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SU(2)×SU(2) ELECTROWEAK THEORY I: THE B^3 FIELD ON THE PHYSICAL VACUUM

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Nonlinear optics confronts the $U(1)$ theory of electrodynamics with the dilemma of the existence of nonlinear fields. The $U(1)$ group is completely linear and Abelian and causes consideration of an $SU(2)$ theory of electrodynamics. An $SU(2)$ theory of electrodynamics, with a B^3 magnetic field, means that physics is forced to consider an $SU(2) \times SU(2)$ electroweak theory. It is then demonstrated that the B^3 field exists on the physical vacuum defined by the Higgs symmetry breaking of this extended electroweak theory.

Key words: $SU(2) \times SU(2)$, electroweak theory, $\mathbf{B}^{(3)}$ theory.

1. INTRODUCTION

The first attempts at extending $SU(2) \times U(1)$ electroweak theory to incorporate the fundamental B^3 field of $SU(2)$ electrodynamics were made in [1] in which an $SU(2) \times SU(2)$ electroweak theory was suggested but not developed. In this paper, a more rigorous, but incomplete, $SU(2) \times SU(2)$ electroweak theory is developed to find several results which are missing from the original electroweak theory, because the latter did not incorporate B^3 and restricted consideration to a $U(1)$ symmetry electromagnetic sector. The extension is achieved in such as way as to maintain agreement with experimental data on weakly interacting vector boson and their masses, while still assuming for the sake of argument, a massless photon. The $SU(2) \times SU(2)$ electroweak theory developed in this Letter reproduces the fundamental relation between B^3 and the conjugate product of nonlinear optics, the relation responsible for the potentially very useful technique of radiation induced fermion resonance [2] and suggests the existence of a longitudinal E^3 field dual to B^3.

Nonlinear optics causes physics to consider electrodynamics to be an $SU(2)$ field theory with a magnetic field defined by the conjugate product[1]

$$\mathbf{B}^3 = -i\frac{e}{\hbar}\mathbf{A}^1 \times \mathbf{A}^2, \quad (1)$$

where $\mathbf{A}^1$ and $\mathbf{A}^2$ are conjugates or duals of each other. This magnetic field will couple to a Fermi spin-1/2 field according to the Hamiltonian[1]

$$H = -\frac{e\hbar}{2m}\boldsymbol{\sigma}^3 \cdot \mathbf{B}^3. \quad (2)$$

The occurrence of the B^3 field, in addition to the observation of nonlinear effects due to the conjugate product of connection coefficients, should be apparent through the spin resonance effect with a Fermi field.

We will examine the $SU(2) \times SU(2)$ electroweak model first with an analysis that is similar to that used with the $SU(2) \times U(1)$ electroweak model. Here we will have one Higgs field for both parts of the twisted bundle. We will then look at the consequences of that and determine what is wrong. From there the requirements to fix the theory are discussed. These corrections to the theory are then to be presented in the second paper.

2. THE SU(2) × SU(2) EXTENDED STANDARD MODEL

Consider an extended standard model to determine what form the electromagnetic and weak interactions assume on the physical vacuum defined by the Higgs mechanism. Such a theory would then be $SU(2) \times SU(2)$. The covariant derivative will then be

$$\mathcal{D}_\mu = \partial_\mu + ig'\boldsymbol{\sigma} \cdot \mathbf{A}_\mu + ig\boldsymbol{\tau} \cdot \mathbf{b}_\mu, \quad (3)$$

where $\boldsymbol{\sigma}$ and $\boldsymbol{\tau}$ are the generators for the two $SU(2)$ gauge fields represented as Pauli matrices and $\mathbf{A}$, $\mathbf{b}$ are the gauge connections defined on the two SU(2) principal bundles. There is an additional Lagrangian for the the ϕ^4 scalar field[2],

$$\mathcal{L}_\phi \;=\; \frac{1}{2}|\mathcal{D}_\mu(\phi)|^2 \;-\; \frac{1}{2}\mu^2|\phi|^2 \;+\; \frac{1}{4}|\lambda|\Big(|\phi|^2\Big)^2. \tag{4}$$

The expectation value for the scalar field is then

$$\langle\phi_0\rangle \;=\; \big(0,\; \frac{v}{\sqrt{2}}\big) \tag{5}$$

for $v \;=\; \sqrt{-\mu^2/\lambda}$. At this point, the generators for the theory on the broken vacuum are:

$$\begin{aligned} \langle\phi_0\rangle\sigma_1 \;&=\; \big(\frac{v}{\sqrt{2}},\; 0\big)\,, \\ \langle\phi_0\rangle\sigma_2 \;&=\; \big(i\frac{v}{\sqrt{2}},\; 0\big)\,, \\ \langle\phi_0\rangle\sigma_3 \;&=\; \big(0,\; -\frac{v}{\sqrt{2}}\big). \end{aligned} \tag{6}$$

These hold similarly for the generators of the other $SU(2)$ sector of the theory. There is a formula for the hypercharge, due to Nishijima, which, when applied directly, would lead to an electric charge

$$Q\langle\phi_0\rangle \;=\; \frac{1}{2}\langle\phi_0\rangle(\boldsymbol{\sigma}_3 \;+\; \boldsymbol{\tau}_1) \;=\; \big(0, -\frac{v}{\sqrt{2}}, 0, \frac{v}{\sqrt{2}}\big). \tag{7}$$

This would mean that there are two photons that carry a $\pm$ charge, respectively. We are obviously treating the hypercharge incorrectly. It is then proposed that the equation for hypercharge be modified as

$$Q\langle\phi_0\rangle \;=\; \frac{1}{2}\langle\phi_0\rangle\Big(\mathbf{n}_2\cdot\boldsymbol{\tau}_3 \;+\; \mathbf{n}_1\cdot\boldsymbol{\sigma}_1\Big) \;=\; 0, \tag{8}$$

where the vectors $\mathbf{n}_1$ and $\mathbf{n}_2$ are unit vectors on the doublet defined by the two eigenstates of the vacuum. This projection onto $\boldsymbol{\sigma}_1$ and $\boldsymbol{\tau}_3$ is an ad hoc change to the theory that is required since we are using a single Higgs field on both bundles on both $SU(2)$ connections. This condition, an artifact of using one Higgs field, will be relaxed later. Now the generators of the theory have a broken symmetry on the physical vacuum. Therefore the photon is defined according to the

$\boldsymbol{\sigma}_1$ generator in one $SU(2)$ sector of the theory, while the charged neutral current of the weak interaction is defined on the $\boldsymbol{\tau}_3$ generator.

We now consider the role of the ϕ^4 scalar field with the basic Lagrangian containing the electroweak Lagrangians

$$\begin{aligned}\mathcal{L} \;=\; -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} \;-\; \frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu}\\ +|\mathcal{D}_\mu\phi|^2 \;-\; \frac{1}{2}\mu^2|\phi|^2 \;+\; \frac{1}{4}\lambda\Big(|\phi|^2\Big)^2.\end{aligned} \tag{9}$$

Here $G^a_{\mu\nu}$ and $F^a_{\mu\nu}$ are elements of the field strength tensors for the two $SU(2)$ principal bundles. In further work the Dirac and Yukawa Lagrangians that couple the Higgs field to the leptons and quarks will be included. It will be pointed out how this will modify the B^3 field. The ϕ^4 field may be written according to a small displacement in the vacuum energy

$$\phi' \;=\; \phi \;+\; \langle\phi_0\rangle \;\simeq\; (v \;+\; \xi \;+\; i\chi)/\sqrt{2}. \tag{10}$$

The fields ξ and χ are orthogonal components in the complex phase plane for the oscillations due to the small displacement of the scalar field. The small displacement of the scalar field is then completely characterized. The scalar field Lagrangian then becomes

$$\begin{aligned}\mathcal{L}_\phi \;=\; \frac{1}{2}\Big(\partial_\mu\xi\partial^\mu\xi - 2\mu^2\xi^2\Big)\\ +\frac{1}{2}v^2\left[g'\mathbf{A}_\mu + g\mathbf{b}_\mu + \Big(\frac{1}{gv} + \frac{1}{g'v}\Big)\partial_\mu\boldsymbol{\chi}\right]\\ \times\left[g'\mathbf{A}^\mu + g\mathbf{b}^\mu + \Big(\frac{1}{gv} \;+\; \frac{1}{g'v}\Big)\partial^\mu\boldsymbol{\chi}\right]\end{aligned} \tag{11}$$

The Lie algebraic indices are implied. The Higgs field is described by the harmonic oscillator equation where the field has the mass $M_H \;\simeq\; 1.0\ TeV/c^2$. On the physical vacuum the gauge fields are

$$g'\mathbf{A}_\mu + g\mathbf{b}_\mu \rightarrow\ , g'\mathbf{A}_\mu + g\mathbf{b}_\mu + \frac{1}{gv}\partial_\mu\boldsymbol{\chi} \;=\; g'\mathbf{A'}_\mu + g\mathbf{b}'_\mu, \tag{12}$$

which corresponds to a phase rotation induced by the transition of the vacuum to the physical vacuum. Let us now break the Lagrangian, now expanded about the minimum of the scalar potential, out into components:

$$\mathcal{L}_\phi \;=\; \frac{1}{2}\Big(\partial_\mu\xi\partial^\mu\xi \;-\; 2\mu^2\xi^2\Big) \;+\; \frac{1}{8}v^2$$

$$\times \left[g'^2|\mathbf{b}^3|^3 + g'^2\Big(|\,\boldsymbol{W}^+|^2 + |\,\boldsymbol{W}^-|^2\Big) + g^2|\boldsymbol{A}^1|^2 + g^2|\boldsymbol{A}^3 + i\boldsymbol{A}^2|^2\right], \tag{13}$$

where we have identified the charged weak gauge fields as

$$W^{\pm}_{\mu} \; = \; \frac{1}{\sqrt{2}}\Big(\mathbf{b}^1_{\mu} \; \pm \; i\mathbf{b}^2_{\mu}\Big). \tag{14}$$

The mass of these two fields are then $gv/2$. From what is left, we are forced to define the fields

$$A_{\mu} \; = \; \frac{1}{\sqrt{g^2 \; + \; g'^2}}\Big(g\boldsymbol{A}^3_{\mu} \; + \; g'\mathbf{b}^3_{\mu} \; - \; g\boldsymbol{A}^1_{\mu}\Big), \tag{15a}$$

$$Z^0_{\mu} \; = \; \frac{1}{\sqrt{g^2 \; + \; g'^2}}\Big(g'\boldsymbol{A}^3_{\mu} \; + \; g\mathbf{b}^3_{\mu} \; + \; g'\boldsymbol{A}^1_{\mu}\Big). \tag{15b}$$

In order to make this consistent with the $SU(2) \times U(1)$ electroweak interaction[4] theory we initially require that ${A^3}_{\mu} \; = \; 0$ everywhere on scales larger than at unification. If this were nonzero then Z_0 would have a larger mass or there would be an additional massive boson along with the Z_0 neutral boson. The first case is not been observed, and the second case is to be determined. This assumption, while ad hoc at this point, is made to restrict this gauge freedom and will be relaxed later when a more complete discussion of the 3-photon is given. This is condition is relaxed in the following letter. This leads to the standard result that the mass of the photon is zero and that the mass of the Z_0 particle is

$$M_{Z^0} \; = \; (v/2)\sqrt{g^2 \; + \; g'^2} \; = \; M_W\sqrt{1 \; + \; \Big(g'/g\Big)^2}. \tag{16}$$

The weak angles are defined trigonometrically by the terms $g/(g^2 + g'^2)$ and $g'/(g^2 + g'^2)$. This means that the field strength tensor $F^3_{\mu\nu}$ satisfies

$$\begin{aligned} F^3_{\mu\nu} \; &= \; \partial_{\nu}A^3_{\mu} - \partial_{\mu}A^3_{\nu} \; - \; i\frac{e}{\hbar}\Big[A^1_{\nu}, \; A^2_{\mu}\Big] \\ &= \; -i\frac{e}{\hbar}\Big[A^1_{\nu}, \; A^2_{\mu}\Big]; \end{aligned} \tag{17}$$

it further implies that the third component of the magnetic field in the $SU(2)$ sector is

$$\begin{aligned} B^3_j \; &= \; {\epsilon_j}^{\mu\nu}{F^3}_{\mu\nu} \\ &= \; -i\frac{e}{\hbar}\Big(\mathbf{A}^1 \times \mathbf{A}^2\Big)_j. \end{aligned} \tag{18}$$

This is the form of the B^3 magnetic field, which also implies that the E^3 electric field then is

$$E^3_j \;=\; \epsilon_j{}^{0\mu}F^3_{0\mu} \;=\; -i\frac{e}{\hbar}\Big(\mathbf{A}^1\times\mathbf{A}^2\Big)_j. \qquad (19)$$

This demonstrates that $E^3 \;=\; B^3$ in naturalized units.

The duality between these electric and magnetic field means that the Lagrangian vanishes. The vanishing of this Lagrangian on symmetry principles means that there can not be any dynamics determined. This would indicate that this particular model simply reproduces $U(1)$ electrodynamics. One possiblity is that the electrodynamic vacuum will continue to exhibit nonabelian symmetries even if we impose $E^3 \;=\; B^3 \;=\; 0$.

However it can be demonstrated that the $B^3 = E^3$ field duality is broken when we consider the Lagrangian for the 3-field with the massive A^3 field introduced as

$$\begin{aligned}\mathcal{L} \;&=\; \frac{1}{2}F^3_{\mu\nu}F^{3\mu\nu} \;+\; \frac{1}{2}\mu A^3{}_\mu A^{3\mu} \;-\; (1/c)j^3{}_\mu A^{3\mu},\\ \mathcal{L} \;&=\; \frac{1}{2}(E^{32} \;-\; B^{32}) \;+\; \frac{1}{2}\mu A^3{}_\mu A^{3\mu} \;-\; (1/c)j^3_\mu A^{3\mu}.\end{aligned} \qquad (20)$$

The middle term is a Proca Lagrangian for a massive photon. Here the mass of this photon is assumed to be larger than the masses of the $W^\pm$ and W^0 bosons. The current $j^3{}_\mu$ is determined by the charged fermions with masses given by the Yukawa interactions with the Higgs field. These are yet to be explored. Now consider the term in the Euler-Lagrange equation

$$\frac{\partial L}{\partial(D^\mu A^{3\nu})} \;=\; [A_\mu, A_\nu], \qquad (21)$$

with covariant derivatives that enter into the Euler-Lagrange equation as

$$D_\mu A^{3\nu} \;=\; \partial_\mu A^{3\nu} \;+\; i(e/\hbar)\epsilon^{3ab}[A^a_\mu, A^b_\nu] \qquad (22)$$

and the subsequent setting of $A^3 \;\rightarrow\; 0$. The full Euler-Lagrange equation

$$D^\mu\frac{\partial L}{\partial(D^\mu A^{3\nu})} \;-\; \frac{\partial L}{\partial A^{3\nu}} \;=\; 0 \qquad (23)$$

is then

$$\nabla\times \boldsymbol{B}^3 \;+\; \mu^2\boldsymbol{A}^3 \;-\; \boldsymbol{j}^3 \;=\; \frac{\partial \boldsymbol{E}^3}{\partial t}, \qquad (24)$$

which is just a form of the Faraday-Maxwell equation. However, the Hodge-star dual of this equation, the Maxwell equation, does not contain the current term

$$\nabla \times \boldsymbol{E}^3 + \mu^2 \boldsymbol{A}^3 = -\frac{\partial \boldsymbol{B}^3}{\partial t}. \tag{25}$$

The nonvanishing A^3 field at high energy will then break the duality between the E^3 and B^3 fields.

It has been demonstrated that there is an $SU(2) \times SU(2)$ electroweak theory that gives rise to the $Z_0, W^{\pm}$ gauge vector bosons plus electromagnetism with the photon theory with the cyclic condition for the B^3 fields. What has not been worked out are the implications for quark and lepton masses by inclusion of Yukawa coupling Lagrangians. However, that sector of the theory has little bearing upon this examination of the electromagnetic theory, with $A^3 = 0$, that emerges from the $SU(2) \times SU(2)$ gauge theory. We now have a theory for electromagnetism on the physical vacuum that is

$$\begin{aligned}\mathcal{L} = & -(1/4)F^{\mu\nu}F_{\mu\nu} + -(1/4)G^{a\mu\nu}G^a_{\mu\nu} + \frac{1}{2}\Big((E^3)^2 - (B^3)^2\Big) \\ & + M_0|Z_0|^2 + M_w|W^{\pm}|^2 + \frac{1}{2}\Big(|\partial\xi|^2 - 2\mu^2|\xi|^2\Big) \\ & + \text{Dirac Lagrangians} + \text{Yukawa (Fermi} - \text{Higgs)},\end{aligned} \tag{26}$$

where $F_{\mu\nu}$ and $G^a_{\mu\nu}$ are the field tensor components for standard electromagnetism and the weak interaction, and the cyclic electric and magnetic fields define the Lagrangian in the third term. The occurrence of the massive Z_0 and $W^{\pm}$ particles obviously breaks the gauge symmetry of the $SU(2)$ weak interaction.

3. DUALITY AND CHIRAL BREAKING

Physically we have the fields E^3, B^3 which are easily demonstrated to be longitudinal fields. Longitudinal fields result from the breaking of gauge invariance. What is unique is that the B^3 and $\mathbf{e}^3$ fields are massless; unlike most fields resulting from symmetry breaking that are massive. That these fields are divergences and are equal up to c, before the breaking of duality, suggests that there are clues to field duality and monopole physics[5,6,7,8]. So we have on the physical vacuum three fields, where two are equivalent and all three are massless. This mechanism of breaking the $E^3 = B^3$ duality needs to be further explored.

Electrodynamics may be more fundamentally nonabelian leads to the existence of B^3 field that gives rise to nonlinear optical effects. This means that in a nonlinear optical system there should be exhibited a Lorentz force on a moving charge that is not predicted by $U(1)$ electrodynamics. The coupling of a Fermi field to the B^3 field is just such an interaction, and empirical evidence for this coupling can be found in the inverse Faraday effect [9]. The $\boldsymbol{\sigma} \cdot \mathbf{A} \times \mathbf{A}^*$ term is given in Eq.(8.6) of Ref. 9. That this field exists, and apparently the E^3 field does not, should be determined more explicitly by an extended standard model.

Now let us perform a gauge transformation $A^1 \rightarrow UA^1U^{-1} + U\partial U^{-1}$. The electric field in the 3-sector is then transformed as

$$B^{3i} = \epsilon^{ijk} U\left[A^1_j,\ A^2_k\right]U^{-1}. \tag{27}$$

This can be easily demonstrated by the antisymmetry of j and k with the commutators with A^1 or A^2. So fortunately the theory appears to be gauge invariant. So there is a general situation of proper gauge transformation if self-duality holds, or if it is broken. In effect if self-duality, and the broken duality due to the A^3 potential, is true for flat gauge connections then it is true for all gauge connections.

This duality is an artifact of excluding the masses predicted by the Higgs mechanism and their role in the decay processes of the charged and neutral weak currents, and the decay of $SU(2)$ massive photon A^3. Setting $A^3 = 0$ allowed us to ignore this problem to examine the basic electroweak issues. The explicit inclusion of massive fermions in the decay of these fields will break field duality[10]. The condition that $A^3 = 0$ everywhere is relaxed and a 3-photon is defined. It can be demonstrated that the currents contain vector and axial vector components that obey the $SU(2) \times SU(2)_C$ algebra. On the physical vacuum fields acquire masses that violate the current conservation of the axial vector current. Within this context a better understanding of duality breaking can be derived.

4. DISCUSSIONS OF THE THEORY, ITS PROBLEMS, AND THEIR REMEDIES

So here we have constructed, in some ways rather artificially, an $SU(2) \times SU(2)$ gauge theory that is able to reproduce the standard model $U(1) \times SU(2)$ with the additional cyclic magnetic field given by equation 18. However, we are left with two uncomfortable conditions imposed on the theory to make this work. The first is that the electric charge is computed in an ad hoc fashion so that we do not have the massless photons $\mathbf{A}^1$ and $\mathbf{A}^2$ that carry a unit of opposite electric charges. The second problem is that we have by hand eliminated the $\mathbf{A}^3$ vector potential. If this were nonzero we would have the following gauge potential

$$\omega'^3_\mu = \frac{g}{\sqrt{g^2 + g'^2}} A^3_\mu. \tag{28}$$

This field would have a mass equal to $(v/2)\sqrt{g^2 + g'^2}$ and would then contribute a large decay signal at the same scattering transverse momenta where the Z_0 is seen.

The problem is that we have a theory with two $SU(2)$ algebras that both act on the same Fermi spinor fields. We further are using one Higgs field to compute the vacuum expectation values for both fields. The obvious thing to do is to first consider that each $SU(2)$ acts on a separate spinor fields doublets. Next the theory demands that we consider that there be two Higgs fields that compute separate physical vacuums for each $SU(2)$ sector independently. This means that the two Higgs fields will give 2×2 vacuum expectations, which may be considered to be diagonal. If two entries in each of these matrices are equal then we conclude that the resulting massive fermion in each of the two spinor doublets are the same field. Further, if the spin or in one doublet assumes a very large mass, then at low energies this doublet will appear as a singlet and the gauge theory that acts on it will be $O(3)$, with the algebra of singlets

$$\mathbf{e}_i = \epsilon_{ijk}[\mathbf{e}_j, \mathbf{e}_k]. \tag{29}$$

This will leave a theory on the physical vacuum that involves transformations on a singlet according to a broken $O(3)$ gauge theory, and transformations on a doublet according to a broken $SU(2)$ gauge theory. The broken $O(3)$ gauge theory reflects the occurence of a very massive $\mathbf{A}^3$ photon, but massless $\mathbf{A}^1$ and $\mathbf{A}^2$ fields. This broken $O(3)$ gauge theory then reduces to electromagnetism with the cyclicity condition. The broken $SU(2)$ theory reflects the occurrence of massive charged and neutral weakly interacting bosons.

To take this theory further would be to embed it into an $SU(4)$ gauge theory. The gauge potentials are described by 4×4 traceless Hermitian matrices and the Dirac spinor has 16 components. The neutrality of the photon is then given by the sum over charges, which vanishes by the tracelessness of the theory. The Higgs field is described by a 4×4 matrix of entries

5. CONCLUSION

It is concluded within the above "toy model" that the B^3 field is consistent with an extended $SU(2) \times SU(2)$ model of electroweak interactions. A more complete formalism of the $SU(2) \times SU(2)$ theory with fermion masses will yield more general results. A direct measurement of B^3 should have a major impact on the future of unified field theory and superstring theories. The first such measurement was reported in Ref. 9, (see also Refs. 1 and 2).

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