438

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THE CONNECTION BETWEEN PHOTON MASS AND $\boldsymbol{B}^{(3)}$: THE POINCARÉ GROUP

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The longitudinal vacuum field $\boldsymbol{B}^{(3)}$ is an experimental observable which produces by magnetization a well-defined square-root beam power density dependence. Its longitudinal polarization implies that the helicities of the photon are +1, 0, and -1, and that the little group of the Poincaré group is the rotation group O(3) of a massive boson. The mass of the photon (m) is therefore related directly to $\boldsymbol{B}^{(3)}$ through the Proca equation, and it is concluded that experimental evidence for $\boldsymbol{B}^{(3)}$ is also evidence for finite m.

Key words: $\boldsymbol{B}^{(3)}$ field, photon mass.

1. INTRODUCTION

Systems invariant under the Poincaré group [1] are characterized by mass and spin, which form the two Casimir invariants of the group. Spin corresponds to a rotation group symmetry if and only if $m^2 > 0$, where m is the mass of the particle being subjected to the general Lorentz transformation (boost, rotation and spacetime translation). In this view, first proposed by Wigner [2], and later discussed by Weinberg [3] for particles of any spin, the little group of a particle with mass *defines* the concept of spin itself. The concept of a particle without mass, corresponding to the condition $m^2 = 0$, results in a non-compact little group, E(2), which is unphysical. *Therefore, the idea of a particle* *without mass is also unphysical*. The conclusion that E(2) is unphysical has been reached repeatedly [1–3], and can be traced to the original 1939 paper by Wigner [2]. The reason is that E(2) is a group that describes rotations *and* translations which must simultaneously be taking place in a plane embedded in three dimensional space. On the other hand, the presence even of a minute amount of mass means that the little group becomes well defined, for a massive fermion it is SU(2), and for a massive boson O(3), the ordinary group of three dimensional rotation, whose infinitesimal rotation generators are, to a factor $\hbar$, the angular momentum operators of quantum mechanics. The inference that an unphysical E(2) means an unphysical particle has, in contrast, been carefully avoided, because in the Lagrangian approach to field theory, a massless photon is needed to keep the mass term $mA_\mu A_\mu$ invariant under gauge transformations. Even in this context, however, the Higgs mechanism, well accepted in unified field theory [1], provides the photon with mass from a Lagrangian which is originally *based* on gauge invariance of the second kind.

In this Letter, the emergence of the longitudinal field $\boldsymbol{B}^{(3)}$ of vacuum electrodynamics [4–12] is used to show conclusively that if there is a particulate photon, it is massive. A direct link is forged between $\boldsymbol{B}^{(3)}$ and m, the mass of the photon, and experimental evidence for $\boldsymbol{B}^{(3)}$ is obtained from plasma magnetization using microwave pulses, evidence based on the application of the Hamilton-Jacobi and Dirac equations to evaluate the spinning trajectory of one electron, e, in the electromagnetic field, represented by the usual classical four-potential A_μ. Weinberg [3] has shown that A_μ cannot be quantized for the massless photon because it corresponds to a (1/2, 1/2) irreducible representation of the Poincaré group. Such a representation is not allowed [3], however, for $m^2 = 0$, because in this case the helicity of the massless particle must be $\lambda = A - B$, where the irreducible representations are (A, B). Thus, we encounter the familiar difficulties in canonical quantization of A_μ in, for example, the Lorentz gauge, an indefinite metric, negative energies, unwelcome c numbers, and so forth. These are usually dealt with in textbooks with the Gupta-Bleuler formalism [1] of the early days of quantum field theory. Because of these well known difficulties, Weinberg [3] in his S matrix formalism chooses to avoid the use of the principle of least action and the Lagrangian formalism, and reaches the conclusion that all field equations, apart from the Klein-Gordon equation, are simply relations between particle spin components. If spin be *defined* through the Wigner little group, therefore, the only well defined field equations are those for a massive particle. In other words, if the E(2) little group of a hypothetically massless neutrino or photon is unphysical, as it must surely be, the Weyl and Maxwell equations *themselves* become unphysical. Expressed in yet another way, if the particle spin itself is unphysical (E(2) little group), any relation between components of the unphysical spin (Weyl or vacuum Maxwell equations) is also unphysical. The Weyl equations must be replaced by the Dirac equations, and the vacuum Maxwell equations by the Proca equations. This means that both the neutrino and photon are massive.

In Sec. 2, the emergence of $\boldsymbol{B}^{(3)}$ [4–12] is shown to make the $m^2 = 0$ assertion untenable, so that the concept of a massless photon must finally be abandoned. Sec. 3 discusses the ramifications within the Poincaré group of the idea of a massive photon. Finally, a discussion of precise conditions is given under which $\boldsymbol{B}^{(3)}$ and, by implication m are isolated unequivocally.

2. $\boldsymbol{B}^{(3)}$, LONGITUDINAL POLARIZATION, AND THREE PHOTON HELICITIES

The defining algebra of the vacuum $\boldsymbol{B}^{(3)}$,

$$\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)} = iB^{(0)}\boldsymbol{B}^{(3)*}, \quad \boldsymbol{B}^{(2)} \times \boldsymbol{B}^{(3)} = iB^{(0)}\boldsymbol{B}^{(1)*}, \\ \boldsymbol{B}^{(3)} \times \boldsymbol{B}^{(1)} = iB^{(0)}\boldsymbol{B}^{(2)*}, \tag{1}$$

is cyclically symmetric, non-Abelian [4–12], compact and semi-simple. Here, $\boldsymbol{B}^{(1)} = \boldsymbol{B}^{(2)*}$ is the vacuum plane wave, and $B^{(0)}$ is the scalar magnitude of the magnetic flux density (Tesla) of vacuum electromagnetic radiation. It is well known that the Lie algebra of E(2), on the other hand, is not cyclically symmetric, contains an Abelian subalgebra [1–3], is non-compact, and is non semi-simple. These troublesome features make E(2) unphysical. Equations (1) may appear a little unfamiliar because they are in the circular basis (1), (2) and (3), defined by the unit vectors

$$\boldsymbol{e}^{(1)} = \boldsymbol{e}^{(2)*} = \frac{1}{\sqrt{2}}(\boldsymbol{i} - i\boldsymbol{j}), \quad \boldsymbol{e}^{(3)} = \boldsymbol{k}. \tag{2}$$

Here $\boldsymbol{i}$, $\boldsymbol{j}$, and $\boldsymbol{k}$ are the usual Cartesian unit vectors.

However, the circular and Cartesian bases are both representations of ordinary three dimensional space, and Eqs. (1) form a Lie algebra of the Poincaré group [4–12], that of its rotation generators. In quantum mechanics this is also the algebra of the fundamental angular momentum operators. Any attempt to assert that $\boldsymbol{B}^{(3)}$ is unphysical or zero means throwing away a rotation generator or an angular momentum operator and destroying the cyclical symmetry of Eqs. (1). This is another way of saying that E(2) is unphysical, whereas O(3) and SU(2) are physical. The fundamental angular momentum operators of quantum mechanics are therefore *defined* by an O(3) or SU(2) little group. The angular momentum of the hypothetically massless photon, on the other hand, is defined by the E(2) little group, which is non-compact (because one commutator ($\{L_1, L_2\}$) is zero [1–3], and zero is not itself an E(2) group generator). However, Eqs. (1) are relations between angular momenta of the O(3) group, which is compact (the cyclically symmetric Lie algebra contains only rotation generators which produce each other in a symmetric manner). Therefore $\boldsymbol{B}^{(3)}$ is incompatible with the existence of the massless photon. We must abandon either the former or the latter. Since $\boldsymbol{B}^{(3)}$ was unknown prior to about 1992 [4], there is a vast amount of literature based on implicit acceptance of identically zero photon mass. There is, however, a persistent echo among several generations of thought [13] and experiment [14] that leads to the opposite conclusion.

We abandon the notion $m^2 =? 0$ because there is irrefutable, conclusive evidence that $\boldsymbol{B}^{(3)}$ can be observed experimentally [6] in principle through its characteristic $I_0^{1/2}$ profile, where I_0 is the power density (W m^{-2}) of a microwave pulse used to magnetize an electron plasma [15] in the condition

$$\omega \leq \frac{e}{m_0} B^{(0)} . \tag{3}$$

Here ω is the angular frequency (in rad s^{-1}) of the circularly polarized beam (e.g. 30 GHz microwaves [15]) and e/m_0 is the charge to mass ratio of the electron (about 2×10^{11} C kgm^{-1}). The $I_0^{1/2}$ profile in this condition is the direct result of the fundamental Hamilton-Jacobi equation itself, and so it is overwhelmingly probable that it will be duly observed experimentally. It has never been observed to date, as far as the author is aware at the time of writing, because the condition (3) has yet to be satisfied. As explained elsewhere [6] condition (3) can be achieved by straightforward modifications of the apparatus used by Deschamps *et al.* [15], and it is of *central* importance to carry out this experiment. No $I_0^{1/2}$ profile can be obtained from the plane wave $\boldsymbol{B}^{(1)} = \boldsymbol{B}^{(2)*}$ because its amplitude averages to zero at first order, and the $I_0^{1/2}$ profile therefore conclusively isolates $\boldsymbol{B}^{(3)}$ from $\boldsymbol{B}^{(1)} = \boldsymbol{B}^{(2)*}$. Non-observation of the $I_0^{1/2}$ profile under condition (3) would signal an unprecedented and inexplicable failure of the principle of least action. Thus, $\boldsymbol{B}^{(3)}$ turns out to be deeply rooted in classical dynamics, and its properties have by now been developed extensively [4–12]. It is a novel spin field of electromagnetism in the vacuum, and as just explained, an experimental observable.

The hypothetically massless photon, on the other hand, can never be an experimental observable, because its concomitant fields are infinite in range. The observable radius of the universe is finite because the radius can be measured, tautologically, only by using radiation from the most distant observable sources. The range of electromagnetic radiation is very great, but there can be no evidence that it is infinite, and there can never be experimental evidence that the photon mass is identically zero. Similar remarks apply to the neutrino and other hypothetically massless particles.

The three physical magnetic fields $\boldsymbol{B}^{(1)}$, $\boldsymbol{B}^{(2)}$ and $\boldsymbol{B}^{(3)}$ are directly proportional to the three rotation generators of O(3), and, within a factor $\hbar$, to the three angular momentum operators of quantum mechanics [4–12]. They are concomitant with the three space-like axes of a massive particle, which is the photon. Under the general Lorentz transformation this particle is described by a Wigner little group which is the physical O(3). To know the representations of the Lorentz group for $m^2 > 0$ we need to know [1–3] only the representations of O(3), which are directly proportional to $\boldsymbol{B}^{(1)}$, $\boldsymbol{B}^{(2)}$ and $\boldsymbol{B}^{(3)}$. There is therefore a direct link between the existence of $\boldsymbol{B}^{(3)}$ and that of m, because if m were identically zero the Wigner little group would be E(2). The latter's characteristic Abelian sub-algebra cannot occur in O(3). The experimentally verifiable existence of $\boldsymbol{B}^{(3)}$ therefore means that the photon's helicities are +1, 0, and -1. Furthermore, to know *all* the irreducible representations of the Lorentz group for the massive photon we need to know [1–3] *only* those proportional to $\boldsymbol{B}^{(1)}$, $\boldsymbol{B}^{(2)}$ and $\boldsymbol{B}^{(3)}$, i.e., the three rotation generators. If we throw away one of these, however, we no longer know the representations of the

Lorentz group, i.e., the structure of spacetime itself is destroyed if $\boldsymbol{B}^{(3)}$ is set to zero. From Eqs. (1), this procedure has the catastrophic result that $\boldsymbol{B}^{(1)}$ and $\boldsymbol{B}^{(2)}$ also disappear, so that we lose all electromagnetism. It has been demonstrated [6], moreover, that the interaction Hamiltonian of intrinsic electron spin ($\boldsymbol{S}$) with the electromagnetic field from the Dirac equation is, within a proportionality constant, simply the product of $\boldsymbol{S}$ and $\boldsymbol{B}^{(3)}$. Therefore if $\boldsymbol{B}^{(3)}$ were zero, $\boldsymbol{S}$ would not be measurable experimentally through the interaction of the quantized e and A_μ. Similarly, the interaction of the classical e with A_μ is governed entirely by $\boldsymbol{B}^{(3)}$ [6, 12] from the Hamilton-Jacobi equation. The very existence of $\boldsymbol{S}$ depends on that of $\boldsymbol{B}^{(3)}$ in the vacuum, because the anomalous magnetic moment of the electron emerges correctly from the Dirac equation [6, 16] *only* in one way, through the formation of an interaction Hamiltonian with $\boldsymbol{B}^{(3)}$. Thus $\boldsymbol{B}^{(3)}$ is to the photon as $\boldsymbol{S}$ is to the electron, an intrinsic and irremovable property.

We conclude therefore that $\boldsymbol{B}^{(1)}$, $\boldsymbol{B}^{(2)}$ and $\boldsymbol{B}^{(3)}$ are each, within a proportionality factor, irreducible representations of the O(3) little group of the Poincaré group, a little group which leaves the momentum-energy four-vector p_μ of the massive photon invariant under the most general Lorentz transformation. The latter applied to the massive photon is therefore defined by the concomitant fields $\boldsymbol{B}^{(1)}$, $\boldsymbol{B}^{(2)}$ and $\boldsymbol{B}^{(3)}$, and by these *alone*.

3. RAMIFICATIONS WITHIN THE POINCARÉ GROUP

The boost generators of the Poincaré group do not occur in the Wigner little group O(3) for the photon with mass, however minute [14] the latter may be in kilograms. (Photon mass is very small because the range of electromagnetism is very great experimentally; light reaches earth from far distant sources.) The fundamental reason for this is that the boost generators [1–3] cannot form a cyclically symmetric Lie algebra akin to (1). In simple vector language, the cross product of two polar vectors is an *axial* vector, not another polar vector, whereas the cross product of two axial vectors is another axial vector. The magnetic fields $\boldsymbol{B}^{(1)}$, $\boldsymbol{B}^{(2)}$ and $\boldsymbol{B}^{(3)}$ are axial vectors. The matrix form of the O(3) infinitesimal generator is a representation of an axial vector [4–12]. To emphasize this point, the algebra of vacuum electric fields akin to (1) is [4–12]

$$\boldsymbol{E}^{(1)} \times \boldsymbol{E}^{(2)} = -E^{(0)}(i\boldsymbol{E}^{(3)})^{*},$$
$$\boldsymbol{E}^{(2)} \times (i\boldsymbol{E}^{(3)}) = -E^{(0)}\boldsymbol{E}^{(1)*}, \quad (i\boldsymbol{E}^{(3)}) \times \boldsymbol{E}^{(1)} = -E^{(0)}\boldsymbol{E}^{(2)*}, \tag{4}$$

in which the longitudinal ((3)) component is pure imaginary and *unphysical*. Although several well known magneto-optic effects due to the conjugate product $iB^{(0)}\boldsymbol{B}^{(3)*}$ $(= \boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)})$ are known [7], there appears, significantly, to be no known effect due to a longitudinal, phase free, electric field in the vacuum. It has been shown [4–12] that electric fields must be proportional to boost generators within the Poincaré group associated with a massive photon, and since the latter cannot form an O(3) little group, neither can the electric fields. Clearly, the spin character (i.e., the little group) of the electromagnetic field is governed by its magnetic properties. The algebra (1) corresponds [4–12] to the rotation generator algebra [1],

$$[J_X, J_Y] = iJ_Z, \text{ et cyclicum,} \tag{5}$$

but (4) corresponds [4–12] to

$$[K_X, K_Y] = -iJ_Z, \text{ et cyclicum,} \tag{6}$$

where the $K's$ denote boost generators. Fundamentally, therefore, the emergence of $\boldsymbol{B}^{(3)}$ has had the effect of splitting the Lie algebra of fields into two distinct parts, namely (1) and (4). Without $\boldsymbol{B}^{(3)}$, the only fields present are $\boldsymbol{B}^{(1)} = \boldsymbol{B}^{(2)*}$, and $\boldsymbol{E}^{(1)} = \boldsymbol{E}^{(2)*}$, the ordinary transverse plane waves of the vacuum electromagnetic field, and Lie algebras of type (1) and (4) cannot exist. They collapse on to the Lie algebra that defines E(2), which, as first seen by Wigner [2], is *unphysical*. The E(2) algebra is made up of a mixture of J and K generators in such a way that if we define [1–3] the group generators

$$L_1 := K_1 - J_2, \qquad L_2 := K_2 + J_1, \tag{7}$$

we obtain

$$[L_1, L_2] = 0, \qquad [J_3, L_1] = iL_2, \qquad [L_2, J_3] = iL_1. \tag{8}$$

This is non semi-simple [2], non-compact, and has lost cyclical symmetry, in contrast to the Lie algebras (1) and

(4). The awkward and obscure algebra (8) is that of the Wigner little group for a massless photon, and by implication, is accepted daily in vast numbers of publications based on the assertion that the only fields in vacuum electromagnetism are the transverse plane waves. With the advent of $\boldsymbol{B}^{(3)}$ [4–12] this assertion becomes untenable, and the overall conclusion is the obvious one, that without $\boldsymbol{B}^{(3)}$ and $i\boldsymbol{E}^{(3)}$, the isolated existence of plane waves is also unphysical – the transverse $\boldsymbol{B}^{(1)}$ and $\boldsymbol{B}^{(2)}$ cannot exist in isolation of the longitudinal $\boldsymbol{B}^{(3)}$ and the transverse $\boldsymbol{E}^{(1)}$ and $\boldsymbol{E}^{(2)}$ in isolation of the longitudinal $i\boldsymbol{E}^{(3)}$. The axial vectors $\boldsymbol{B}^{(3)}$ and $i\boldsymbol{E}^{(3)}$ are both relativistically invariant but one is physical (i.e., real) and the other unphysical (i.e., pure imaginary). The ordinary plane waves $\boldsymbol{B}^{(1)} = \boldsymbol{B}^{(2)*}$ and $\boldsymbol{E}^{(1)} = \boldsymbol{E}^{(2)*}$ are complex (i.e., contain both real and imaginary parts).

It is necessary to emphasize the unphysical nature of E(2) because this is an irrecoverable fault in the special relativity of massless particles. If we try to associate such particles with fields, relativistic field theory becomes unphysical. The specifically Abelian feature of the Lie algebra of E(2), Eq. (8), shows up through the fact that the commutator of L_1 and L_2 does not produce a generator in the 3 axis orthogonal to the plane (1,2) of L_1 and L_2. This is so despite the fact that the generator J_3 appears in the other two commutators, and as shown elsewhere [4–12], is proportional to the field $\boldsymbol{B}^{(3)}$ in the vacuum. This demonstrates the internal inconsistency of E(2) and the concomitant Abelian electrodynamics it represents, because $\boldsymbol{B}^{(3)}$ is, as argued already, an experimental observable. This inference emerges from basic equations of motion in relativistic classical and quantum mechanics (Hamilton-Jacobi and Dirac equations respectively). The rotation generator J_3 appears in two out of three commutators of E(2), and $\boldsymbol{B}^{(3)}$ is directly proportional [6] to J_3. Therefore $\boldsymbol{B}^{(3)}$ also appears in these commutators, but does not appear in the first commutator on the right hand side. Since L_1 and L_2 are two of the basic generators of E(2) (J_3 being the third), this group cannot produce $\boldsymbol{B}^{(3)}$ self-consistently, and this group is also unphysical. The inference here is that $\boldsymbol{B}^{(3)}$ is a *physical* field, as deduced already from the fundamental relativistic equations of motion. The physical little group O(3) produces $\boldsymbol{B}^{(3)}$ self-consistently through the defining algebra (1).

4. SUMMARY

The existence of the magnetic field $B^{(3)}$ in the vacuum can be measured experimentally through the characteristic $I_0^{1/2}$ profile that it generates on magnetizing an electron plasma. Therefore $B^{(3)}$ is an observable field of vacuum electromagnetism, and its longitudinal polarization means that O(3) becomes a physically meaningful Wigner little group. The concept of E(2) and the massless photon are therefore unphysical.

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