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440

THE BIOT-SAVART-AMPÈRE LAW AND THE VACUUM FIELD $\boldsymbol{B}^{(3)}$

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It is shown that the longitudinal vacuum field $\boldsymbol{B}^{(3)}$ emerges from the Biot-Savart-Ampère law governing the motion of an electron with intrinsic spin moving at the speed of light, in which case the expression for $\boldsymbol{B}^{(3)}$ is identical with that obtained from the Dirac equation of one electron accelerated to the speed of light by an electromagnetic field. Use of an O(3), non-Abelian, gauge geometry for $\boldsymbol{B}^{(3)}$ identifies the quantized photon momentum $\hbar\kappa$ appearing in the Dirac equation with $eA^{(0)}$, where e is the charge on the electron and $A^{(0)}$ the amplitude of the vector potential. The condition $\hbar\kappa = eA^{(0)}$ can be obtained in turn from the relativistic Hamilton-Jacobi equation of an electron accelerated to the speed of light by an electromagnetic field.

Key words: $\boldsymbol{B}^{(3)}$ Field, Biot-Savart law.

1. INTRODUCTION

Detailed theoretical development [1–9] has supported the inference that there exists in the vacuum a longitudinal magnetic flux density $\boldsymbol{B}^{(3)}$, which is a spin field linked to the usual plane wave $\boldsymbol{B}^{(1)} = \boldsymbol{B}^{(2)*}$ by the cyclically symmetric Lie algebra,

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$$\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)} = iB^{(0)}\boldsymbol{B}^{(3)*}, \qquad \boldsymbol{B}^{(2)} \times \boldsymbol{B}^{(3)} = iB^{(0)}\boldsymbol{B}^{(1)*}, \qquad (1)$$
$$\boldsymbol{B}^{(3)} \times \boldsymbol{B}^{(1)} = iB^{(0)}\boldsymbol{B}^{(2)*}.$$

Here $B^{(0)}$ is the scalar amplitude of the magnetic flux density (tesla) of an electromagnetic plane wave propagating in axis (3) of a complex, circular representation [5,6] of three-dimensional space. It is shown in this Letter that the algebra (1) can be put in the form of the Biot-Savart-Ampère (BSA) law [10,11] for the source of a magnetic field. The source of $\boldsymbol{B}^{(3)}$ in this form of the BSA law is an electron with intrinsic spin moving at the speed of light, in which case its radiated electromagnetic fields become indistinguishable from those accompanying the (uncharged) photon. The magnetic components of these fields are $\boldsymbol{B}^{(1)}$, $\boldsymbol{B}^{(2)}$ and $\boldsymbol{B}^{(3)}$, each of Eqs. (1) being BSA laws. Thus $\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)}$, the conjugate product of nonlinear optics [1–9], can be considered as the source of $\boldsymbol{B}^{(3)}$. Similarly, $\boldsymbol{B}^{(2)} \times \boldsymbol{B}^{(3)}$ is the source of $\boldsymbol{B}^{(1)}$, and $\boldsymbol{B}^{(3)} \times \boldsymbol{B}^{(1)}$ that of $\boldsymbol{B}^{(2)}$.

In Sec. 2, the source of the experimentally observable [6,8] $\boldsymbol{B}^{(3)}$ is considered by hypothesis to take the analytical form of a BSA law, and it is shown that this hypothesis leads to the same expression for $\boldsymbol{B}^{(3)}$ as that derivable from the Dirac equation of an electron (e), with intrinsic spin, accelerated to the speed of light by an electromagnetic field (A_μ). In Sec. 3, $\boldsymbol{B}^{(3)}$ is expressed using an O(3) gauge geometry in terms of the vector cross product of complex vector potentials $\boldsymbol{A}^{(1)} = \boldsymbol{A}^{(2)*}$. Comparing this expression for $\boldsymbol{B}^{(3)}$ with that from the BSA law and Dirac's equation gives the charge quantization condition, $\hbar\kappa = eA^{(0)}$, in which the quantized momentum of the photon is identified with $eA^{(0)}$. Finally, in Sec. 4, it is shown that the charge quantization condition emerges from the relativistic Hamilton-Jacobi equation of e in A_μ when the electron is accelerated by the electromagnetic field to the velocity of light.

Therefore, an electron with intrinsic spin moving at the velocity of light through the vacuum radiates electromagnetic fields which are indistinguishable from those accompanying the uncharged photon. Under this condition, the novel vacuum field $\boldsymbol{B}^{(3)}$ is described by a BSA law equivalent to a spinning electron moving at the speed of light. The form of this law is shown to be that of Eq. (1a).

2. THE BSA HYPOTHESIS FOR $B^{(3)}$

Since $\boldsymbol{B}^{(3)}$ is an experimentally observable [6,8] magnetic flux density, we make the hypothesis that its source is a moving electron, and that in consequence it is described by a BSA law [12]

$$i\boldsymbol{B}^{(3)*} = \frac{1}{c^2}\boldsymbol{v}^{(1)} \times \boldsymbol{E}^{(2)}, \tag{2a}$$

where, in S.I. units, $\boldsymbol{v}^{(1)}$ is a transverse velocity in the complex basis,

$$i\boldsymbol{e}^{(3)*} = \boldsymbol{e}^{(1)} \times \boldsymbol{e}^{(2)}, \text{ et cyclicum}, \tag{2b}$$

and where $\boldsymbol{E}^{(2)}$ is a transverse electric field, a plane wave. As shown by Jackson [12], an electron accelerated to the speed of light radiates electromagnetic fields which are indistinguishable from those accompanying the photon, which is itself considered to be uncharged. The transverse velocity $\boldsymbol{v}^{(1)}$ is associated with the spinning motion of this electron, an intrinsic spin which also emerges from the Dirac equation [6].

Using a transverse gauge [12], the plane wave $\boldsymbol{E}^{(2)}$ can be expressed as

$$\boldsymbol{E}^{(2)} = i\omega\boldsymbol{A}^{(2)} = -\frac{\partial \boldsymbol{A}^{(2)}}{\partial t} \tag{3}$$

in terms of the plane wave $\boldsymbol{A}^{(2)}$, a vector potential [1-9] in the vacuum. Similarly, the complex conjugate $\boldsymbol{E}^{(1)}$ can be expressed through the complex conjugate $\boldsymbol{A}^{(1)}$,

$$\boldsymbol{E}^{(1)} = -i\omega\boldsymbol{A}^{(1)} = -\frac{\partial \boldsymbol{A}^{(1)}}{\partial t}, \tag{4}$$

and therefore Eq. (1a) of the defining Lie algebra of $\boldsymbol{B}^{(3)}$ can be expressed as a cross product of electric fields or as a cross product of vector potentials in the vacuum,

$$\boldsymbol{B}^{(3)*} = -\frac{i}{c^2 B^{(0)}}\boldsymbol{E}^{(1)} \times \boldsymbol{E}^{(2)} = -i\frac{\kappa^2}{B^{(0)}}\boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)}. \tag{5}$$

Using Eq. (2b) in Eq. (2a), our BSA hypothesis becomes expressible as

$$B^{(3)*} = -\frac{\kappa}{c} v^{(1)} \times A^{(2)} \tag{6}$$

where κ is the scalar magnitude of the classical wave-vector. The factor $-\kappa v^{(1)}/c$ in Eq. (6) therefore has the units of a wavevector, which, using the de Broglie wave-matter duality [6], can be expressed as a momentum operator in quantum mechanics ($\hbar\kappa = ip$),

$$B^{(3)*} = \frac{i}{\hbar} p^{(1)} \times A^{(2)}, \tag{7}$$

where $\hbar$ is the Dirac constant. Therefore the BSA hypothesis results in Eq. (17) with the use of the de Broglie wave-matter relation between the classical wavevector and the quantized momentum operator. This is, of course, a fundamental axiom of quantum mechanics.

In our hypothesis (2a), the quantized transverse momentum $p^{(1)}$ is that [6] of a spinning electron moving at the speed of light.

The standard demonstration of the existence of intrinsic electron spin in relativistic quantum field theory rests on the application [6, 13] of the Dirac equation to the motion of e in A_μ. It has been shown [6, 14] that the intrinsic electron spin forms an interaction Hamiltonian with $B^{(3)}$, and with $B^{(3)}$ *only*. No other component field of vacuum electromagnetism is involved. Moreover, the field $B^{(3)}$ is given by an expression which is identical with (7), in which $p^{(1)}$ is, however, the original momentum operator of an electron in the field-free Dirac equation. This shows that the BSA hypothesis gives a result for $B^{(3)}$ which is identical with that obtained [6, 14] from the Dirac equation describing the interaction of the quantized intrinsic spin of the electron in the classical electromagnetic field. When this electron translates at c, the source of $B^{(3)}$ becomes the BSA Eq. (1a), which is also derivable from the Dirac equation of an electron accelerated to c by an electromagnetic field.

3. THE CHARGE QUANTIZATION CONDITION

Since $B^{(3)}$ is longitudinal and $B^{(1)} = B^{(2)*}$ is transverse, it follows that an O(3) gauge group [6, 15] is necessary for self-consistency, because the conventional O(2) gauge group [13] necessarily prohibits consideration of a physical field such as $B^{(3)}$ perpendicular to the plane defined by O(2). The non-Abelian theory [6] of vacuum electrodynamics then leads directly to an expression for $\boldsymbol{B}^{(3)}$ of the form

$$\boldsymbol{B}^{(3)*} = -i\frac{e}{\hbar}\boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)}, \tag{8}$$

in which appears the electron charge e. Comparison of this with Eq. (7) introduces the charge quantization condition [6, 16]

$$\hbar\kappa = eA^{(0)}, \tag{9}$$

in which the quantized vacuum photon momentum $\hbar\kappa$ is identified with $eA^{(0)}$. The physical meaning of this is elucidated in Sec. 4; suffice it to mention here that it emerges as a direct result of the BSA hypothesis and the use of a self-consistent gauge geometry O(3) for vacuum electromagnetism. Condition (9) emerges independently from Eq. (5), which expresses $\boldsymbol{B}^{(3)*}$ in terms of the conjugate product $\boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)}$ of vector potentials. For an electron moving at c, therefore, the BSA law is equivalent to the expression of $\boldsymbol{B}^{(3)*}$ in terms of conjugate *field* products such as $\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)}$, $\boldsymbol{E}^{(1)} \times \boldsymbol{E}^{(2)}$ or $\boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)}$. Therefore, these products become the *source* of $\boldsymbol{B}^{(3)}$ in the vacuum, as discussed in the introduction. Note that cross products such as these can act as sources only for a magnetic field, which is consistent with the fact that there is no real longitudinal electric field akin to $\boldsymbol{B}^{(3)}$ [6]. Similarly, Jackson [12] has shown that an electron moving at c cannot radiate a measurable longitudinal electric field in its direction of propagation. Jackson, however, did not consider intrinsic electron spin, because his theory is classical [12] and did not consider $\boldsymbol{B}^{(3)}$.

4. PHYSICAL MEANING OF THE CHARGE QUANTIZATION CONDITION (9)

Through the use of the transverse gauge for $\boldsymbol{A}^{(1)}$ and $\boldsymbol{A}^{(2)}$, the relativistic Hamilton-Jacobi equation can be used [17] to show that the electromagnetic field induces an orbital electronic angular momentum which appears in addition to the intrinsic spin angular momentum used in Secs. 2 and 3. This is also governed *entirely* by $\boldsymbol{B}^{(3)}$,

$$J_{el}^{(3)} = \frac{e^2c^2}{\omega^2}\left(\frac{B^{(0)}}{(m^2\omega^2 + e^2B^{(0)2})^{\frac{1}{2}}}\right)B^{(3)}, \tag{10}$$

where m is the mass of the electron. We consider the condition

$$\omega \ll \frac{e}{m}B^{(0)}, \tag{11}$$

in which the field is intense but of low angular frequency. In condition (11), the total energy [18] acquired by the electron becomes

$$En = \omega\left|J_{el}^{(3)}\right| \rightarrow \frac{ec^2}{\omega}B^{(0)}, \tag{12a}$$

$$J_{el}^{(3)} \rightarrow \frac{ec^2}{\omega^2}B^{(0)} \tag{12b}$$

and has been given up to the electron by the field. Considering the limit in which energy is given up *entirely* to the electron by the field, and representing the field energy by the quantized photon $\hbar\omega$, we obtain [17]

$$\frac{ec^2B^{(0)}}{\omega} = \hbar\omega. \tag{13}$$

Conservation of energy means that the photon energy has been lost and given up entirely to the electron. Conservation of linear momentum means that the electron must be accelerated to the speed of light in an elastic collision with the photon of energy $\hbar\omega$ and momentum magnitude $\hbar\kappa$. Conservation of angular momentum [18] means that the angular momentum of the photon has been given up entirely to the electron, resulting in Eq. (12b) from the relativistic Hamilton-Jacobi description [17] of this process.

Using [6] $B^{(0)} = \kappa A^{(0)}$ and $\kappa = \omega/c$, Eq. (13) emerges as the charge quantization condition (9), which therefore describes the magnitude of the quantized linear momentum $\hbar\kappa$ of a photon in terms of that of the classical linear momentum $eA^{(0)}$ of an electron accelerated to the speed of light.

5. DISCUSSION

It has been shown that the Lie algebra (1) is equivalent to three BSA laws tied together in a circular basis in free space. This inference identifies the source of $\boldsymbol{B}^{(3)}$ in terms accessible to classical magnetism [12]. The charge associated with these three BSA laws is the electron with intrinsic spin moving at the speed of light, in which case its radiated fields are indistinguishable from the electromagnetic fields (including $\boldsymbol{B}^{(3)}$) accompanying the uncharged photon in vacuo. The BSA hypothesis (2a) produces an equation for the vacuum $\boldsymbol{B}^{(3)}$ that is identical with that obtained [6] from the Dirac equation of e in A_μ. The latter in turn produces the charge quantization condition (9) when care is taken to use a self-consistent, non-Abelian gauge geometry for vacuum electrodynamics [6]. The charge quantization condition is derived independently from the Hamilton-Jacobi equation for the relativistic trajectory of e in A_μ, provided that the field energy is quantized to give the quantum of energy $\hbar\omega$ (the photon), which is annihilated in a purely elastic collision with an electron. Conservation laws lead directly to the condition $eA^{(0)} = \hbar\kappa$, in which charge occurs in units of $\hbar$ in the same way as energy and linear momentum.

The major conclusion of this Letter is that the source of $\boldsymbol{B}^{(3)}$ in the vacuum is the conjugate product $\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)}$; a statement of the BSA law.

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