Paper 8

Relativistic Magneto-Optics and the Evans-Vigier Field

Relativistic effects in magneto-optics are discussed in order to isolate experimentally the newly inferred Evans-Vigier field of vacuum electromagnetism. The discussion is reduced to its simplest form by considering the interaction of one photon with one electron over the complete range of photon energy and momenta transfer. It is shown that there are three regions: a) quadratic, b) linear and c) region of saturation; into which the characteristics of the interaction process can be divided. In region b), the angular momentum imparted to the electron by the photon is linearly dependent on the Evans-Vigier field $\boldsymbol{B}^{(3)}$, and this region is accessible experimentally with microwave pulses of sufficient power density. In magneto-optics with visible lasers, only region a) is accessible, and this is shown to be the non-relativistic limit.

Key words: relativistic magneto-optics, Evans-Vigier field.

8.1 Introduction

Magneto-optic effects received great impetus from the discovery of the laser, but their existence was inferred earlier [1]. They have recently been reviewed comprehensively by Zawodny [2], using the accepted semi-classical approach [3]. It is shown in this Letter that this is the non-relativistic limit, suitable for application in what we infer here to be the *quadratic region* (a). This region is characterized by a relatively low beam power density (I_0 in W m^{-2}) and high beam angular frequency (ω in rad sec^{-1}), conditions which obtain in the visible for all but the most powerful pulses. Under these conditions, the semi-classical approach (which is based on the non-relativistic Schrödinger equation [4]) shows that magnetization by light (the inverse Faraday effect [5,6]) is proportional to I_0. This has been verified experimentally in glasses and liquids [6] and in an electron plasma [7]. For many years, therefore, the theory has been thought of as complete, and magnetization by circularly polarized light has been ascribed to the well known conjugate product [8], the antisymmetric component of the light intensity tensor.

In this Letter it is shown that the above represents only the non-relativistic limit of magneto-optics. It is shown by considering the collision of a photon with an electron that as the beam power density is increased, and the beam angular frequency decreased to the microwave range, the magnetization of the inverse Faraday effect becomes proportional to $I_0^{1/2}$, which means that it is proportional at first order to a magnetic field $\boldsymbol{B}^{(3)}$, the newly inferred [9—14] Evans-Vigier field of vacuum electromagnetism. This is labeled as *region b)* or *linear region*, and can be described only with a correctly relativistic theory. As the beam power density is increased further for a given angular frequency, the curve of magnetization versus I_0 saturates, and we enter the *region of saturation*, region c). In this limit the magnetization is constant for all I_0. This is a purely relativistic phenomenon, with no non-relativistic meaning, and can be understood simply because the maximum angular momentum that the photon (at any beam frequency) can transfer to the electron is $\hbar$, the Dirac constant. In a

perfectly elastic transfer of angular momentum, conservation demands that the angular momentum, $\hbar$, of the photon be annihilated and given up completely to the electron. This process, although allowed theoretically, requires enormous beam power densities.

In Sec. 8.2, the classical, but relativistic, Hamilton-Jacobi equation of one electron (e) in the electromagnetic field, represented by the potential four-vector A_μ, is used to illustrate the existence of regions a) and b), and to infer that region a) is the non-relativistic limit. Sec. 8.3 describes the development of region c), by quantizing the field into energy quanta, photons. Thereafter, Sec. 8.4 uses simple Compton effect theory to illustrate the existence of regions a), b) and c) in energy and linear momentum transfer from photon to electron.

8.2 Classical, Relativistic, Magneto-Optics

The correctly relativistic, but classical, theoretical basis for magneto-optics can be developed in a relatively simple way by using the Hamilton-Jacobi equation [15]. The trajectory of one electron in the electromagnetic field can be shown [15] to be governed by a classical, orbital, angular momentum,

$$\boldsymbol{J}^{(3)} = \frac{e^2c^2}{\omega^2}\left(\frac{B^{(0)}}{\left(m_0^2\omega^2 + e^2B^{(0)2}\right)^{1/2}}\right)\boldsymbol{B}^{(3)} . \tag{2.8.1}$$

Here e/m_0 is the charge to mass ratio of the electron, and $B^{(0)}$ the magnetic flux density amplitude of the beam. The magnetization induced by the beam is therefore due entirely to the Evans-Vigier field $\boldsymbol{B}^{(3)}$

$$\boldsymbol{M}^{(3)} = -\frac{e}{2m_0}\boldsymbol{J}^{(3)}. \tag{2.8.2}$$

In the condition,

$$\omega = \frac{e}{m_0} B^{(0)}, \tag{2.8.3}$$

this result becomes a sum of two terms [15],

$$\boldsymbol{M}^{(3)} = \frac{1}{2\sqrt{2}} \left(\chi' + B^{(0)} \beta'' \right) \boldsymbol{B}^{(3)}, \tag{2.8.4}$$

where

$$\chi' := -\frac{e^2 c^2}{2 m_0 \omega^2} \tag{2.8.5}$$

is the one electron susceptibility and where

$$\beta'' := -\frac{e^3 c^2}{2 m_0^2 \omega^3} \tag{2.8.6}$$

is the one electron hyperpolarizability. The relativistic factor of the Hamilton-Jacobi equation can be expressed in terms of the momentum magnitude $p = eA^{(0)}$, where $A^{(0)}$ is the amplitude of the vector potential of the beam,

$$\gamma := \frac{c}{\omega} \left(m_0^2 \omega^2 + e^2 B^{(0)2} \right)^{1/2} = \frac{1}{c} \left(m_0^2 c^4 + c^2 p^2 \right)^{1/2}. \tag{2.8.7}$$

In the non-relativistic limit, the speed of light is much greater than the speed, v, imparted to the electron (Sec. 8.4) by a collision with the photon. This means that

$$m_0 c \gg p, \tag{2.8.8}$$

a limit which corresponds to

$$\omega \gg \frac{e}{m_0} B^{(0)}, \tag{2.8.9}$$

i.e., the angular frequency of the beam is much greater than $eB^{(0)}/m_0$. For all but the most enormous laser power densities, this is always true in the visible, whereupon the expression for the magnetization in the one electron inverse Faraday effect becomes

$$\boldsymbol{M}^{(3)} \underset{c \gg v}{\rightarrow} \frac{\beta''}{2} B^{(0)} \boldsymbol{B}^{(3)}, \tag{2.8.10}$$

which is quadratic in $B^{(0)}$. Our quadratic region a) is therefore defined as the non-relativistic region of the interaction of one electron with the classical electromagnetic field. The result a) becomes recognizable in the conventional semi-classical theory [8] when the one electron hyperpolarizability is replaced by a semi-classical atomic or molecular hyperpolarizability, usually calculated [8] from a perturbation theory, using a quantum approach for the atom or molecule and a classical view of the field [4]. The only conceptual difference is that the atomic or molecular property tensor contains resonance features, and the free electron equivalent does not.

The classical version, $\boldsymbol{B}^{(3)}$, of the Evans-Vigier field therefore governs the inverse Faraday effect in atoms and molecules as well as in an electron plasma.

In the usual, non-relativistic, theory [8], however, the term linear in $\boldsymbol{B}^{(3)}$ is missing completely. This term becomes *dominant,* however, in the condition,

$$\omega \ll \frac{e}{m_0} B^{(0)}, \tag{2.8.11}$$

which obtains with microwave pulses [17,15] of sufficient power density. Under condition (2.8.11) we expect $\boldsymbol{M}^{(3)}$ to be proportional to $I_0^{1/2}$, and not to I_0 as in the visible. This expectation has yet to be verified experimentally, but it is nevertheless based on first principles. When $\boldsymbol{M}^{(3)}$ is linearly proportional to $\boldsymbol{B}^{(3)}$, we enter the linear region b). We cannot enter this region if our theory is non-relativistic, i.e., based on perturbation theory applied to the non-relativistic Schrödinger equation. The latter represents the usual semi-classical theory of the inverse Faraday effect. A more plausible semi-classical approach is one based on perturbation theory applied to the relativistic Dirac equation for free electrons, atoms and molecules. If done properly, this should correctly quantize our classical result (2.8.1), and extend it to atoms and molecules as well as the single free electron for which it is valid.

8.3 The Region of Saturation, c)

As the power density of the electromagnetic beam is increased in region b), it might be expected that $\boldsymbol{M}^{(3)}$ will simply increase indefinitely with $\boldsymbol{B}^{(3)}$. Special relativity shows that this is not the case, however, because when the electromagnetic field is quantized, the angular momentum of the photon is constant, $\hbar$ [15]. The law of conservation of angular momentum asserts that in a photon electron collision, the angular momentum transferred to the electron from the photon cannot exceed $\hbar$. Similarly, the energy and linear momentum transferred cannot exceed $\hbar\omega$ and $\hbar\kappa$ respectively, where $\kappa = \omega/c$. Anticipating the quantized field theory, the maximum $\boldsymbol{J}^{(3)}$ from Eq. (2.8.1) is $\hbar\boldsymbol{e}^{(3)}$, where $\boldsymbol{e}^{(3)}$ is a unit vector [12],

$$\boldsymbol{J}^{(3)}_{\max} \rightarrow \hbar\boldsymbol{e}^{(3)}, \tag{2.8.12}$$

and the theory enters region c), the region of saturation. In region c), the magnetization in the inverse Faraday effect no longer depends on I_0. The physical meaning of this is that in the photon-electron collision, the $\hbar$ of the photon has been given up entirely to the electron in a perfectly elastic transfer of angular momentum.

Equation (2.8.12) can be obtained from Eq. (2.8.1) by using the charge quantization condition [15],

$$\hbar\kappa = eA^{(0)}, \tag{2.8.13}$$

in Eq. (2.8.1). In Eq. (2.8.13), the classical momentum magnitude $eA^{(0)}$ imparted to the electron by the field is identified with the quantized photon momentum, $\hbar\kappa$. Equation (2.8.1) becomes

$$\boldsymbol{J}^{(3)} = \frac{\hbar\kappa}{\left(m_0^2c^2 + \hbar^2\kappa^2\right)^{1/2}} \frac{ec^2}{\omega^2}\boldsymbol{B}^{(3)} . \tag{2.8.14}$$

This result clearly identifies the Evans-Vigier field $\boldsymbol{B}^{(3)}$ as solely responsible for the inverse Faraday effect. In the limit $\hbar\kappa \gg m_0c$, which corresponds with $eB^{(0)} \gg m_0\omega$, Eq. (2.8.14) becomes

$$\boldsymbol{J}^{(3)} \rightarrow \frac{e}{\kappa^2}\boldsymbol{B}^{(3)}, \tag{2.8.15}$$

and using $B^{(0)} = \kappa A^{(0)}$

$$J^{(3)} \rightarrow \frac{eA^{(0)}}{\kappa} e^{(3)} = \hbar e^{(3)} . \tag{2.8.16}$$

This identifies the transition from region b) (Eq. (2.8.15)) to region c) (Eq. (2.8.16)). Note that this transition comes about through Eq. (2.8.13), which means that the maximum magnitude of the linear momentum imparted to the electron by the field is that of the photon. The law of conservation of momentum shows that this occurs in a perfectly elastic transfer of linear momentum $\hbar\kappa$ from the photon to the electron. The physical meaning of this is developed in the next section.

8.4 Regions A), B) and C) in Compton Theory

In this section, energy and linear momentum transfer in the photon-electron collision is analyzed with the simplest type of Compton theory. The key feature of the Compton effect is that the collision changes the frequency of the photon from ω_i to ω_f, providing early evidence for the light quantum hypothesis. Accordingly, the conservation of energy demands that [16]

$$\Delta En = \hbar\left(\omega_i - \omega_f\right) = \left(p^2c^2 + m_0^2c^4\right)^{1/2} - m_0c^2 , \tag{2.8.17}$$

where $\boldsymbol{p}$ is the linear momentum given to the electron (initially at rest) by the photon. Quantities on the left hand side of Eq. (2.8.17) refer to the photon, and those on the right hand side to the electron, whose rest energy is m_0c^2. Similarly, conservation of linear momentum demands that

$$\hbar\left(\kappa_i - \kappa_f\right) = p . \tag{2.8.18}$$

If we consider the limit $\hbar\omega_i \gg m_0c^2$, i.e., the collision of a very energetic photon with an initially stationary electron; and if the photon gives

up its energy entirely to the electron, then $\omega_f = 0$. The translational kinetic energy acquired by the electron is such that $pc \gg m_0c^2$, and so

$$\hbar\omega_i \sim pc . \tag{2.8.19}$$

The same result is obtained from Eq. (2.8.18) by assuming that $\kappa_f = 0$. i.e that the linear momentum of the photon is zero after collision and has been transferred elastically to the electron. (In an elastic collision, kinetic energy and linear momentum are both conserved.) In this limit,

$$\hbar\kappa_i \sim p , \tag{2.8.20}$$

and using $\kappa = \omega/c$ for the photon, Eq. (2.8.19) emerges from Eq. (2.8.20). These two equations show that in this limit, corresponding to the region of saturation, c), the photon and electron have become kinematically indistinguishable. This occurs when the linear momentum and energy transfer is such that the electron is accelerated towards c. In this condition, its momentum can no longer be related to its velocity through an equation such as $p \stackrel{?}{=} m_0c$. If such a relation is tried in Eq. (2.8.19), there emerges $\hbar\omega_i \stackrel{?}{=} m_0c^2$, which contradicts the initial assumption $\hbar\omega_i \gg m_0c^2$. The electron traveling at c loses its mass, but retains the momentum $\hbar\kappa_i = p$, and so Newtonian concepts become inapplicable. This difficulty is inherent in the axioms of special relativity themselves - the concept of mass loses meaning in a particle traveling at c. The obvious conclusion is that neither the photon nor the electron can be regarded as particles without mass, and neither can travel at c, only infinitesimally near c. This conclusion is reinforced rigorously by the emergence [15] of the Evans-Vigier field $\boldsymbol{B}^{(3)}$, which implies that the photon has three degrees of space polarization. Its Wigner little group [15] is therefore *O(3)* and it cannot be massless. An electron traveling infinitesimally near c is well known [17] to be concomitant with electromagnetic plane waves which are indistinguishable at these velocities from those concomitant with the photon. A plot of electron kinetic energy verses its momentum is a constant in region c).

Region a), the quadratic region, refers in this context to very low energy photons, so $\omega_i \sim \omega_f$ in Eq. (2.8.17) and $v \ll c$ where v is the electron's speed, acquired in a collision with the incoming photon. If $\omega_i \sim \omega_f$, Eq. (2.8.17) shows that

$$m_0 c^2 \sim \left(p^2c^2 + m_0^2 c^4\right)^{1/2}, \tag{2.8.21}$$

and so $p \ll m_0 c$, i.e., $v \ll c$ if $p = m_0 v$. Therefore the electron's kinetic energy in region a) is the non-relativistic $1/2\left(m_0 v^2\right)$, and the plot of kinetic energy against linear momentum is quadratic.

An intermediate region b) develops in which the kinetic energy is linearly proportional to its momentum. Therefore simple Compton theory parallels the qualitative features of the relativistic inverse Faraday effect. The existence of region b) can be inferred as follows. The relativistic kinetic energy is [18]

$$T = En - m_0 c^2, \tag{2.8.22}$$

where

$$En^2 = c^2 p^2 + m_0^2 c^4, \tag{2.8.23}$$

and where the relativistic momentum is $\boldsymbol{p} = \gamma_{|} m_0 \boldsymbol{v}$. If we assume that the rest energy, $m_0 c^2$, is small compared with cp, then

$$T \sim cp = \gamma_{|} m_0 v c := \left(1 - \frac{v^2}{c^2}\right)^{-1/2} m_0 v c, \tag{2.8.24}$$

so that the kinetic energy is directly proportional to the relativistic momentum, and approximately proportional to the classical momentum, $m_0 \boldsymbol{v}$, if v/c is still fairly small. The latter condition holds in region b).

8.5 Discussion

The inverse Faraday effect and related magneto-optic effects, when detected with visible frequency light, appear to be proportional to the light intensity I_0, because we are in region a), the non-relativistic limit. The nearest approach to region b) to date appears to have been the experiment of Deschamps *et al.* [7], with microwave pulses. It has been shown [19] that the conditions in this experiment correspond, for 3 GHz pulse, to

$$\omega \sim 5\frac{e}{m_0}B^{(0)}, \tag{2.8.25}$$

and so (from Eq. (2.8.1)) we are still in region a). The experimental demonstration of the existence of region b), and of the characteristic $I_0^{1/2}$ profile of the Evans-Vigier field, requires an experiment of the type carried out by Deschamps *et al.* [7], but with a peak microwave pulse power density about two orders of magnitude greater. With contemporary technology this is entirely feasible. Related magneto-optic phenomena should also enter region b) under the right experimental conditions, i.e., high beam power density and low beam frequency. Laser sources in the visible have the opposite characteristics, i.e., high frequency compared with power density, and so produce region a), the non-relativistic limit. So magneto-optic phenomena of this kind [2] have always been seen to be proportional to I_0.

The approach to region c) appears with contemporary technology to be very difficult, because it requires enormous pulse power density in comparison to beam frequency.

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