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THE PHOTOMAGNETON $\hat{B}^{(3)}$ AND ELECTRODYNAMIC CONSERVATION LAWS

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It is shown that the basic electrodynamical conservation laws are unaffected by the presence in free space of the photomagneton of light, $\hat{B}^{(3)} = B^{(0)}\hat{J}/\hbar$, the fundamental photon property responsible for magnetization by light. The expectation value $\boldsymbol{B}^{(3)} = \langle \hat{B}^{(3)} \rangle$ does not affect the Poynting vector, so that it does not contribute to electromagnetic flux density. The electromagnetic energy density can be expressed in terms of $\boldsymbol{B}^{(3)}$ through the equation

$$\hbar\omega = \frac{1}{\mu_0}\int \boldsymbol{B}^{(3)} \cdot \boldsymbol{B}^{(3)*} dV.$$

When light magnetizes matter, the unit $\boldsymbol{B}^{(3)}$ of magnetic flux density per photon is transferred from light to matter. This is equivalent to an elastic transfer of angular momentum. Experimental indications for the existence of $\boldsymbol{B}^{(3)}$ are discussed.

Key words: phtomagneton, electrodynamics, conservation laws.

1. INTRODUCTION

It is well known that the electrodynamic conservation laws are fundamental to any consideration of light in free space or the interaction of light with matter. The momentum and energy of a radiation pulse totally contained within a finite volume V has the same Lorentz transformation properties [1–3] as a material point particle, and so the laws of conservation of energy and momentum must be similar to those for a particle, the photon. In free space, the photon, if considered to be a particle without mass, travels at the speed of light, c, and the conservation laws can be expressed in classical relativistic field theory as

$$\frac{\partial T^{ij}}{\partial x^i} = 0, \tag{1}$$

where T^{ij} is the energy momentum four-tensor and x^i the metric. This implies that the quantity [1]

$$G^i G_i = W^2 - c^2 G^2 = 0 \tag{2}$$

must be a Lorentz invariant. Here W denotes classical electromagnetic energy density and G classical electromagnetic linear momentum density. In the quantum field we have

$$\hbar\omega = \int W dV, \qquad \frac{\hbar\omega}{c} = \int G dV, \tag{3}$$

so that $\hbar\omega$ and $\hbar\omega/c$ are the energy and linear momentum of one photon in free space.

Additionally, it is well known [2] that in classical relativistic field theory the following two Lorentz invariants vanish in free space:

$$F^{kl}F_{kl} = 2(c^2B^2 - E^2) = 0, \qquad F^{kl}G_{kl} = 0, \tag{4}$$

where F_{kl} is the four-curl of the potential four-vector A_k in free space, and where G_{kl} is its dual tensor (i.e., G_{kl} is F_{kl} with $\boldsymbol{E}$ and $c\boldsymbol{B}$ interchanged in S.I. units).

The purpose of this Letter is to show that these relations remain valid in the presence of the magnetic field $\boldsymbol{B}^{(3)}$ [4–6], the expectation value of the novel photomagneton $\hat{B}^{(3)}$ of light, the basic photon property responsible for the magnetization of matter by circularly polarized electromagnetic radiation. In Sec. 2 it is shown that the invariants (2) and (4) remain zero in free space in the presence of $\boldsymbol{B}^{(3)}$ and its dual $-i\boldsymbol{E}^{(3)}/c$, and expressions are given for the light quantum of energy $\hbar\omega$ in terms of $\boldsymbol{B}^{(3)}$, which therefore conforms in every respect with the conservation laws of electrodynamics. In Sec. 3 it is shown that the Planck radiation law, and its classical limits, are unaffected by the presence of $\boldsymbol{B}^{(3)}$, which magnetizes matter in an elastic transfer of photon angular momentum from electromagnetic radiation.

2. THE FIELD $\boldsymbol{B}^{(3)}$ AND FUNDAMENTAL LORENTZ INVARIANTS IN FREE SPACE

It has been shown [4–6] recently that the photomagneton $\hat{B}^{(3)}$ is defined by

$$\hat{B}^{(3)} = B^{(0)}\frac{\hat{J}}{\hbar}, \tag{5}$$

where $\hat{J}$ is the angular momentum operator of one photon, whose expectation values are $\pm\hbar$ if the photon is considered to be without mass [7]. Here $B^{(0)}$ is the scalar amplitude of magnetic flux density in vacuo. Therefore $\hat{B}^{(3)}$ is generated from the angular momentum of the photon. In the classical theory of fields, the expectation value of $\hat{B}^{(3)}$, the axial vector $\boldsymbol{B}^{(3)}$, is defined through the cyclically symmetric [4], field algebra in the circular basis (1), (2), (3):

$$\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)} = iB^{(0)}\boldsymbol{B}^{(3)*}, \tag{6}$$

where $\boldsymbol{B}^{(1)}$ is a plane wave in vacuo and where $\boldsymbol{B}^{(2)}$ is its complex conjugate,

$$\boldsymbol{B}^{(1)} = \boldsymbol{B}^{(2)*} = \frac{B^{(0)}}{\sqrt{2}}(i\boldsymbol{i} + \boldsymbol{j})\,e^{i\phi}. \tag{7}$$

Thus, $\boldsymbol{B}^{(3)}$ is free of phase, ϕ and for this reason is not a wave. It propagates in free space, however, with photon spin. The field $\boldsymbol{B}^{(3)}$ is responsible for the phase free magnetization of matter by circularly polarized light; for example, in the inverse Faraday effect [8–10] this magnetization can be expressed to second order as [4–6]

$$\boldsymbol{M} = AB^{(0)}\boldsymbol{B}^{(3)}, \tag{8}$$

where A is an ensemble averaged molecular property tensor. *Therefore the existence of* $\boldsymbol{B}^{(3)}$ *is consistent with experimental data from the inverse Faraday effect.*

Basic symmetry considerations [4–6] show that there can be no real $\boldsymbol{E}^{(3)}$ in free space, but the duality transformation of special relativity [1–3] implies that $\boldsymbol{B}^{(3)}$ must be dual to a pure imaginary $-i\boldsymbol{E}^{(3)}/c$ which, however, has no physical significance as a field. This result is consistent with the fact that, in a circular basis,

$$\boldsymbol{e}^{(1)} = \frac{1}{\sqrt{2}}(\boldsymbol{i} - i\boldsymbol{j}) = \boldsymbol{e}^{(2)*}, \qquad \boldsymbol{e}^{(3)} = \boldsymbol{k}, \tag{9}$$

the following are Lorentz invariants in free space:

$$\begin{aligned} F^{kl}F^{*}_{kl} &= 2(c^2\boldsymbol{B}^{(i)}\cdot\boldsymbol{B}^{(i)*} - \boldsymbol{E}^{(i)}\cdot\boldsymbol{E}^{(i)*}) = 0, \\ F^{kl}G^{*}_{kl} &= 0; \qquad i = 1, 2, 3, \end{aligned} \tag{10}$$

as required from fundamental considerations [1–3]. Also, since $\boldsymbol{B}^{(3)}$ is generated directly from photon spin, it cannot affect the linear momentum of the photon, i.e., in classical terms it cannot affect the Poynting vector. This is easily verified from results such as

$$\begin{aligned} (\boldsymbol{E}^{(1)} - i\boldsymbol{E}^{(3)}) \times (\boldsymbol{B}^{(1)} + \boldsymbol{B}^{(3)}) &= \boldsymbol{E}^{(1)} \times \boldsymbol{B}^{(1)}, \\ (\boldsymbol{E}^{(2)} + i\boldsymbol{E}^{(3)}) \times (\boldsymbol{B}^{(2)} + \boldsymbol{B}^{(3)}) &= \boldsymbol{E}^{(2)} \times \boldsymbol{B}^{(2)}, \end{aligned} \tag{11}$$

with $\boldsymbol{E}^{(1)} \times \boldsymbol{B}^{(3)} = i\boldsymbol{E}^{(3)} \times \boldsymbol{B}^{(1)}$, $i\boldsymbol{E}^{(3)} \times \boldsymbol{B}^{(2)} = -\boldsymbol{E}^{(2)} \times \boldsymbol{B}^{(3)}$. This means that $\boldsymbol{B}^{(3)}$ has no extra effect on light intensity, the time averaged value of the Poynting vector, and therefore does not affect the Planck law and its classical limits. If it adds nothing to photon linear momentum, it can add nothing to photon energy, which is c times photon linear momentum in free space. In classical terms, electromagnetic energy density can be expressed equally well in terms of $\boldsymbol{B}^{(1)}$, $\boldsymbol{B}^{(2)}$ or $\boldsymbol{B}^{(3)}$:

$$\hbar\omega = \frac{1}{\mu_0}\int \boldsymbol{B}^{(i)}\cdot\boldsymbol{B}^{(i)*}dV; \qquad i = 1, 2, 3, \tag{12}$$

where μ_0 is the permeability of free space and V is a finite volume occupied by electromagnetism. Using $E^{(0)} = cB^{(0)}$, the above relations can be re-expressed as

$$\hbar\omega = \epsilon_0 \int \boldsymbol{E}^{(i)} \cdot \boldsymbol{E}^{(i)*} dV, \qquad i = 1, 2, 3, \tag{13}$$

in which $\boldsymbol{E}^{(3)} \cdot \boldsymbol{E}^{(3)*}$ is the square of the (real) magnitude of the imaginary $-i\boldsymbol{E}^{(3)}$. (In the analytical theory of complex numbers, the magnitude of a complex quantity z is defined as a pure real quantity, the square root of z multiplied by its modulus.)

3. EFFECT ON THE PLANCK LAW AND SPECTRAL ABSORPTION LAW OF $\boldsymbol{B}^{(3)}$

The total electromagnetic energy per unit time per unit area, or energy flux density at temperature T,

$$I(T) = \int_0^\infty I_\nu(T)\, d\nu, \tag{14}$$

is not augmented by the presence of $\boldsymbol{B}^{(3)}$ in free space, because I is the light intensity in watts per unit area, and is the time average of the Poynting vector, which is in turn directly proportional to the linear momentum density of light. This is consistent with the fact that $\boldsymbol{B}^{(3)}$ is generated by the spin *angular* momentum of one photon, whose magnitude is $\hbar$ and therefore independent of frequency ν. The total electromagnetic energy density,

$$U = \frac{4}{c} I, \tag{15}$$

is therefore also unchanged by $\boldsymbol{B}^{(3)}$. These results are expressed through Eqs. (12) and (13) of Sec. 2.

The essence of Planck's hypothesis is that the total electromagnetic energy at a particular frequency is the sum of identical energy elements, or light quanta, $\hbar\omega$. Clearly, the light quantum can be expressed through each of $\boldsymbol{B}^{(1)}$, $\boldsymbol{B}^{(2)}$, or $\boldsymbol{B}^{(3)}$ and is the same for each. The fundamental photon energy is therefore defined through the *square* of $\boldsymbol{B}^{(3)}$:

$$\hbar\omega = \frac{1}{\mu_0} \int \boldsymbol{B}^{(3)} \cdot \boldsymbol{B}^{(3)*} dV. \tag{16}$$

Unlike the waves $\boldsymbol{B}^{(1)}$ and $\boldsymbol{B}^{(2)}$, the spin field $\boldsymbol{B}^{(3)}$ has no specific dependence on frequency outside of the amplitude $B^{(0)}$, which is the same in $\boldsymbol{B}^{(1)}$, $\boldsymbol{B}^{(2)}$ and $\boldsymbol{B}^{(3)}$ and which is defined by

$$B^{(0)} = \left(\frac{\mu_0 \hbar \omega}{V} \right)^{\frac{1}{2}}. \tag{17}$$

Similarly, the photon angular momentum is an intrinsic property of the photon whose magnitude is always $\hbar$, whatever the frequency of the electromagnetic wave. In other words $\boldsymbol{B}^{(3)}$ is not a wave field, and *has no phase dependence*. A spectral absorption process in which a light quantum $\hbar\omega$ is transferred from radiation to atom can be described through Eq. (16); but, since $\boldsymbol{B}^{(3)}$ is not a wave, it has no specific frequency, and therefore spectral absorption must be described through the *phase-dependent electromagnetic waves* $\boldsymbol{B}^{(1)}$ *and* $\boldsymbol{B}^{(2)}$, which are transverse as usual. In other words a spectral absorption is a resonant process which must involve waves, and only the total (or integrated) electromagnetic density can be described through $\boldsymbol{B}^{(3)}$ using Eq. (16). The only way to detect the *specific* effect of $\boldsymbol{B}^{(3)}$ is to measure experimentally its phase-free magnetization of matter, and this is discussed further as follows.

4. DISCUSSION

Equation (8) expresses the fact that the process of magnetization of matter by light (the inverse Faraday effect) can be described directly in terms of $\boldsymbol{B}^{(3)}$, which is therefore a real and physical magnetic flux density, generated by the intrinsic spin of the photon. Equation (8) therefore represents a transfer of photon angular momentum from field to matter. It is important to realize, however, that *resonant optical absorption is not necessary for this transfer to take place, and that the inverse Faraday effect is observed experimentally [8–10] in the absence of resonant absorption, i.e., at frequencies far from optical resonance.* This is an experimental manifestation of the fact that the field $\hat{B}^{(3)}$ is phase free, i.e., is an intrinsic photon property whose eigenvalues are the eigenvalues of the photon angular momentum multiplied by the scalar flux density magnitude $B^{(0)}$ in free space. The difference between a photon absorption at a definite optical resonance frequency and magnetization by $\hat{B}^{(3)}$ is that absorption requires a definite phase dependence, i.e., an electromagnetic wave of a definite frequency (e.g., visible), while magnetization occurs through the ghost or spin field $\hat{B}^{(3)}$ which has no definite frequency dependence (apart from the "blackbody" distribution of $B^{(0)}$ described by Eq. (17). This distribution is the same, of course, in the usual transverse waves). *Magnetization by light can therefore occur without any resonant* absorption.

The recently demonstrated [11, 12] ability of circularly polarized light to shift nuclear magnetic resonances is definitive evidence for a novel property of light. In a recent technical comment [13] on the remarkable optical NMR experiments reported by Warren *et al*. [11], Harris and Tinoco give a conventional calculation of optically induced NMR shifts. In their reply [12] Warren *et al*. report careful and repeated experimental checks which confirm beyond doubt that the experimentally observed shifts [11,12] are *fifteen to sixteen orders of magnitude greater than allowed for in conventional [13] theory*. The observed shifts are of the order 0.1 to 1.0 Hz. for laser intensities of 1 to 3 W cm^{-2}, while Harris and Tinoco [13] assert using conventional theory that the expected shift is no larger than 10^{-15} Hz for 10 W cm^{-2}. Essentially, therefore, large optical NMR phenomena [11] *cannot occur in conventional understanding*.

For a beam intensity of 1.0 W cm^{-2}, however, $B^{(0)}$ is about 10^{-6} tesla, which for a 270 MHz (order of 10 tesla) NMR spectrometer is expected to shift resonances by roughly ±30 Hz, depending on the sense of circular polarity in the laser

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[14]. Warren *et al*. [11, 12] report that the experimental shifts, although difficult to measure, do indeed move in different directions for right and left circularly polarized laser light. The $\hat{B}^{(3)}$ theory of optical NMR therefore appears to produce shifts which are too large by about an order, but the conventional theory [13] produces a result fifteen orders of magnitude too small. The over-estimate by the $\hat{B}^{(3)}$ theory could be due to multiple competing mechanisms, as discussed by Warren *et al*. [12], or perhaps be due to the fact that only a fraction of the beam intensity at the

circular polarity of the laser. It is significant that $\hat{B}^{(3)}$ changes sign from left to right circular polarization [4–6], and that the ONMR data were obtained *far from optical resonance*. In terms of $\hat{B}^{(3)}$, the torque imparted to the sample by the beam is therefore $-\boldsymbol{m}\times\boldsymbol{B}^{(3)}$, where $\boldsymbol{m}$ is the nuclear magnetic dipole moment, and the interaction energy between $\boldsymbol{m}$ and $\boldsymbol{B}^{(3)}$ is $-\boldsymbol{m}\cdot\boldsymbol{B}^{(3)}$. The angular momentum imparted from light to matter is therefore an integral over the torque $-\boldsymbol{m}\times\boldsymbol{B}^{(3)}$, and this is non-zero far from optical resonance and changes sign with the sense of circular polarization. Harris and Tinoco [13] have shown clearly that *any* process second order in $B^{(0)}$ will produce shifts many orders of magnitude too small, and the ONMR results are strongly indicative of a process to first order in $B^{(0)}$. Finally, this cannot be a time dependent process, i.e. cannot be due to the usual transverse $\boldsymbol{B}^{(1)}$ (or $\boldsymbol{B}^{(2)}$), because it would time average to zero, in the same way that $\boldsymbol{B}^{(1)}$ and $\boldsymbol{B}^{(2)}$ time average to zero at first order. (The time averaged torque due to $\boldsymbol{B}^{(1)}$ is $-\boldsymbol{m}\times\boldsymbol{B}^{(1)}$, i.e. zero.)

We conclude that ONMR shifts are strongly indicative of the presence of a mechanism based on $\hat{B}^{(3)}$, and in this Letter we have shown that the latter is consistent with the fundamental conservation laws of electrodynamics in vacuo.

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