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The photomagneton and photon helicity

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Abstract

It is shown that the real and physical Pauli–Lubansky pseudo-four-vector (V_μ) dual to the electromagnetic four-tensor in free space is proportional to photon helicity. Therefore V_μ is rigorously non-zero, showing that the photomagneton $\hat{B}^{(3)}$ is rigorously non-zero in the theory of special relativity. In the conventional approach to free space electromagnetism, the photomagneton is at best not considered (because it is not a transverse wave), and at worst asserted to be zero. Physical effects are expected from the interaction of $\hat{B}^{(3)}$ with matter. The vector V_μ contains the expectation value (B_z) of $\hat{B}^{(3)}$ as its space-like and time-like component, i.e. $V_\mu = \varepsilon_0 c(0, 0, B_z, \mathrm{i}B_z)$. The imaginary pseudo-four-vector $W_\mu = -\mathrm{i}\varepsilon_0(0, 0, E_z, \mathrm{i}E_z)$ is dual to V_μ in free space, but its components are unphysical because they are imaginary. Therefore B_z in free space is dual to an imaginary and unphysical longitudinal electric field.

1. Introduction

In the conventional theory of free-space electromagnetism, electromagnetic waves are transverse to the direction of propagation (Z). It has been demonstrated recently [1–5] however that the transverse waves are linked geometrically to a spin field, which has been denoted $\boldsymbol{B}^{(3)}$ in the circular basis (1), (2), (3) defined by the unit vectors;

$$\boldsymbol{e}^{(1)} \times \boldsymbol{e}^{(2)} = \mathrm{i}\boldsymbol{e}^{(3)*}, \quad \text{and cyclic permutations,} \tag{1}$$

where $\boldsymbol{e}^{(1)} = \boldsymbol{e}^{(2)*} = 1\sqrt{2}(\boldsymbol{i} - \mathrm{i}\boldsymbol{j})$, $\boldsymbol{e}^{(3)} = \boldsymbol{k}$, and where $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$ are Cartesian unit vectors of the laboratory frame (X, Y, Z). The spin field $\boldsymbol{B}^{(3)}$ is a magnetic field which is phase free and directly proportional to the angular momentum of the photon in free space. It is responsible for the phenomenon of magnetization by light, the inverse Faraday effect [6–12], which is expressible at second order as:

$$\boldsymbol{M}^{(3)} = AB^{(0)}\boldsymbol{B}^{(3)}, \quad \boldsymbol{B}^{(3)} = B^{(0)}\boldsymbol{k}, \tag{2}$$

where A is a molecular property and where $B^{(0)}$ is the scalar magnitude of $\boldsymbol{B}^{(3)}$. There are available experimental indications that $\boldsymbol{B}^{(3)}$ can act at first order through its

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ability to shift NMR resonances in the phenomenon known as optical NMR spectroscopy [13–16]. The field $\boldsymbol{B}^{(3)}$ is also responsible for other types of observed magnetic phenomena produced by circularly polarized light, such as the optical Faraday effect [17]. A discussion of the experimental evidence for $\boldsymbol{B}^{(3)}$ is given in Ref. [5]. It is clear from a simple equation such as Eq. (2) that if $\boldsymbol{B}^{(3)}$ were zero, there would be no inverse Faraday effect at any order in this spin field of light. It is also expected [5] that $\boldsymbol{B}^{(3)}$ will produce the optical equivalent of the Aharonov–Bohm effect, i.e. circularly polarized laser radiation sent through an optical fiber between two interfering electron beams is expected to shift the interference fringes through the action of the vector potential associated with $\boldsymbol{B}^{(3)}$ [5] in free space.

It has been argued that the clear theoretical existence of $\boldsymbol{B}^{(3)}$ is compatible with the existence of photon mass, and the Proca equation [5] does indeed provide a $\boldsymbol{B}^{(3)}$ which is a very slow exponential decay. This finding is, turn, compatible with the Einstein–de Broglie interpretation of quantum mechanics [18], in which the photon and concomitant wave are simultaneously physical. This interpretation has recently been given experimental support in interference experiments such as those of Mizobuchi and Ohtaké [19]. It has been shown that in the basis (1), there exists a cyclically symmetric Lie algebra between the three space-like fields $\boldsymbol{B}^{(1)}$, $\boldsymbol{B}^{(2)}$ and $\boldsymbol{B}^{(3)}$ in free space, an algebra which is isomorphic with that of the infinitesimal rotation generators of the Lorentz group of special relativity [20], generators that are all physical, and which are angular momentum operators of quantum mechanics within a factor $\hbar$, the reduced Planck constant. Therefore $\boldsymbol{B}^{(1)}$, $\boldsymbol{B}^{(2)}$ and $\boldsymbol{B}^{(3)}$ are themselves directly proportional to angular momentum operators in quantum field theory. In particular [1–5],

$$\hat{B}^{(3)} = B^{(0)}\frac{\hat{J}}{\hbar}, \tag{3}$$

where $\hat{B}^{(3)}$ is an operator which has been identified as the fundamental photomagneton of circularly polarized radiation of all frequencies. Here $\hat{J}$ is the angular momentum operator in the propagation axis Z, an operator whose eigenvalues for the free photon are $\pm\hbar$. This defines the helicity ± 1 of the free photon without mass.

Therefore it is expected in the quantum field theory that the photomagneton is directly proportional to the photon helicity, and as such must be non-zero in free space. In this paper this is confirmed using the classical relativistic field by directly considering the basic four-tensor $F_{\mu\nu}$ of electromagnetism [21], defined as usual as the antisymmetric four-curl,

$$F_{\mu\nu} = \frac{\partial A_\nu}{\partial X_\mu} - \frac{\partial A_\mu}{\partial X_\nu}, \tag{4}$$

where A_μ is the potential four-vector. Throughout this paper we adhere to Minkowski's notation, in which

$$X_\mu \equiv (X, Y, Z, \mathrm{i}ct). \tag{5}$$

In Section 2, it is shown that $F_{\mu\nu}$ is dual in free space to the pseudo-four vectors V_μ and W_μ, whose space-like components are respectively $\varepsilon_0 c B_z$ and $-\mathrm{i}\varepsilon_0 E_z$. Thus, B_z is dual to the pure imaginary (and therefore unphysical) phase free electric field $-\mathrm{i}E_z/c$. It is experimentally significant that the latter appears never to have been observed to produce physical effects at first order. (At second order, the effect of $c^2 B_z^2$ is evidently the same as that of E_z^2 in free space because of the Maxwellian result $E^{(0)} = cB^{(0)}$. Thus $\boldsymbol{E}^{(1)} \times \boldsymbol{E}^{(2)}$ is observed at second order in the inverse Faraday effect, for example in the recent work of Woźniak et al. [22]. It has also been demonstrated recently that the inverse Cotton–Mouton effect, observed experimentally in Ref. [23], can be described in terms of B_z^2, or equivalently, E_z^2/c^2.)

In Section 3, it is shown that the pseudo-four-vector V_μ is directly proportional to the well known Pauli–Lubansky angular momentum pseudo-four-vector J_μ of special relativity, defined in the Poincaré group (the inhomogeneous Lorentz group) by use of the linear momentum four-vector p_μ (the generator of translations in space-time). For the free photon, the latter is the light-like,

$$p_\mu = \hbar\kappa(0, 0, 1, \mathrm{i}), \tag{6}$$

where κ is the scalar magnitude of the wave vector of the photon, and the helicity of the photon is given by the ratio of J_μ to p_μ, as in standard texts in field theory [21]. Therefore, the line of reasoning which asserts in Section 2 that V_μ is dual to $F_{\mu\nu}$ in the theory of tensors leads to the result in Section 3 that $\boldsymbol{B}^{(3)}$ (whose magnitude is B_z) is also directly proportional to the free photon helicity, and is therefore non-zero and physical. This is the result indicated by Eq. (3) of the quantum field theory.

The paper ends with a discussion of this fundamentally significant result, which ties together $\boldsymbol{B}^{(3)}$ and helicity. The theory of special relativity has been shown to predict the physically meaningful phase free magnetic field $\boldsymbol{B}^{(3)}$, which is tied geometrically to the usual transverse wave fields $\boldsymbol{B}^{(1)}$ and $\boldsymbol{B}^{(2)}$ in the circular basis (1). There are clear experimental indications that $\boldsymbol{B}^{(3)}$ is observable in the inverse Faraday effect (Eq. (2)), (the inverse Cotton–Mouton effect, and the optical Faraday effect (and in general whenever the well known conjugate product is observed experimentally, as reviewed in Ref. [4]). There are signs that it can act at first order in optical NMR, but there is a need to demonstrate conclusively that $\boldsymbol{B}^{(3)}$ can act at first order in the optical equivalent of the Aharonov–Bohm effect [5]. If so, it would have been demonstrated that a novel fundamental field of electromagnetism exists in free space. If $\boldsymbol{B}^{(3)}$ is not observed, however, to act at first order as well as second order, a paradox would have been demonstrated in special relativity, in that the latter would have produced a magnetic field without symmetry violation, but a field that is not observable at first order in the interaction of photons and matter.

2. The dual pseudo-four-vectors of $F_{\mu\nu}$ in free space

Though consideration of the space-like, cyclically symmetric, algebra

$$\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)} = \mathrm{i}B^{(0)}\boldsymbol{B}^{(3)*}, \quad \boldsymbol{B}^{(2)} \times \boldsymbol{B}^{(3)} = \mathrm{i}B^{(0)}\boldsymbol{B}^{(1)*}, \boldsymbol{B}^{(3)} \times \boldsymbol{B}^{(1)} = \mathrm{i}B^{(0)}\boldsymbol{B}^{(2)*} \tag{7}$$

it is inferred that there exists a type of four-vector

$$B_\mu \equiv (B_X, B_Y, B_Z, \mathrm{i}B^{(0)}), \tag{8}$$

in free-space electromagnetism. Similarly, there is a priori a four-vector,

$$E_\mu = -\mathrm{i}(E_X, E_Y, E_Z, \mathrm{i}E^{(0)}). \tag{9}$$

The properties of this class of four-vector under Lorentz transformation must be such that the individual components transform as magnetic and electric fields, components of the well known four-tensor $F_{\mu\nu}$. In S.I. units,

$$B'_X = \gamma\left(B_X + \frac{v}{c^2}E_Y\right),\quad E'_X = \gamma(E_X + vB_Y),\quad B'_Y = \gamma\left(B_Y - \frac{v}{c^2}E_X\right),$$

$$E'_Y = \gamma(E_Y - vB_X),\quad B'_z = B_z, E'_z = E_z, \tag{10}$$

where $\gamma = (1 - v^2/c^2)^{-1/2}$; and where v is the velocity of frame K' with respect to K as usual [21]. Therefore B_μ and E_μ are not ordinary four-vectors such as x_μ, because the latter are defined through the Lorentz transformation,

$$X'_\mu = a_{\mu\nu}X_\nu, \tag{11}$$

where $a_{\mu\nu}$, as usual [21] is the transformation matrix.

The Lorentz transformation of the space components of the magnetic field, as in Eq. (10), is reminiscent of that of the space components of angular momentum in special relativity, i.e. of

$$J'_Z = J_Z,\quad J'_X = \frac{J_X}{\gamma},\quad J'_Y = \frac{J_Y}{\gamma}. \tag{12}$$

It is seen that B_z and J_z are invariant under Lorentz transformation, i.e. are the same in all Lorentz frames of reference, a result which is consistent with the fact that in the quantum field the angular momentum of the free photon has eigenvalues $\pm\hbar$, a universal constant, invariant under the Lorentz transformation of the classical electromagnetic field. Thus B_μ and E_μ cannot have the same transformation properties as a four-vector such as X_μ. However, both B_μ and $F_{\mu\nu}$ contain the same magnetic field space components, suggesting that they are inter-related. Therefore, despite the fact that axial vectors such as $\boldsymbol{B}$ cannot [24] form the spatial part of an ordinary four-vector such as X_μ the theory of tensors [21] allows a *pseudo* four-vector (V_μ) to be defined rigorously from an antisymmetric four-tensor such as $F_{\mu\nu}$. Maxwell's equations in free space apply equally well to the components of $F_{\mu\nu}$ or V_μ, but the latter has a non-zero time-like component whose presence in $F_{\mu\nu}$ is implied only indirectly. Obviously, the pseudo four-vector B_μ does not transform as Eq. (*11*), i.e. does not transform in the same way as the ordinary four-vector X_μ. These considerations can be amplified by reference to the definition of angular momentum in special relativity, following a standard text such as Barut [21],

$$J_{\mu\nu} = X_\mu p_\nu - X_\nu p_\mu. \tag{13}$$

The tensor $J_{\mu\nu}$ is antisymmetric, in the same way a $F_{\mu\nu}$ in electromagnetism is antisymmetric in four dimensions. Its components are

$$J_{\rho\sigma} = \begin{bmatrix} 0 & J_Z & -J_Y & -\mathrm{i}J_{10} \\ -J_Z & 0 & J_X & -\mathrm{i}J_{20} \\ J_Y & -J_X & 0 & -\mathrm{i}J_{30} \\ \mathrm{i}J_{10} & \mathrm{i}J_{20} & \mathrm{i}J_{30} & 0 \end{bmatrix}, \tag{14}$$

where the space-like elements transform according to Eq. (12). The time-like components are

$$J_{k0} = X_k p_0 - p_k X_0, \tag{15}$$

and their physical interpretation is more difficult [21]. In the theory of tensors [21] there exists a dual tensor defined by:

$$J^{\mathrm{D}}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}J_{\rho\sigma} = \begin{bmatrix} 0 & -\mathrm{i}J_{30} & \mathrm{i}J_{20} & J_X \\ \mathrm{i}J_{30} & 0 & -\mathrm{i}J_{10} & J_Y \\ -\mathrm{i}J_{20} & \mathrm{i}J_{10} & 0 & J_Z \\ -J_X & -J_Y & -J_Z & 0 \end{bmatrix}, \tag{16}$$

and also a dual pseudo four-vector, the Pauli–Lubansky angular momentum four-vector [20], denoted J_μ. The latter, however, cannot be defined from the tensor $J_{\mu\nu}$ without the use of a one index unit four-vector ε_ν,

$$J_\mu = -\frac{\mathrm{i}}{2}\varepsilon_{\mu\nu\rho\sigma}J_{\rho\sigma}\varepsilon_\nu, \tag{17}$$

the fundamental reason for this is inherent in the mathematical structure of these tensors. It is well known that ε_ν is the unit translation generator in space-time [20].

Using Eqs. (16) and (17) gives the result:

$$J_\mu \equiv -\mathrm{i}J^{\mathrm{D}}_{\mu\nu}\varepsilon_\nu = (J_{20} + J_{X'} - J_{10} + J_{Y'}J_{Z'}\mathrm{i}J_Z), \tag{18}$$

so that in general, the pseudo four-vector J_μ of angular momentum in special relativity has four non-zero components. The longitudinal space component J_z is equal to the time component. If the free photon is considered as a particle of zero mass propagating in the axis Z, then

$$J_\mu = (0, 0, J_Z, \mathrm{i}J_Z) = \hbar(0, 0, 1, \mathrm{i}), \tag{19}$$

and the four-vector J_μ has just two components because the eigenvalues of the photon angular momentum must be $\pm\hbar$. There can be no non-zero components of photon angular momentum perpendicular to Z, i.e.

$$J_{20} + J_X = 0, \quad -J_{10} + J_Y = 0. \tag{20}$$

The physical meaning of this result is discussed later (Section 3). The J_μ vector of a free photon is therefore light-like, and directly proportional to the photon's linear momentum four-vector,

$$p_\mu = \hbar\kappa(0,0,1,\mathrm{i}), \tag{21}$$

another light-like vector. The ratio of the unit four-vector $J_\mu/\hbar$ to the unit four-vector corresponding to $p_\mu/(\hbar\kappa)$ is therefore $+1$ and defines the helicity. Applying the parity operator reverses the sign of p_μ but not of J_μ, so that the ratio becomes -1. The helicity of the free photon in special relativity is therefore 1 and -1. If the photon has mass, the helicity becomes 1, 0 and -1. Multiplication of the photon helicity by $\hbar$ gives the eigenvalues of the photon angular momentum in quantum field theory.

In direct analogy to Eq. (17), the pseudo four-vector V_μ can be defined as the dual of the electromagnetic four-tensor $F_{\rho\sigma}$,

$$V_\mu \equiv -\frac{\mathrm{i}}{2}\varepsilon_{\mu\nu\rho\sigma}F_{\rho\sigma}\varepsilon_\nu, \tag{22}$$

so that the components of V_μ must be electric and magnetic field components. Since ε_ν and the antisymmetric unit tensor $\varepsilon_{\mu\nu\rho\sigma}$ are Lorentz invariants, the components of V_μ must transform in the same way as the components of $F_{\rho\sigma}$. In general, the latter in S.I. units is well known to be

$$F_{\rho\sigma} \equiv \varepsilon_0 \begin{bmatrix} 0 & cB_Z & -cB_Y & -\mathrm{i}E_X \\ -cB_Z & 0 & cB_X & -\mathrm{i}E_Y \\ cB_Y & -cB_X & 0 & -\mathrm{i}E_Z \\ \mathrm{i}E_X & \mathrm{i}E_Y & \mathrm{i}E_Z & 0 \end{bmatrix}, \tag{23}$$

in free space. (Here ε_0 is the free-space permittivity in S. I. units.) There is no indication in this definition that the longitudinal components E_Z and cB_Z must be zero, or otherwise unphysical, as frequently asserted in the electrodynamic literature. The tensor dual to $F_{\rho\sigma}$ is

$$G_{\mu\nu} \equiv \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}F_{\rho\sigma} = \varepsilon_0 \begin{bmatrix} 0 & -\mathrm{i}E_Z & \mathrm{i}E_Y & cB_X \\ \mathrm{i}E_Z & 0 & -\mathrm{i}E_X & cB_Y \\ -\mathrm{i}E_Y & \mathrm{i}E_X & 0 & cB_Z \\ -cB_X & -cB_Y & -cB_Z & 0 \end{bmatrix}, \tag{24}$$

and the pseudo four-vector V_μ in general is easily evaluated as

$$\begin{aligned} V_\mu &= -\mathrm{i}G_{\mu\nu}\varepsilon_\nu = \varepsilon_0(E_Y + cB_X, -E_X + cB_Y, cB_Z, \mathrm{i}cB_Z) \\ &= \varepsilon_0(cB_\mu + (E_Y, -E_X, 0, 0)). \end{aligned} \tag{25}$$

In free space, however, the transverse components of the electric and magnetic fields are

$$\boldsymbol{E}^{(1)} = \boldsymbol{E}^{(2)*} = \frac{E^{(0)}}{\sqrt{2}}(\boldsymbol{i} - \mathrm{i}\boldsymbol{j})\varepsilon^{\mathrm{i}\varphi}, \quad \boldsymbol{B}^{(1)} = \boldsymbol{B}^{(2)*} = \frac{B^{(0)}}{\sqrt{2}}(\mathrm{i}\boldsymbol{i} + \boldsymbol{j})\varepsilon^{\mathrm{i}\varphi}, \tag{26}$$

so that

$$E_Y + cB_X = 0, \quad -E_X + cB_Y = 0, \tag{27}$$

a result which is directly analogous to Eq. (20) for angular momentum of the free photon. This point is discussed further in Section 3.

We therefore arrive at the result that in free space, V_μ has only one non-zero component

$$V_\mu = \varepsilon_0(0, 0, cB_Z, icB_Z), \tag{28}$$

and is directly proportional to both J_μ and ε_μ. This result is the classical relativistic counterpart of the result (3) from quantum field theory of the free photon. *In particular, if we assert that B_Z is zero in the everyday four-tensor $F_{\mu\nu}$, then the dual vector V_μ vanishes in free space, and is no longer proportional to J_μ. This means that the photon helicity cannot be defined in terms of concomitant electromagnetic field components if B_Z is asserted to be zero.* Such an assertion also violates the Lie algebra (7) and is incompatible with the structure of he Lorentz and Poincaré groups. It would violate the $\hat{C}\hat{P}\hat{T}$ Theorem [5] because the cyclic relations (7) conserve $\hat{C}$, $\hat{P}$, and $\hat{T}$, and thus $\hat{C}\hat{P}\hat{T}$. Finally, the assertion that B_Z is zero is incompatible with the experimental observation of the inverse Faraday effect (Eq. (2)), because if $B_Z = 0$, the observed magnetization vanishes. Similar deductions accrue from the experimental existence of the optical Faraday and inverse Cotton-Mouton effects, and the recent observation of optical NMR spectroscopy.

In general, V_μ can always be written as the sum of two pseudo four-vectors, in free space, or in matter. In free space

$$V_\mu = \varepsilon_0 c B_\mu + \varepsilon_0(E_{Y'} - E_X, 0, 0). \tag{29}$$

Similarly, there exists a pseudo four-vector which is dual to the four-tensor $G_{\rho\sigma}$ in free space,

$$W_\mu \equiv -\frac{i}{2}\varepsilon_{\mu\nu\rho\sigma}G_{\rho\sigma}\varepsilon_\nu = -iF_{\mu\nu}\varepsilon_\nu\,, \tag{30}$$

and W_μ is imaginary. In general it can be expressed as the sum

$$W_\mu \equiv \varepsilon_0 E_\mu - i\varepsilon_0 c(-B_Y, B_X, 0, 0) \tag{31}$$

of two pseudo four-vectors. Note that V_μ is also dual to W_μ, because $F_{\mu\nu}$ is dual to $G_{\rho\sigma}$, so that the real B_Z is dual to the imaginary $-iE_Z/c$. Thus B_Z is physical and its imaginary dual is unphysical, in accordance with experimental observations. This is consistent with the fact that a real, polar $\boldsymbol{E}^{(3)}$ cannot be formed from the vector cross product of two axial vectors $\boldsymbol{B}^{(1)}$ and $\boldsymbol{B}^{(2)}$ and with the result [1–5]

$$\boldsymbol{E}^{(1)} \times \boldsymbol{E}^{(2)} = icE^{(0)}\boldsymbol{B}^{(3)*}, \tag{32}$$

which links $\boldsymbol{B}^{(3)}$ to the conjugate product $\boldsymbol{E}^{(1)} \times \boldsymbol{E}^{(2)}$. It is well known [4] that the antisymmetric part of light intensity is directly proportional to the conjugate product, so that the assertion $\boldsymbol{B}^{(3)} = 0$ results in the disappearance of this fundamental property of light, and of fundamental phenomena such as antisymmetric light scattering which depend on the fact that light intensity is a tensor with a non-zero antisymmetric component proportional to the conjugate product. Finally, $\boldsymbol{B}^{(3)}$ is simply the amplitude $B^{(0)}$ (a scalar invariant) multiplied by the invariant unit four-vector $(0,0,1,\mathrm{i})$, which is light-like. Therefore $B^{(3)}$ is non-zero in general in special relativity.

With these considerations we conclude that B_μ is a Pauli-Lubansky pseudo four-vector, and that $B_\mu B_\mu$ is a spin Casimir invariant [20] of the Poincaré group. Proper definition of B_μ in special relativity therefore requires consideration of the unit space-time translation generator, a deduction which is consistent with the fact that B_μ needs for its definition a travelling plane wave. In a standing wave the conjugate product vanishes, and B_μ is not defined. Similarly, the helicity of the photons in a standing wave cannot be defined.

3. The link between V_μ and J_μ in free space

The following quantities can be defined through the unit vector ε_μ in free space,

$$p_\mu = \hbar\kappa\varepsilon_\mu, \quad J_\mu = \hbar\varepsilon_\mu, \quad E_\mu = -\mathrm{i}E^{(0)}\varepsilon_\mu, \quad B_\mu = B^{(0)}\varepsilon_\mu, \tag{33}$$

three of which are real and one of which is imaginary and unphysical. These equations give the direct relativistic equivalent of Eq. (3),

$$\hat{B}_\mu = B^{(0)}\frac{\hat{J}_\mu}{\hbar}, \tag{34}$$

which shows that B_μ is directly proportional to J_μ in free space. The photon helicity can be defined formally as the ratio,

$$\lambda = \frac{\hbar\kappa}{B^{(0)}}\frac{B_\mu}{p_\mu}, \tag{35}$$

which with Eq. (34) becomes the formal ratio of four vectors,

$$\lambda = \kappa\frac{J_\mu}{p_\mu}. \tag{36}$$

If B_μ were zero in free space, the helicity of the photon would vanish, and this is a clearly incorrect and unphysical result. Therefore B_μ is rigorously non-zero in free space.

A comparison of Eqs. (20) and (27) for the photon suggests that E_Y is proportional to J_{20}; E_X is proportional to J_{10}; cB_X is proportional to J_X; and cB_Y is proportional to J_Y. Such comparisons lead to the inference [1–5] that in the free-space Poincaré group of electromagnetism, magnetic fields are rotation generators, and electric fields are boost generators. The Lie algebra of magnetic and electric field components is therefore isomorphic with that of infinitesimal generators of the Poincaré group. This is discussed in detail elsewhere [4, 5].

4. Discussion

The existence of the four-tensor $F_{\mu\nu}$ is of course well accepted in conventional electrodynamics, both in free space and in matter (where B and E components are replaced by H and D components as usual). The dual pseudo four-vectors V_μ and W_μ (and their equivalents in matter) are not used conventionally, but appear to be implied (if not actually derived explicitly) in the work of Einstein circa 1916 [24]. The structure of V_μ and W_μ in free space is also implied in the well known Gupta–Bleuler method of electromagnetic field quantization, a method which produces the result:

$$\hat{a}^{(0)}|\psi\rangle = \hat{a}^{(3)}|\psi\rangle. \tag{37}$$

Here $\hat{a}^{(0)}$ and $\hat{a}^{(3)}$ are time-like and longitudinal annihilation operators and $|\psi\rangle$ is an eigenfunction of the electromagnetic field [20]. This result is the admixture condition in Lorentz gauge quantization, and is conventionally interpreted, obscurely, to imply that admixtures of time-like and longitudinal photon states may be physically meaningful, but that these states have no independent physical significance [20].

The structure of the vector V_μ is such that its Z space-like component is equal in magnitude to that of its time-like component. This is the classical equivalent of the Gupta-Bleuler admixture condition (37), and the structure of the physical V_μ shows that the longitudinal $\hat{a}^{(3)}$ and time-like $\hat{a}^{(0)}$ must both be physically meaningful. The timelike component of V_μ is the scalar magnitude of its longitudinal component in free space. Thus, in the equation

$$\boldsymbol{B}^{(3)} = B^{(0)}\boldsymbol{k}, \tag{38}$$

the time-like component appears as $B^{(0)}$, and longitudinal magnetic flux density vector in free space is $\boldsymbol{B}^{(3)}$, being the longitudinal component B_Z multiplied by the axial unit vector $\boldsymbol{k}$. The vector $\boldsymbol{B}^{(3)}$ is a physical magnetic flux density in the theory of special relativity, and as such should produce observable physical effects, such as magnetization. Therefore the conventional interpretation of the Gupta–Bleuler admixture condition (37), i.e. that only admixtures of $\hat{a}^{(0)}$ and $\hat{a}^{(3)}$ are physically meaningful, is not tenable.

In this paper it has been proven that $\boldsymbol{B}^{(3)}$ is non-zero in free space and that the helicity, λ, of the free photon can be defined in terms of it. Since λ is physical, and non-zero, then the real $\boldsymbol{B}^{(3)}$ is also physical and non-zero, and defines the four-vector V_μ and the spin Casimir invariant of the Poincaré group, $V_\mu V_\mu$. The latter is the product of two light-like vectors for the photon without mass, but is non-zero for the photon with mass, and more generally, for any particle with mass. Similarly $\varepsilon_\mu\varepsilon_\mu$ is the mass Casimir invariant of the Poincaré group, and is non-zero for the photon with mass. For the free photon without mass, V_μ is light-like and its norm or metric is zero. The Lorentz transformation leaves it invariant, and $\boldsymbol{B}^{(3)}$ is also invariant and non-zero. Since $\boldsymbol{B}^{(3)} \neq 0$ then B_Z must be directly proportional to λ in free space, a proportionality which defines the scalar helicity of the photon.

The fact that B_Z is directly proportional to the helicity of the photon means that helicity can be defined in terms of a phase independent, electromagnetic spin field, $\boldsymbol{B}^{(3)}$, rather than in terms of the angular and linear momenta of a particle (the photon). Such an inference lends support to the view that the particle and field components of electromagnetic radiation are simultaneously physical – as in the experiment of Mizobuchi and Ohtaké [19], an experiment which shows that the photon and concomitant field are both observable in one experiment. This is not possible in the Copenhagen interpretation but is a cornerstone of the Einstein–de Broglie interpretation of light and matter waves [5], i.e. of quantum mechanics. It seems clear in this view that $\boldsymbol{B}^{(3)}$ is the piloting field of photon spin, and is itself a spin field, defined directly by the spin or intrinsic angular momentum of the free photon. It is important to note therefore that $\boldsymbol{B}^{(3)}$ is not an electromagnetic wave, such as $\boldsymbol{B}^{(1)}$ or $\boldsymbol{B}^{(2)}$, which remain transverse in free space. The usual electrodynamic view that waves are transverse therefore remains the same, but these waves are accompanied by the spin field $\boldsymbol{B}^{(3)}$ which is the third space component. It is also possible, of course, to interpret $\hat{B}^{(3)}$ in the Copenhagen view of quantum mechanics, because it is directly proportional to the ordinary photon angular momentum operator $\hat{J}$, which can be interpreted in terms of commutators and the Heisenberg Uncertainty Principle as usual. If the results of Mizobuchi and Ohtaké [19] are accepted however, it becomes difficult to maintain the Copenhagen point of view, because it prohibits the simultaneous observation and existence of particle and wave (photon and field).

Above all, the $\boldsymbol{B}^{(3)}$ field, being phase independent, real, and physical, is an ordinary magnetic flux density in free-space electrodynamics, and as such ought to act upon matter at first order. If it does so (for example in the proposed optical Aharonov–Bohm effect [5]), then a major step forward in electrodynamics would have been achieved, in that the existence of a free-space spin field would have been observed unequivocally for the first time to act at first order on matter. If it does not, a major paradox would have developed, because electrodynamics would have contradicted itself at the most fundamental level, in that the theory would have produced a field $\boldsymbol{B}^{(3)}$ that is physical but which cannot act upon matter. This is a paradox in special relativity itself, indicating a need to modify the Maxwell equations. It is therefore of key importance to devise unequivocal experimental tests for the field $\boldsymbol{B}^{(3)}$ acting at first order.

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Appendix. Inverse relations

The core relations,

$$V_\mu = -\mathrm{i}G_{\mu\nu}\varepsilon_\nu = -\frac{\mathrm{i}}{2}\varepsilon_{\mu\nu\rho\sigma}F_{\rho\sigma}\varepsilon_\nu, \quad W_\mu = -\mathrm{i}F_{\mu\nu}\varepsilon_\nu = -\frac{\mathrm{i}}{2}\varepsilon_{\mu\nu\rho\sigma}G_{\rho\sigma}\varepsilon_\nu, \tag{A.1}$$

derived in the text can be inverted in order to express the electromagnetic field four-tensors $F_{\mu\nu}$ and $G_{\mu\nu}$ in terms of the pseudo four-vector V_μ and W_μ. The relevant expressions are

$$G_{\mu\nu} = \mathrm{i}(V_\mu\varepsilon_\nu^{-1} - V_\nu G_\mu^{-1}), \quad F_{\mu\nu} = \mathrm{i}(W_\mu\varepsilon_\nu^{-1} - W_\nu\varepsilon_\mu^{-1}), \tag{A.2}$$

where

$$\varepsilon_\mu^{-1} = \tfrac{1}{2}(0, 0, 1, -\mathrm{i}), \tag{A.3}$$

is the inverse of ε_μ. Eq. (A.2) and (A.3) show that if the components of V_μ were each zero, as in the conventional point of view, then the tensors $F_{\mu\nu}$ and $G_{\mu\nu}$ would vanish in free space, implying the disappearance of electromagnetism in free space. In other words, the pseudo four-vector V_μ is structured in such a way that if B_Z were zero, then $F_{\mu\nu}$ and $G_{\mu\nu}$ would disappear. This is conclusive mathematical evidence that a rigorously non-zero B_Z is essential for a self consistent theory of free-space electrodynamics, and that if B_Z is asserted to be zero, then the structure of the theory fails at the most basic level.

The pseudo four-vector V_μ contains elements that under Lorentz transformation have the properties of a longitudinal magnetic field in free space, a field that is directly proportional to the helicity of the free photon in the particle interpretation of light and electromagnetic radiation in general. The field B_Z, the expectation value of the photomagneton $\hat{B}^{(3)}$, can be interpreted therefore as the piloting field of the photon helicity. Within the structure of the Einstein–de Broglie theory of light, such an interpretation has a transparent physical meaning, the photon helicity is accompained by a field helicity. Since helicity cannot be defined if the particle has no forward (linear) momentum, this interpretation naturally accounts for the fact that $\hat{B}^{(3)}$ can be defined only in a travelling wave, a wave in which the conjugate product is imaginary but rigorously non-zero. In the Copenhagen interpretation, on the other hand, the field B_Z is an expectation value akin to the expectation value of longitudinal (specified) angular momentum, and the Heisenberg uncertainty principle can be written as

$$\delta B^{(1)}\delta B^{(2)} \geqslant \tfrac{1}{2} B^{(0)}B^{(3)}, \tag{A.4}$$

showing that there is no uncertainty if $\boldsymbol{B}^{(3)}$ is zero, meaning that specific quantum phenomena such as light squeezing [4] could not occur. This chain of reasoning implies, therefore, that light squeezing is experimental evidence for the existence of $\boldsymbol{B}^{(3)}$, as discussed more fully elsewhere in the literature [4, 5].

Therefore there are several deeply interesting implications of $\boldsymbol{B}^{(3)}$ theory in natural philoscopy, not least of which is the implication which it holds for the interpretation of quantum mechanics, which arose out of de Broglie's matter waves. The interpretation of quantum mechanics is based on a suggestion by Born that the wave function is probabilistic in nature, so that matter waves are asserted to be 'waves of probability', loosely speaking, with only indirect physical significance. This view was never accepted by Einstein or de Broglie, and it is well known that the former referred to the wave function as the psi function, a mathematical necessity whose ultimate physical interpretation is not clear, but which has physical meaning. It is well known that these two famous interpretations, and paradoxes therein, have been the cornerstones of a discussion which has persisted to this day, generating an enormous and brilliant literature. Despite the assured success of the quantum theory, its interpretation is by no means agreed upon universally, and radical thinkers continue to challenge the original developments of the 'golden age' of twentieth century physics. This is of course interesting and essential for ultimate progress in the subject. Recent experiments such as those of Mizobuchi et al. [19] appear to have provided firm experimental evidence against the Copenhagen interpretation. As reviewed by Vigier [5, 18], these data find a transparent physical interpretation within the Einstein–de Broglie theory, in which waves and photons are physically concomitant and effective.

Eq. (A.1) and (A.2) are rigorous in the theory of tensors, and in natural philoscophy, Eq. (A.1) shows that there is a pseudo four-vector V_μ which can be defined from the usual four-tensor $F_{\mu\nu}$ of field theory and the unit momentum four-vector ε_ν. This procedure is mathematically isomorphic [5] with the definition of the Pauli–Lubansky pseudo four vector J_μ from the angular momentum four-tensor $J_{\mu\nu}$ of special relativity, a procedure which defines the particle helicity within the inhomogeneous Lorentz group, or Poincaré group. The latter is made up of boost and rotation generators which are essentially geometric in nature. The conventional approach to free-space electrodynamics makes the assertion that $\boldsymbol{B}^{(3)}$ is zero, or otherwise irrelevant. Equations (7) show if $\boldsymbol{B}^{(3)}$ were zero, so are $\boldsymbol{B}^{(1)}$ and $\boldsymbol{B}^{(2)}$. This is echoed in the relativistic theory by the finding that if V_μ were zero, the four-tensor $G_{\mu\nu}$ and its dual $F_{\mu\nu}$ both vanish identically. Also, if $\boldsymbol{B}^{(3)}$ were zero, there would be no light squeezing, as we have argued, and no inverse Faraday effect, because $\boldsymbol{M}^{(3)}$ in Eq. (2) would be zero. There are many other arguments of this kind that can be made [4, 5] in support of a non-zero, real, and physically effective $\boldsymbol{B}^{(3)}$. The inverse Faraday effect as embodied in Eq. (2) would vanish if $\boldsymbol{B}^{(3)}$ were non-zero, contrary to observation and theory of the last thirty years [6–12]. A careful reading of standard texts in classical electrodynamics usually reveals the assertion that plane waves are transverse. This remains true, because $\boldsymbol{B}^{(3)}$ is not a wave, it is a spin field, and so is its dual, the imaginary and unphysical $-\mathrm{i}\boldsymbol{E}^{(3)}/c$. It can be shown [5] that Lorentz invariants such as $L_1 = F_{\mu\nu}F^{*}_{\mu\nu}$ and $L_2 = F_{\mu\nu}G^{*}_{\mu\nu}$ remain null in the presence of $\boldsymbol{B}^{(3)}$ and $-\mathrm{i}\boldsymbol{E}^{(3)}/c$. In other words $\boldsymbol{B}^{(3)}$ and its dual violate none of the laws of radiation, and are, of course, solutions of Maxwell's equations in free space because both are time-independent, solenoidal and divergentless.

Since $\boldsymbol{B}^{(3)}$ contributes at second order to the inverse Faraday effect through Eq. (2), it should produce effects at first order, proportional to the square root of light intensity. If these first order effects are not observed, Maxwell's equations themselves will have to be modified to ensure that Eq. (2) remains valid, but that $\boldsymbol{B}^{(3)}$ cannot act at first order.

Eq. (A.1a) an also be formally inverted as follows. Using

$$\varepsilon_\nu = \frac{V_\nu}{\varepsilon_0 c B_Z}, \quad V_\mu = \varepsilon_0 c B_Z \varepsilon_\mu, \tag{A.5}$$

the unit momentum four-vector ε_ν is given by

$$\varepsilon_\nu = \mathrm{i} G_{\mu\nu} \frac{V_\mu}{\varepsilon_0^2 c^2 B_Z^2}. \tag{A.6}$$

Note the parallel result from relativistic angular momentum:

$$\varepsilon_\nu = -\mathrm{i} J^{\mathrm{D}}_{\mu\nu} \frac{J_\mu}{J_Z^2}. \tag{A.7}$$

The free photon linear momentum can therefore be expressed as

$$p_\nu = -\hbar\kappa\varepsilon_\nu = \frac{-\mathrm{i}\hbar\kappa}{2\varepsilon_0^2 B_Z^2 c^2} \varepsilon_{\mu\nu\rho\sigma} F_{\rho\sigma} V_\mu, \tag{A.8}$$

and the free photon angular momentum by

$$j_\nu = -\hbar\varepsilon_\nu = \frac{-\hbar}{2\varepsilon_0^2 B_Z^2 c^2} \varepsilon_{\mu\nu\rho\sigma} F_{\rho\sigma} V_\mu. \tag{A.9}$$

The last two equations interestingly relate particulate and undulatory qualities of light, because they express a particle linear and angular momentum in terms of fields. In the conventional interpretation of electrodynamics B_Z is either unconsidered or asserted to be zero in free space, and if so, the linear and angular momenta of the free photon become indeterminate.

Thus, if there is a pseudo four-vector V_μ dual to the tensor $F_{\rho\sigma}$ in free space, it must be light like and non-zero for a determinate ε_ν. In exact analogy, if there is a Pauli-Lubansky pseudo four-vector J_μ dual to $J_{\rho\sigma}$ in relativistic angular momentum theory, then ε_ν must be determinate. In other words the helicity of the free photon is not properly defined within the Lorentz group, only within the Poincaré group. Similarly, V_μ is properly defined only within the Poincaré group, i.e. if and only if the linear momentum of the free photon is considered explicitly through the generator of space-time translation. If $B_Z = 0$, then $V_\mu = 0$ and ε_ν becomes indeterminate. Thus both the linear and angular momenta of the free photon become indeterminate if $B_Z = 0$.

This conclusion is yet another way of illustrating that conventional classical and quantum electrodynamics are incomplete without a proper consideration of the magnetic field $\boldsymbol{B}^{(3)}$ in free space.

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