol{B}_\Pi$ in terms of S_3, and shows that the former is a real axial vector that changes sign with the laser circular polarization (left to right). The transition from classical to quantised field theory is made through the third Stokes operator, $\hat{S}_3$, recently introduced by Tanaś and Kielich [6]:

$$\hat{S}_3 \equiv -\left(\frac{2\pi\hbar\omega}{n^2(\omega)V}\right) \mathrm{i}(\hat{a}_X^+\hat{a}_Y - \hat{a}_Y^+\hat{a}_X) . \tag{15}$$

Here $n(\omega)$ is the refractive index, V the quantisation volume, and $\hat{a}^+$ and $\hat{a}$ denote respectively the creation and annihilation operator. Defining a coherent state of a laser beam of N photons by the Schrödinger equation [6],

$$\hat{a}|\alpha\rangle = \alpha|\alpha\rangle \tag{16}$$

provides the expectation value

$$\langle\alpha|\hat{S}_3|\alpha\rangle = |\alpha_+|^2 - |\alpha_-|^2 \tag{17}$$

with

$$\alpha_\pm = \frac{1}{\sqrt{2}} (\alpha_X \mp \mathrm{i}\alpha_Y) . \tag{18}$$

We define the operator $\hat{\boldsymbol{B}}_\Pi$ using eqs. (14) and (15):

$$\hat{\boldsymbol{B}}_\Pi \equiv \left(\frac{\varepsilon_0}{8I_0c}\right)^{1/2} \hat{S}_3 \tag{19}$$

and rewrite eq. (19) as

$$\hat{\boldsymbol{B}}_\Pi = \left(\frac{\varepsilon_0}{8I_0c}\right)^{1/2} \left(\frac{2\pi\omega}{n^2(\omega)V}\right)(\hbar(\hat{a}_+^+\hat{a}_+ - \hat{a}_-^+\hat{a}_-)) \tag{20}$$

with

$$\hat{a}_\pm = \frac{1}{\sqrt{2}} (\hat{a}_X \mp \mathrm{i}\hat{a}_Y) . \tag{21}$$

The quantity

$$\hbar(\hat{a}_+^+\hat{a}_+ - \hat{a}_-\hat{a}_-) \equiv \hbar(\hat{n}_+ - \hat{n}_-) \tag{22}$$

has the units of quantised angular momentum because $(\hat{n}_+ - \hat{n}_-)$ is dimensionless. Here

$$\hat{n}_+ \equiv \hat{a}_+^+\hat{a}_+ , \qquad \hat{n}_- \equiv \hat{a}_-^+\hat{a}_- \tag{23}$$

are the number of photons operators [6].

The total angular momentum of a beam of N photons propagating in Z is known independently [17] to be $NM_J\hbar$, where M_J is the azimuthal quantum number associated with the photon's angular momentum operator $\hat{J}$. Defining the angular momentum eigenfunction of a single photon by $|JM_J\rangle$ we arrive at the Schrödinger equation

$$\hbar(\hat{n}_+ - \hat{n}_-)|JM_J\rangle = \hbar M_J N|JM_J\rangle . \tag{24}$$

From eq. (20) in eq. (24),

$$\frac{n^2(\omega)V}{2\pi\omega} \hat{S}_3|JM_J\rangle = \hbar NM_J|JM_J\rangle \tag{25}$$

and with the identity

$$\hat{J} = \left(\frac{n^2(\omega)V}{2\pi\omega N}\right)\hat{S}_3 \tag{26}$$

eq. (25) becomes the standard Schrödinger equation describing the angular momentum of one photon:

$$\hat{J}|JM_J\rangle = \hbar M_J|JM_J\rangle . \tag{27}$$

From eq. (19) in eq. (26),

$$\hat{\boldsymbol{B}}_\Pi = \left(\frac{\varepsilon_0}{8I_0c}\right)^{1/2} \left(\frac{2\pi\omega N}{n^2(\omega)V}\right)\hat{J} \equiv \zeta\hat{J} , \tag{28}$$

showing that $\hat{B}_{\Pi}$ is directly proportional to $\hat{J}$. Considerable insight into the nature of the constant ζ is obtained with the results of Tanaś and Kielich [6]:

$$S_0 = \left(\frac{2\pi\omega\hbar}{n^2(\omega)V}\right)\langle\alpha|\hat{S}_0|\alpha\rangle\,, \tag{29}$$

$$N = \langle\alpha|\hat{S}_0|\alpha\rangle\,, \tag{30}$$

for the zeroth Stokes operator $\hat{S}_0$ and its classical equivalent, the Stokes parameter S_0. Furthermore we make use of the classical result [18]

$$S_0 = 2E_0^2 \tag{31}$$

which follows from our definitions (5) of E and E^*. From eqs. (28) to (31),

$$E_0^2 = \frac{\pi\omega\hbar N}{n^2(\omega)V} \tag{32}$$

Using eqs. (10) and (32) in eq. (28) gives, finally, the fundamental and simple operator equation we seek to prove

$$\hat{B}_{\Pi} = B_0\left(\frac{\hat{J}}{\hbar}\right). \tag{33}$$

3. A key experiment for $\hat{B}_{\Pi}$

The theoretical existence of $\hat{B}_{\Pi}$ implies many different things experimentally, because a circularly polarised laser acts as a simple magnet and delivers equivalent (or 'latent' or 'potential') static magnetic flux density through a vacuum, a flux density which is able to form a scalar interaction hamiltonian with a dipole operator $\hat{m}$:

$$\Delta H = -\hat{m}\cdot\hat{B}_{\Pi}\,.$$

The electron carries an elementary $\hat{m}$:

$$\hat{m} = g_e\gamma_e\hat{I} \tag{34}$$

where g_e is the electron's g factor [1] (2.002) and $\hat{I}$ is the electron's angular momentum operator. The key experiment devised in this section isolates the effect of the hamiltonian in eq. (2) on a circularly polarised visible frequency laser reflected at right angles from an electron beam. It is shown that there is a frequency shift

$$\Delta f = \frac{\langle\hat{m}\cdot\hat{B}_{\Pi}\rangle}{h} \tag{35}$$

in the reflected beam, which provides a method of measuring $\hat{B}_{\Pi}$ spectrally with a high resolution spectrometer. The derivation of eq. (35) is based on conservation of momentum and kinetic energy when a circularly polarised laser beam of N photons is reflected at right angles from the electron beam. Consider the collision of one photon of the beam with one electron, whereby the former is reflected by an angle θ and the latter by an angle θ'. Initially, the electron is at rest with relativistic energy $m_e c^2$, where m_e is its mass [1]. After the collision, the electron's linear momentum magnitude is p and its translational kinetic energy is $(p^2c^2 + m_e^2c^4)^{1/2}$. The initial linear momentum of the photon is h/λ_i where λ_i is its wavelength, and its initial energy is hc/λ_i. The photon strikes the electron, considered stationary [1], is deflected through an angle θ and emerges with linear momentum h/λ_f and translational kinetic energy hc/λ_f. The electron after collision moves off at an angle θ' to the incident photon's trajectory. Conserving linear momentum and translational kinetic energy gives the following three equations:

$$p\cos\theta' + \frac{h}{\lambda_f}\cos\theta = \frac{h}{\lambda_i}\,, \tag{36}$$

$$p\sin\theta' + \frac{h}{\lambda_f}\sin\theta = 0\,, \tag{37}$$

$$m_e c^2 + \frac{hc}{\lambda_i} = (p^2c^2 + m_e^2c^4)^{1/2} + \frac{hc}{\lambda_f}\,, \tag{38}$$

which can be solved simultaneously to give the standard Compton equation for the wavelength shift

$$(\lambda_f - \lambda_i) = \frac{2h}{m_e c}\sin^2\frac{\theta}{2} + \frac{h}{2m_e c}\,\frac{\lambda_f}{\lambda_i}\cos^2\theta\,. \tag{39}$$

At $\theta = 90°$,

$$(\lambda_f - \lambda_i) = \frac{h}{m_e c}\,, \tag{40}$$

a result which has no classical counterpart [1]. The wavelength shift is in the X-ray region of the spectrum.

The theory of the Compton effect, embodied in eqs. (36) to (38) takes no account of the interaction energy

$$\Delta E_{\Pi} = -\langle \hat{m} \cdot \hat{B}_{\Pi} \rangle\,. \tag{41}$$

As a consequence, eq. (38) for the conservation of kinetic energy must be modified to

$$m_e c^2 + \frac{hc}{\lambda_i} = (p^2c^2 + m_e^2 c^4)^{1/2} + \frac{hc}{\lambda_f} + \Delta E_{\Pi}\,, \tag{42}$$

i.e. ΔE_{Π} contributes to the total energy after collision. Also the theory embodied in eqs. (36) to (38) takes no account of the intrinsic angular momenta of either photon or electron, and clearly, there must also be conservation of rotational kinetic energy and angular momentum. However, eqs. (36), (37) and (42) suffice to solve for $\lambda_f - \lambda_i$ in the presence of ΔE_{Π}. For an electromagnetic beam of N photons (a circularly polarised laser) reflected at 90° off the electron beam, solving eqs. (36), (37) and (42) gives

$$(\lambda_f - \lambda_i) = \frac{\left[\dfrac{h}{m_e c} + \lambda_i \lambda_f \dfrac{\Delta E_{\Pi}}{hc}\left(1 - \dfrac{\Delta E_{\Pi}}{2m_e c^2}\right)\right]}{(1 - \Delta E_{\Pi}/m_e c^2)} \tag{43}$$

We consider the order of magnitude of ΔE_{Π} compared with $m_e c^2$. The magnitude of the observable associated with the operator $\hat{m}$ is the Bohr magneton multiplied by the electron g factor (2.002), and is about 10^{-23} joules per tesla. The magnitude of the observable with the elementary photon operator $\hat{B}_{\Pi}$ [7] is

$$|\boldsymbol{B}_{\Pi}| \sim 10^{-7}\, I_0^{1/2} \tag{44}$$

and for an I_0 of 1.0 watt per square centimetre (10 000 watts per square metre), it is of the order 10^{-5} tesla. Therefore ΔE_{Π} is of the order 10^{-28} joules for this intensity. However, $m_e c^2$ is of the order 10^{-13} joules per electron, so that to an excellent approximation

$$\frac{\Delta E_{\Pi}}{m_e c^2} \ll 1\,, \tag{45}$$

and eq. (43) reduces to

$$\lambda_f - \lambda_i = \frac{h}{m_e c} + \lambda_i \lambda_f \frac{\Delta E_{\Pi}}{hc}\,. \tag{46}$$

which is consistent with eq. (40) for $\Delta E_{\Pi} = 0$. It is useful to express eq. (46) in terms of wavenumbers:

$$\bar{\nu}_f = \frac{m_e c \bar{\nu}_i}{m_e c + h\bar{\nu}_i} - \frac{m_e \Delta E_{\Pi}}{m_e hc + h^2 \bar{\nu}_i}\,. \tag{47}$$

We now compare the order of magnitude of $m_e hc$ (about 10^{-55} kg m J m) with $h^2\bar{\nu}_i$, which is about $4.4 \times 10^{-67}\bar{\nu}_i$ kg J m, and find that for $\bar{\nu}_i \leqslant 10^9\ \mathrm{m}^{-1}$ ($10^7\ \mathrm{cm}^{-1}$),

$$\bar{\nu}_f - \bar{\nu}_i = -\frac{\Delta E_{\Pi}}{hc} \tag{48}$$

to a very good approximation. In terms of frequency in hertz

$$\Delta f = -\frac{\Delta E_{\Pi}}{h} = \frac{\langle \hat{m} \cdot \hat{B}_{\Pi} \rangle}{h} \tag{49}$$

which is eq. (35).

This equation shows that at electromagnetic frequencies well below $10^7\ \mathrm{cm}^{-1}$ (wavenumbers), for example at visible frequencies, the change in frequency in hertz in a circularly polarised laser reflected at right angles off an electron beam is given by equation (49). This is based on the interaction energy $\langle \hat{m} \cdot \hat{B}_{\Pi} \rangle$ between $\hat{m}$ of the electron and $\hat{B}_{\Pi}$ of the photon, two elementary properties of quantised matter. In general, the frequency shift is proportional to the square root of the incident laser intensity I_0. The interaction energy is quantised according to the well-known quantum theory [19–21] of operator products, and can be expressed as the expectation value

$$\Delta E_{\Pi} = -\langle IJFM_F|\hat{m}\cdot\hat{B}_{\Pi}|I'J'F'M_F'\rangle\ ,$$
$$\hat{m} = -2.002\gamma_e\hat{I} \equiv -g_e\gamma_e\hat{I}\ , \tag{50}$$

where I is the angular momentum quantum number of the electron ($I=\frac{1}{2}$) and J is the angular momentum quantum number of the photon (a positive integral quantity greater than zero [1]). Here γ_e is the gyromagnetic ratio and g_e the electronic g factor [1]. The quantum number F is given by the Clebsch–Gordan series

$$F = J + I, \ldots, |J-1| \tag{51}$$

and the expectation value of the Z component of the resultant angular momentum operator $\hat{F}$ is given by $M_F\hbar$ with the selection rule

$$\Delta M_F = \pm 1 \tag{52}$$

with M_F having $(2F+1)$ values from $-F$ to F as usual. Therefore, depending on the value of J, there are several different possible values of the frequency shift Δf, i.e. an analysis of the reflected laser beam will reveal their presence as a spectrum. The diagonal matrix elements of eqs. (50) can be worked out analytically [19–21], giving the frequency shift

$$\Delta f = -\frac{10^{-7}I_0^{1/2}g_e g\gamma_e}{(2\pi)}\ ,$$
$$g = [3(2F+1)I(I+1)(2I+1)J(J+1) \times (2J+1)]^{1/2}\begin{bmatrix} I & I & 1 \\ J & J & 1 \\ F & F & 0 \end{bmatrix}. \tag{53}$$

Here the quantity in braces is the well known 9-j symbol. For $I=\frac{1}{2}$, $J=1$ and $F=\frac{3}{2}$, this is -0.05 and for an incident circularly polarised laser intensity of 1.0 watt per square centimeter the frequency shift in hertz from eq. (19) is $\sim 20\,000 M_F$ Hz. For $I=\frac{1}{2}$, $J=1$, $F=\frac{1}{2}$, the shift is $\sim 10\,000 M_F$ Hz.

Therefore, for modest incident laser intensity the shift is already in the kilohertz range, easily measurable, and has the following characteristics.

(1) It should change sign with respect to the incident laser frequency if the incident laser circular polarity is switched from left to right (i.e. if the azimuthal expectation value of the photon static magnetic flux density operator is changed from $B_0|M_J|$ to $-B_0|M_J|$).

(2) There should be no frequency shift or spectral detail if the incident laser is linearly polarised because the net static flux density delivered by the photon beam is zero, being fifty percent $B_0|M_J|$ and fifty percent $-B_0|M_J|$.

(3) The shifts with respect to the incident laser frequency should be proportional to the square root of the laser incident intensity I_0.

These features should provide adequately for the measurement of the novel elementary property $\hat{B}_{\Pi}$, and in principle the method can be extended to other elementary particles by using for example a neutron beam in place of the laser beam. This would allow an experimental determination of whether the neutrons also have an elementary magnetic flux density similar to $\hat{B}_{\Pi}$ of the photon (the elementary 'magneton' of electromagnetic radiation).

4. Discussion

In addition to the frequency-shift phenomenon introduced in this paper, it is possible to predict novel phenomena due to $\hat{B}_{\Pi}$ wherever the photon interacts with matter, one of these being the optical Zeeman effect, another the optical Faraday effect. Both of these have been suggested in a semi-classical context recently [22–26], and their existence is reinforced by that of $\hat{B}_{\Pi}$ on a fundamental level. In the optical Zeeman effect the magneton $\hat{B}_{\Pi}$ plays the role taken by a static magnetic flux density in the conventional Zeeman effect, and its relatives, the anomalous Zeeman effect and Paschen–Back effect. In the optical Faraday effect the magneton rotates the plane of polarisation of a linearly polarised probe. Clearly, the interaction energy in these magneton-based effects must be constructed from the quantum theory of operator products, as in eqs. (53). Off-diagonal components of the matrix elements so obtained are important in general, as well as diagonal elements. In molecules, these off-diagonal elements must probably be worked out numerically, but for this purpose there are many standard ab initio packages available.

Each individual photon of a beam of N photons generates $\hat{B}_\Pi$, and to obtain the expectation value of $\hat{B}_\Pi$ for the beam, i.e. to obtain the classical B_Π, we use the relation (19) between $\hat{B}_\Pi$ and the third Stokes operator $\hat{S}_3$ [6]. Since $\hat{B}_\Pi$ is directly proportional to $\hat{S}_3$, it is possible to discuss the properties of the former in terms of the commutator and uncertainty properties of the four Stokes operators for a beam of N photons [6]:

$$[\hat{S}_1, \hat{S}_2] = 2i\hat{S}_3 \tag{54}$$

in cyclic permutation,

$$[\hat{S}_i, \hat{S}_0] = 0\,, \quad i = 1, 2, 3\,, \tag{55}$$

and

$$(\langle(\Delta\hat{S}_1)^2\rangle\langle(\Delta\hat{S}_2)^2\rangle)^{1/2} \geq |\langle\hat{S}_3\rangle|\,, \tag{56}$$

which shows that the Stokes operators are described by the algebra of angular momentum operators in quantum mechanics. Tanaś and Kielich [6] have shown, furthermore, that the classical Stokes parameters [21] are well-defined expectation values of the Stokes operators in the eigenstates $|\alpha\rangle$, coherent states of a beam of N photons. In other words, it is possible to define the macroscopic S_3 from $\hat{S}_3$ for N photons:

$$S_3 = \frac{S_0}{N}\langle\alpha|\hat{S}_3|\alpha\rangle = \frac{S_0}{N}(|\alpha_+|^2 - |\alpha_-|^2) \tag{57}$$

(cf. eq. (17)). It follows that

$$B_\Pi = \langle\alpha|\hat{B}_\Pi|\alpha\rangle = \frac{B_0}{\hbar}\langle\alpha|\hat{J}|\alpha\rangle\,. \tag{58}$$

This equation shows clearly that the classical vector B_Π is an expectation value of the angular momentum operator $\tilde{J}$ in the eigenstate $|\alpha\rangle$. The latter is defined [6,15] by:

$$|\alpha\rangle = \exp\left(-\frac{|\alpha|^2}{2}\right)\sum_{n=0}\frac{\alpha^n}{\sqrt{n!}}|n\rangle \tag{59}$$

where the mean number of photons is $N = |\alpha|^2$. If $N = 1$, $|\alpha|^2 = 1$ and

$$|1\rangle = e^{-1/2}\sum_{n=0}\frac{|n\rangle}{\sqrt{n!}}\,. \tag{60}$$

For a circularly polarised beam consisting of one photon, therefore,

$$B_\Pi = \frac{B_0}{\hbar}\langle 1|\hat{J}|1\rangle \equiv B_0\frac{\langle\hat{J}\rangle}{\hbar}\,, \tag{61}$$

where $\hat{J}$ is the angular momentum boson operator of one photon. Therefore it is possible to define the expectation value of $\hat{B}_\Pi$ for a beam of N photons in terms of the angular momentum operator $\hat{J}$ of one photon. In precise analogy, it is possible to define the Stokes parameters in terms of the Stokes operators for one photon. This is achieved through the eigenstate $|\alpha\rangle$ which is well defined in the limit $N = 1$. If the beam consists of more than one photon, eq. (59) is used to define the coherent eigenstate $|\alpha\rangle$. Note that for a completely circularly polarised beam, S_1 and S_2 vanish, i.e. the expectation values $\langle\alpha|\hat{S}_1|\alpha\rangle$ and $\langle\alpha|\hat{S}_2|\alpha\rangle$ vanish. In a completely linearly polarised beam,

$$S_3 = \frac{S_0}{N}\langle\alpha|\hat{S}_3|\alpha\rangle = 0\,, \tag{62}$$

and in general all four Stokes parameters are nonzero only in elliptical polarisation [6].

The magnetostatic flux quantum $\hat{B}_\Pi$ is capable also of generating optically induced resonance spectra (optical NMR and ESR), and evidence for this has been obtained recently [27], although a full theoretical description is not yet available. They must probably be generated ab initio, using software packages such as HONDO or GAUSSIAN 90 by taking into account the interaction of the magneton $\hat{B}_\Pi$ with the large and complicated chiral test-molecule used by Warren et al.

[27] to show interesting, site-specific effects of a low-power circularly polarised laser on a conventional one- and two-dimensional NMR spectrum. This technique appears to have considerable promise, especially if the laser intensity could be increased by pulsing. In principle, considerable increase in resolution of conventional resonance spectra is obtainable [7–10,27].

Finally, the optical equivalent of the Stern–Gerlach experiment is possible in principle by using an expanding or focussed laser beam to generate the optical equivalent of a magnetic field *gradient* in the axis of propagation of a circularly polarised laser beam coaxial with a beam of atoms, such as silver atoms. The magneton theory of this will be the subject of future work.

5. Conclusions

This paper has proposed the existence of the elementary magneton $\hat{B}_{\Pi}$, a fundamental property of the photon, and has suggested a key experiment to test the hypothesis.

Appendix

The neutron has a magnetic moment and quantum number $I = \frac{1}{2}$ but is approximately 10 000 times heavier than the electron. The theory of spectral detail in a circularly polarised laser beam reflected off a beam of neutrons can be set up in the same way as for an electron beam, but the expected splitting in the reflected laser beam is much smaller and much more difficult to detect with a spectrometer. However, the presence of such detail would be further evidence for the existence of the magneton $\hat{B}_{\Pi}$. It is also possible to replace the electron beam by a beam of atoms with net electronic dipole moment, for example, and in general it is clear that for any material with net electronic or nuclear dipole moment the interaction with the magneton produces spectral detail in the reflected laser beam, which can be analysed spectroscopically. Furthermore, the experiment is not confined to beams, but can also proceed in principle by reflecting the circularly polarised laser beam from a material of interest with a net magnetic dipole moment. In this context, the behaviour of superconducting surfaces is particularly interesting [28] and the magneton $\hat{B}_{\Pi}$ could well provide an entirely novel way of analysing type I and II superconductors by reflecting a laser beam from the surface of the material at right angles, and looking for the specific magnetic effects due to $\hat{B}_{\Pi}$. Type II superconductors are of particular interest [28] because they remain superconducting in the presence of magnetic flux density, which is known to propagate in such material in the form of quantised flux lines, each carrying one quantum of magnetic flux [28]. In this case it might be expected that the magneton $\hat{B}_{\Pi}$ would be converted in the type II superconductor to the quantum of magnetic flux hc/m_e. Furthermore, Bitter imaging techniques [28] can be utilised in principle in superconductors to detect the presence of magnetisation due to the magneton $\hat{B}_{\Pi}$ of the circularly polarised laser, which can also be used to scan the surface of the sample and induce individual vortices of magnetisation in the superconducting sample.

More generally and fundamentally it is interesting to speculate on the possibility that elementary particles with spin are also capable of generating magnetons of flux akin to $\hat{B}_{\Pi}$ of the photon. The latter is massless and travels at c in vacuo, whereas the neutron for example has mass and does not travel at c. The electron also has mass and does not travel at c. Nevertheless, the electron and neutron both have intrinsic, irremovable spin, essentially in the same way as the photon, and in terms of symmetry and dimensionality, electron and neutron both can generate, in principle, magnetons of flux through equations identical in structure to eq. (1) of the text. However, neither electrons nor neutrons are electromagnetic plane waves, but different types of wave, and the question comes down to whether a beam of electrons or neutrons carries a finite, scalar flux density amplitude akin to $\hat{B}_{\Pi}$ of the photon. It is known that the neutron for example obeys the Planck relation between energy and frequency, but there appears to be no evidence that the Maxwell equations can be written for neutrons or electrons and solved to generate plane waves akin to electromagnetic waves. It appears clearly at present that electrons and neutrons generate elementary magnetic *dipole moments* and that photons generate the elementary *magnetic field* $\hat{B}_{\Pi}$.

These speculations can be extended to other elementary particles with intrinsic spin (i.e. angular momentum operators) and experiments can be devised to test the speculations. For example, if the electron does indeed generate its own magneton, a quantised magnetic flux density operator $\hat{B}_e$, a beam of electrons reflected off a beam of neutrons will generate the interaction hamiltonian

$$\Delta H_1 = -\hat{m}_n \cdot \hat{B}_e \tag{A.1}$$

where $\hat{m}_n$ is the magnetic dipole moment of the neutron. This is quantised as in eqs. (53) of the text, and consequently the energy of the emerging electron beam must record in some way the presence of ΔH_1. If the electron beam has wave properties, it should be analysable spectrally, and the spectral pattern due to the interaction ΔH_1 should be measurable experimentally. Electron diffraction forms an evidence that electrons can behave as waves as well as particles, and this is a result of the de Broglie principle, as is well known. Proceeding with the speculative logic in this way it becomes clear that reflecting a beam of ANY particles with intrinsic elementary spin from any other particle beam with intrinsic elementary magnetic dipole moment could, in principle, result in an interaction energy of type (A.1). In other words, we speculate on the possibility that elementary particles in general can each generate its own magneton.

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