Paper 17

Maxwell's Vacuum Field — a Rotating Charge

The rotating electric field of a propagating Maxwellian plane wave in vacuo is shown to be a rotating dipole, with one tip fixed at the origin. At the other there is located a rotating charge, whose helical trajectory forms the Evans-Vigier field $\left(\boldsymbol{B}^{(3)}\right)$ in perfect analogy with a solenoid. The length of the dipole is $r_0 = \left(V/(4\pi\alpha)\right)^{1/3}$, where V is the radiation volume and α the fine structure constant. The quantum of electromagnetic radiation (the photon) is, consequently, $\hbar\omega = (4\pi\alpha V)^{3/2} e^2/\epsilon_0$, where ϵ_0 is the vacuum permittivity.

Key words: Maxwellian vacuum field, rotating charge, Evans-Vigier field.

17.1 Introduction

The recent emergence [1—12] of the classical Evans-Vigier field, $\boldsymbol{B}^{(3)}$, the spin field of vacuum electromagnetism, has led to the charge quantization condition,

$$\hbar\kappa = eA^{(0)}, \tag{2.17.1}$$

where the de Broglie photon momentum, $\hbar\kappa$, is expressed as $eA^{(0)}$, where e is the charge on the electron and $A^{(0)}$ the scalar potential in vacuo. Here $\hbar$ is the Dirac constant as usual and κ the magnitude of the classical wavevector in vacuo. In this Letter it is shown that Eq. (2.17.1) is a straightforward consequence of the existence of the rotating Maxwellian electric field in the vacuum, and that Eq. (2.17.1), derived independently [6,12], using generalized gauge theory, is a consistent outcome of standard field theory. This proves beyond reasonable doubt that $\boldsymbol{B}^{(3)}$ is also consistent in field theory, and is a novel fundamental property of electromagnetic radiation in the vacuum.

In Sec.17.2, it is shown that Eq. (2.17.1) emerges directly from the link between an electric field strength (V m^{-1}) and a dipole moment. In *S.I.* units, this link is

$$\boldsymbol{\mu} = \epsilon_0 V \boldsymbol{E}, \tag{2.17.2}$$

where V has the units of volume, and where ϵ_0 is the vacuum permittivity. In the vacuum, there is no material polarization, and V is the volume occupied by classical electromagnetic radiation in vacuo. In Sec. 17.3, it is shown that the length of the dipole is

$$r_0 = \frac{1}{\kappa} = \left(\frac{V}{4\pi\alpha}\right)^{1/3} = \frac{\lambda}{2\pi}, \tag{2.17.3}$$

where α is the fine structure constant [13] and λ is the wave length; the quantum of electromagnetic radiation (the photon) is

$$\hbar\omega = \frac{(4\pi\alpha V)^{3/2} e^2}{\epsilon_0}, \tag{2.17.4}$$

where e is the charge on the electron. The photon is therefore proportional to the square of e, and this is the origin of the textbook assertion [14] that the photon is uncharged. This assertion is seen through Eq. (2.17.4) to be true only in the narrowest of senses: the photon only *appears* to be uncharged because it is proportional to the *square* of the electronic charge. Section 17.4 is a discussion of these findings.

17.2 Derivation of the Charge Quantization Condition

An electric dipole moment is a separation of opposite charge [15]. An electric field is generated by separated opposite charge, or by a single charge, as in Coulomb's law. The elementary charge is that on the electron, of magnitude e, and so

$$\boldsymbol{E}^{(1)} = \frac{\boldsymbol{\mu}^{(1)}}{\left(\epsilon_0 V\right)} = \frac{e\boldsymbol{r}^{(1)}}{\epsilon_0 V}, \tag{2.17.5}$$

where $\boldsymbol{E}^{(1)}$ is Maxwell's rotating electric field in electromagnetic radiation propagating through the vacuum [15]. In Eq. (2.17.5), $r^{(0)} := |\boldsymbol{r}^{(1)}|$ is the length of the dipole formed by these separated charges in vacuo. If V is the radiation volume, it is easily checked that Eq. (2.17.5) is dimensionally and physically consistent in the theory of electrodynamics [15]. The electromagnetic field propagates in the axis perpendicular to the plane, and so the negative rotating charge draws out a helical path through the vacuum. In perfect analogy with the solenoid, this movement produces a magnetic flux density $\boldsymbol{B}^{(3)}$ in the axis of propagation, and this is the Evans-Vigier field [6]. The origin of $\boldsymbol{B}^{(3)}$ becomes perfectly clear in classical electrodynamics.

It has been shown elsewhere [6,12] that the existence of $\boldsymbol{B}^{(3)}$ means that

$$\hbar\kappa = eA^{(0)}, \tag{2.17.6}$$

where $\hbar\kappa$ is the magnitude of the de Broglie photon momentum. Equation (2.17.6) appears at first to be unorthodox, in that it expresses $\hbar\kappa$ in terms of e, the charge on the electron. However, Eq. (2.17.6) is easily derived from Eq. (2.17.5) as follows.

From Eq. (2.17.5), the scalar magnitude of the rotating Maxwellian $\boldsymbol{E}^{(1)}$ (V m^{-1}) is

$$E^{(0)} = \frac{er^{(0)}}{\epsilon_0 V} . \tag{2.17.7}$$

The classical electromagnetic energy in the volume V is [14]

$$En = \epsilon_0 E^{(0)2} V, \tag{2.17.8}$$

where

$$V := \int_0^V dV, \tag{2.17.9}$$

and so the energy in volume V is expressible in terms of the radius $r^{(0)}$,

$$En = eE^{(0)} r^{(0)} = \frac{e^2 r^{(0)2}}{\epsilon_0 V} . \tag{2.17.10}$$

Thus far, the development has been entirely classical. Using the classical relation between $A^{(0)}$ and $E^{(0)}$ [6],

$$A^{(0)} = \frac{E^{(0)}}{\omega} , \tag{2.17.11}$$

it is found that

$$En = \left(eA^{(0)} r^{(0)} \right) \omega, \tag{2.17.12}$$

i.e., electromagnetic energy is proportional to electromagnetic angular frequency as radiation propagates in vacuo. Provided we make the identity

$$\hbar := eA^{(0)} r^{(0)} , \tag{2.17.13}$$

Eq. (2.17.12) is the Planck-Einstein relation of quantum theory, the light quantum hypothesis. *It has been shown that the Light Quantum hypothesis has a purely classical origin.*

Finally, the identification of $r^{(0)}$ as κ^{-1} results in the charge quantization condition that we are seeking to derive. The length of the rotating dipole, and therefore of the rotating Maxwellian electric field, is therefore the inverse of the wavevector magnitude of the radiation. *We have identified the origin of the Planck constant itself.*

17.3 The Dipole Radius and Photon

Using Eq. (2.17.6) with the equation

$$A^{(0)} = \left(\frac{c}{\epsilon_0 \omega^2 V} \right) e , \tag{2.17.14}$$

the quantum of electromagnetic energy, $\hbar\omega$, is defined as

$$\hbar\omega = \frac{e^2}{\epsilon_0 \kappa^2 V} , \tag{2.17.15}$$

and is proportional to e^2. This is why the particulate photon is *uncharged*. This result can now be expressed in terms of the fine structure constant [13, 16],

$$\alpha := \frac{e^2}{4\pi c \epsilon_0 \hbar} . \qquad (2.17.16)$$

From Eqs. (2.17.15) and (2.17.16), the fine structure constant becomes expressible as

$$\alpha = \frac{V}{4\pi r_0^3} , \qquad (2.17.17)$$

and so the radius of the rotating dipole is

$$r_0 = \frac{1}{\kappa} = \left(\frac{V}{4\pi\alpha} \right)^{1/3} = \frac{\lambda}{2\pi}, \qquad (2.17.18)$$

so that the photon defined in equation (2.17.15) becomes expressible as

$$\hbar\omega = (4\pi\alpha V)^{3/2} \frac{e^2}{\epsilon_0} . \qquad (2.17.19)$$

17.4 Discussion

The rotating dipole is consistent with special relativity and with Maxwell's equations, because it is derived simply by re-expressing (Eq. (2.17.2)) the usual rotating electric field in terms of separated charge. The positive charge is located at the origin because the rotating electric field rotates around the origin, as described for example by Jackson [15]. If the

rotating electric charge is identified with the charge on the electron, e, and the dipole radius is denoted $r^{(0)}$, Eq. (2.17.12) follows from *classical* electrodynamics. However, Eq. (2.17.12) is the basis of *quantum* theory, and is the light quantum hypothesis suggested originally by Planck in Nov., 1900. Equation (2.17.12) shows that in classical electrodynamics, the energy *En* of a plane wave in vacuo is proportional to its angular frequency, ω, through $eA^{(0)}r^{(0)}$. The Planck-Einstein relation of the quantum theory requires this to be $\hbar$, and so

$$\hbar = eA^{(0)}r^{(0)} = \frac{eA^{(0)}}{\kappa} , \qquad (2.17.20)$$

which is the charge quantization condition [6] provided that the radius $r^{(0)}$ is the inverse of the magnitude of the wavevector. The quantity $eA^{(0)}/\kappa$ is therefore a constant angular momentum in *classical* electrodynamics for the propagating plane wave in vacuo, and is identified as the angular momentum, $\hbar$, of one photon, the latter being the quantum of electromagnetic radiation. The quantum theory is therefore classical in origin, and we have shown that Planck's *assertion* that *En* must be proportional to ω is, in fact, a logical outcome of classical electrodynamics. The photon is a classical entity.

Furthermore, our innocent expression of $\boldsymbol{E}^{(1)}$ as an electric dipole moment results immediately in the Evans-Vigier field $\boldsymbol{B}^{(3)}$ [1—12], which is shown to be an outcome of helical charge motion as in a solenoid. In this respect the positive charge is fixed at the origin, and does not cancel out the induction of $\boldsymbol{B}^{(3)}$ because the positive charge is not rotating, and therefore does not induce a $\boldsymbol{B}^{(3)}$ in the opposite direction.

The usual idea of a *photon* as being uncharged [14], and therefore its own anti-particle, is shown by Eq. (2.17.19) to have the narrowest of meanings. The *photon* is a classical amount of energy, which is proportional to the *square* of the charge on the electron, and is not *uncharged*. The charge quantization condition shows that the origin of the Dirac constant $\hbar$, (or of the Planck constant h) is the electronic charge e multiplied by $A^{(0)}$, which is, within a constant c, the scalar potential [15] of the classical electromagnetic wave. Re-expressing $A^{(0)}$ through Eq. (2.17.15) leads to Eq. (2.17.17), which expresses the fine structure constant of quantum electrodynamics as a simple ratio of volumes. The fine structure constant is the ratio of the volume, V, used in Eq. (2.17.2) to the volume of the sphere defined by the radius $r^{(0)}$, the length of the dipole. This spherical volume is

$$V_0 = \frac{4}{3}\pi r^{(0)3} , \tag{2.17.21}$$

and so the fine structure constant is the ratio

$$\alpha = \frac{1}{3}\frac{V}{V_0} . \tag{2.17.22}$$

Equation (2.17.18) shows that the magnitude of the classical Maxwellian wavevector is defined by the volume V through

$$\kappa = \left(\frac{4\pi\alpha}{V}\right)^{1/3}, \tag{2.17.23}$$

and because α is a universal constant [13], the de Broglie photon momentum becomes

$$p = \hbar\kappa = \left(\frac{4\pi\alpha\hbar^3}{V}\right)^{1/3} = \left(\frac{e^2\hbar^2}{\epsilon_0 c V}\right)^{1/3}, \tag{2.17.24}$$

and the Planck-Einstein photon becomes expressible as

$$En = \hbar\omega = \hbar\kappa c = \left(\frac{e^2\hbar^2 c^2}{\epsilon_0 V}\right)^{1/3} . \tag{2.17.25}$$

These equations show that the quantum of energy, $\hbar\omega$, and the quantum of radiation momentum, $\hbar\kappa$, are both defined in terms only of V and the universal fine structure constant α. This is consistent with the fact that they are fundamental quanta of energy and momentum, and with the fact that the magnitudes of these quanta vary *only* with the volume V introduced in Eq. (2.17.2). On the most fundamental level in this Maxwellian theory, everything depends ultimately on e, the charge on the electron, and the constant h is a consequence of the existence of e. In generalized gauge theory [1—12], from which the charge quantization condition emerges [10—12], e simultaneously plays the role of a gauge scaling factor, i.e., is geometrical in nature, so that an angular momentum such as h becomes understandable in terms of a geometrical entity, e. We know the latter much more familiarly as the elementary unit of charge, an elementary geometrical measure of the known universe.

Finally, the concept of photon mass as envisaged [17] by de Broglie, Bohm, Vigier, and co-workers appears to be consistent with the rotating Maxwellian dipole because if there is mass involved in this motion, *it must be concentrated at the origin*, because the center of mass of the rotating dipole must be there because it is rotating about that point. If the mass were distributed, for example if the negative charge had some mass as well as the positive charge, the center of mass would be somewhere between the two charges, and the dipole could not rotate about one point. The particulate photon can therefore be envisaged with a mass concentrated at the origin. This is still a *classical* picture, because h has been identified as being classical in nature. This picture becomes consistent with special relativity if we use the de Broglie guidance theorem,

$$m_r = \frac{\hbar\omega_r}{c^2} = \frac{\hbar\kappa_r}{c} = \left(\frac{e^2\hbar^2}{\epsilon_0 c^4 V_r}\right)^{1/3} , \tag{2.17.26}$$

and so express the photon rest mass, m_r, in terms of the rest volume V_r. The introduction of mass is necessary because the Evans-Vigier field, $\boldsymbol{B}^{(3)}$, is *longitudinal*, so that the Wigner little group for a massless particle, E(2), becomes untenable [6]. The existence of $\boldsymbol{B}^{(3)}$ shows immediately that the particulate photon is concomitant with *three* degrees of space polarization, which, in the circular basis [6], are (1), (2) and (3). If the theory of special relativity is accepted therefore, the photon cannot be massless because of the existence of $\boldsymbol{B}^{(3)}$.

Acknowledgments

It is a pleasure to acknowledge discussions with several colleagues among whom are: Gareth J. Evans, Stanley Jeffers, Malcolm Mac Gregor, Mikhail Novikov, Sisir Roy, and Jean-Pierre Vigier.

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