EMPIRICAL EVIDENCE FOR NON-ABELIAN ELECTRODYNAMICS AND THEORETICAL DEVELOPMENT

ABSTRACT

There is clear empirical evidence for the fact that classical electrodynamics is not fully described by the Maxwell-Heaviside equations. In this paper, the empirical evidence is used to construct a form of classical electrodynamics which is more self-consistent and able to account for observations where the received Maxwell-Heaviside view fails. It is shown that equations of the new electrodynamics by at least four authors have nearly identical structure. They lead to concepts and results which are not available in the received view, and account for empirical data which cannot be accounted for by the four Maxwell-Heaviside equations. A new view of electrodynamics is emerging in which it is part of a gauge theory with an internal gauge symmetry which is SU(2). This can be broken to the isomorphic O(3) structure with the Higgs field. The structure of both the SU(2) and O(3) electrodynamics is identical with that of the Harmuth equations; and includes the concept of space charge and current introduced by Lehnert and shown to lead to empirical results not accountable for with the Maxwell-Heaviside equations.

INTRODUCTION

Recently it has been shown both empirically {1-15} and theoretically {16-24} that classical electrodynamics is not described by the Maxwell-Heaviside equations, which in modern gauge theory has a U(1) internal gauge group. The evidence is now overwhelming that a new electrodynamics is needed which is more self-consistent and able to account more satisfactorily for observational fact on the classical level. This means that quantum electrodynamics is in need of a similar revision {25}. In Section 2, the empirical evidence for magnetic charge and current is reviewed briefly, together with the Harmuth equations to which these observations lead. In Section 3, it is argued that the Harmuth equations can be constructed from a gauge theory with an internal SU(2) gauge group structure as proposed by Barrett {16, 17}. This structure allows for the existence of Lehnert's space charge and current {20}, with which he has extended the Maxwell-Heaviside equations {18-20}, and has demonstrated advantages of his extended electrodynamics over these received equations. In Section 4, it is argued that the Barrett field equations, which have an SU(2) internal gauge symmetry, can be broken to the O(3) electrodynamics recently developed by Evans et al. {21-24} and shown to have many advantages over the received view {26, 27}. One of the outcomes of the O(3) equations is that there always exists in the classical vacuum, a fundamental, longitudinally directed field, which was first proposed in 1992 {28} to account for the inverse Faraday effect and the third Stokes parameter without phenomenology. The most extensively developed of these new theories of electrodynamics is the O(3) electrodynamics, but the Lehnert, Harmuth, Barrett and O(3) equations are all interlinked, and are all able to describe empirical data which are outside the scope of Maxwell-Heaviside theory {26, 27}.

THE EXISTENCE OF MAGNETIC CHARGE AND CURRENT AND THE HARMUTH FIELD EQUATIONS

Recently, Mikhailov and Mikhailova {1-8} have provided unequivocal experimental evidence for the existence of magnetic charge. Six of these experiments are reviewed in ref (9). These consist of: an experiment with a homogeneous magnetic field; quantitative measurement of the magnetic charge; magnetic charge in a magnetic field of a line conductor's current; observation of a magnetic charge in a diffusion chamber; detection of a discriminating magnetic charge response to light of various polarizations; magnetic charge and optical levitation of ferromagnetic particles in a magnetic field. There were therefore six independent experimental designs showing the existence of magnetic charge. The latter was later interpreted by Barrett {10} in terms of a low energy magnetic monopole or instanton, a minimum action solution of an SU(2) Yang-Mills structure applied to classical SU(2) electrodynamics {16, 17} and resulting in the Barrett field equations to be cited later in this paper. The concept of magnetic charge and current does not exist within the received view of electrodynamics, the Maxwell-Heaviside equations, which are therefore incomplete.

The Barrett field equations are homomorphic with a set of equations developed by Harmuth {11-15} to account for the fact that the received Maxwell-Heaviside equations {26, 27} do not satisfactorily account {15-17} for the propagation velocity of electromagnetic signals. The calculated group velocity is almost always larger than the speed of light for radio frequencies through the atmosphere, and its derivation implies a transmission rate of information equal to zero {16}. The Maxwell-Heaviside equations do not permit the calculation of the propagation velocity of signals with bandwidth propagating in a lossy medium and all the published solutions for propagation velocities assume sinusoidal (linear) signals.

The set of equations proposed by Harmuth, in Gaussian units, are as follows:

$$\nabla \cdot \boldsymbol{D} = 4\pi\rho_e \tag{1}$$

$$\nabla \times \boldsymbol{H} = \left(\frac{4\pi}{c}\right)\boldsymbol{J}_e + \frac{1}{c}\frac{\partial \boldsymbol{D}}{\partial t} \tag{2}$$

$$\nabla \cdot \boldsymbol{B} = \rho_m \tag{3}$$

$$\nabla \times \boldsymbol{E} + \frac{1}{c}\frac{\partial \boldsymbol{B}}{\partial t} + \frac{4\pi}{c}\boldsymbol{J}_m = \boldsymbol{0}. \tag{4}$$

The Coulomb and Ampère-Maxwell laws look to have the same structure but the Gauss and Faraday Laws are extended to include the existence of magnetic charge and current. Here, $\boldsymbol{D}$ is the displacement; ρ_e is the charge density; $\boldsymbol{H}$ is the magnetic field strength; $\boldsymbol{J}$ is the electric current density; ρ_m is the magnetic charge density; $\boldsymbol{J}_m$ is the magnetic current density and c is the speed of light in vacuo, all in Gaussian units. The constitutive relations were modified to {11-16}:

$$\boldsymbol{D} = \varepsilon \boldsymbol{E} \tag{5}$$

$$\boldsymbol{B} = \mu \boldsymbol{H} \tag{6}$$

where ϵ is material permittivity and μ is material permeability. The usual electric Ohm's law was supplemented by Harmuth with a magnetic Ohm's law:

$$\boldsymbol{J} = \sigma \boldsymbol{E} \tag{7}$$

$$\boldsymbol{J}_m = s\boldsymbol{E} \tag{8}$$

where σ is electric conductivity and s is magnetic conductivity. The Harmuth equations provide complete integrability and soliton solutions of the Maxwell-Heaviside equations, introducing into them essentially topological concepts. Barrett {10} later used the topological concept of instanton to describe magnetic charge as observed by Mikhailov et al. {1-8} and earlier by Ehrenhaft {29}.

HARMUTH EQUATION AND SU(2) ELECTRODYNAMICS

Later, Barrett {16, 17} showed that the Harmuth field equations were equations of Yang-Mills theory applied to classical electrodynamics with an SU(2) internal gauge symmetry. In so doing, Barrett also showed the existence of the space charge and current proposed by Lehnert {18, 19} and developed by Lehnert and Roy {20}.

Empirical evidence for the existence of the Lehnert space and current was given by considering internal reflection at a vacuum {18-20}. This evidence supplements that for the Harmuth field equations given in Section 2. In condensed matrix notation, the Barrett field equations in S.I. units {17} are:

$$\nabla \bullet \boldsymbol{E} = J_0 - iq(\boldsymbol{A} \bullet \boldsymbol{E} - \boldsymbol{E} \bullet \boldsymbol{A}) \tag{9}$$

$$\frac{\partial \boldsymbol{E}}{\partial t} - \nabla \times \mathrm{B} + \boldsymbol{J} + iq[A_0, \boldsymbol{E}] - iq(\boldsymbol{A} \times \boldsymbol{B} - \boldsymbol{B} \times \boldsymbol{A}) = \boldsymbol{0} \tag{10}$$

$$\nabla \bullet \mathrm{B} + iq(\boldsymbol{A} \bullet \boldsymbol{B} - \boldsymbol{B} \bullet \boldsymbol{A}) = \boldsymbol{0} \tag{11}$$

$$\nabla \times \mathrm{E} + \frac{\partial \boldsymbol{B}}{\partial t} + iq[A_0, \boldsymbol{B}] + iq(\boldsymbol{A} \times \boldsymbol{E} - \boldsymbol{E} \times \boldsymbol{A}) = \boldsymbol{0}. \tag{12}$$

The Maxwell-Heaviside equations are supplemented by extra terms, which are conserved Noether currents {17}. These terms are a combination of those appearing in the Harmuth and Lehnert field equations and will be identified as such later in this Section. Without these extra Noether currents, the Barrett field equations reduce to the Maxwell-Heaviside equations {26, 27}, which in Barrett's notation are:

$$\nabla \bullet \boldsymbol{E} = J_0 \tag{13}$$

$$\frac{\partial \boldsymbol{E}}{\partial t} - \nabla \times \boldsymbol{B} + \boldsymbol{J} = \boldsymbol{0} \tag{14}$$

$$\nabla \bullet \boldsymbol{B} = 0 \tag{15}$$

$$\nabla \times \mathrm{E} + \frac{\partial \mathrm{B}}{\partial t} = \boldsymbol{0}. \tag{16}$$

The extra terms which appear because of the SU(2) structure of Barrett's equations can be identified as follows:

1) The Harmuth magnetic charge density:

$$\rho_m = -iq(\boldsymbol{A} \bullet \boldsymbol{B} - \boldsymbol{B} \bullet \boldsymbol{A}) \tag{17}$$

2) The Harmuth magnetic current density:

$$\boldsymbol{J}_m = -iq\left([A_0, \boldsymbol{B}] + \boldsymbol{A} \times \boldsymbol{E} - \boldsymbol{E} \times \boldsymbol{A}\right) \tag{18}$$

3) The Lehnert charge density:

$$\rho_e = -iq\left(\boldsymbol{A}\bullet\boldsymbol{E} - \boldsymbol{E}\bullet\boldsymbol{A}\right) \tag{19}$$

4) The Lehnert current density:

$$\boldsymbol{J}_e = -iq\left(-\left[A_0, \boldsymbol{E}\right] + \boldsymbol{A}\times\boldsymbol{B} - \boldsymbol{B}\times\boldsymbol{A}\right) \tag{20}$$

Here, A_0 and $\boldsymbol{A}$ are scalar and vector components of a four potential, which plays a physical role on the classical level. The empirical evidence for this is reviewed by Barrett {16, 17} from six different sources. The factor q in the Barrett equations is the coupling constant which appears in the SU(2) covariant derivative, and couples the radiated SU(2) field to its source, electric and magnetic charge. Therefore the Barrett equations are based on empirical evidence from several sources {16, 17}.

ELECTROMAGNETIC FIELD EQUATIONS OF EVANS ET ALIA

These field equations were developed in an entirely different manner and have an entirely different history from the Barrett equations {17}, but are O(3) Yang-Mills equations isomorphic to those of Barrett. So they contain the same information. One of the distinguishing features of the field equations by Evans et al. {21-24} is that they infer the existence in the vacuum of a longitudinally directed magnetic field component $\boldsymbol{B}^{(3)}$, which is an observable in the third Stokes parameter {24} and in magneto-optical effects such as the inverse Faraday effect {21-24}. The equations were developed in a complex basis ((1), (2), (3)) which describes an O(3) symmetry internal gauge space {21-24}. The $\boldsymbol{B}^{(3)}$ component in the theory is fundamentally defined from the field tensor and is given by:

$$\boldsymbol{B}^{(3)*} \equiv -ig\boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)} \tag{21}$$

where g is a coupling constant derived from the use of an O(3) symmetry covariant derivative {21-24}. It is the equivalent of Barrett's q. In the complex basis ((1), (2), (3)), the complex vector potential $\boldsymbol{A}^{(1)}$ is the complex conjugate of the vector potential $\boldsymbol{A}^{(2)}$. The product $\boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)}$ is proportional {21-24} to the third Stokes parameter in the vacuum and is observable in the inverse Faraday effect. The coupling constant g has the classical units $\kappa/A^{(0)}$ where $A^{(0)}$ is the magnitude of $\boldsymbol{A}^{(1)} = \boldsymbol{A}^{(2)*}$ and where κ is the magnitude of the wave-vector. It follows that Barrett's SU(2) coupling constant q also has these units in a classical context. Therefore both g and q disappear in the static limit and we recover the equations of electro-statics, the Gauss, Coulomb and Ampère laws.

This fact illustrates that, in the Barrett and Evans et al. equations, the origin of magnetic charge and current is topological. Similarly, the origin of the Lehnert space charge and vacuum current is topological - they are physical instantons. Both the Barrett and Evans et alia equations have a Yang-Mills structure, whose minimum action solution is a physical instanton or pseudo-particle. Therefore both the Barrett and Evans et alia equations are significant developments of the Maxwell-Heaviside equations, which give no instantons. Therefore, in the Yang-Mills theory of pure electrodynamics developed by both Barrett {16, 17} and Evans et al {21-24}, the magnetic charge of Harmuth and Mikhailov et alia, and the Lehnert space charge and current, both give rise to physical instantons. This interpretation has already been discussed by Barrett {10} in the context of the Mikhailov/Ehrenhaft experiments.

In their most condensed form, the equations of Evans et al. {21-24} look at first sight to be similar to the Maxwell-Heaviside equations:

$$D_\mu \tilde{\boldsymbol{G}}^{\mu\nu} \equiv \boldsymbol{0} \tag{22}$$

$$D_\mu \boldsymbol{H}^{\mu\nu} \equiv \boldsymbol{J}^\nu \tag{23}$$

The familiar field tensors $\tilde{G}^{\mu\nu}$ and $H^{\mu\nu}$ of the homogenous and inhomogeneous Maxwell-Heaviside equations (a U(1) symmetry gauge theory {26}) become vectors in the O(3) internal space of eqns. (22) and (23), so are denoted as vectors in this space and tensors in Minkowski space-time. The O(3) group is the covering group of Barrett's SU(2) group and so eqns. (22) and (23) have the same physical content as the Barrett equations. The ordinary derivatives of the Maxwell-Heaviside equations are replaced in eqns. (22) and (23) by the O(3) covariant derivatives D_μ {26} which are consistent with special relativity. The isomorphic Barrett and Evans et al. equations are gauge covariant and Lorentz covariant equations of special relativity. They are, in other words, equations of standard gauge theory applied logically to pure classical electrodynamics. The four current J^{ν} is also a vector in the O(3) symmetry gauge space of eqns. (22) and (23). The homogeneous eqn. (22) is a Jacobi identity of the O(3) group, and the tilde denotes dual tensor as usual.

The homogeneous eqn. (22) can be developed by writing out the covariant derivative in terms of its coupling constant g, which classically has the units of $\kappa/A^{(0)}$. This couples the dynamical field to its source, so in the Barrett or Evans et alia equations, the dynamical field is never free of its source, there is no source-free region as in the Maxwell-Heaviside point of view when the electric charge and current are set to zero mathematically. This property of the new equations is a significant advantage because a dynamical field propagating without a source is effect without cause, i.e. a violation of causality. On quantization, the coupling constant g has units of $e/\hbar$, and for one photon in free space:

$$eA^{(0)} = \hbar\kappa \tag{24}$$

signaling that the photon is also always coupled to its source, the electron. In the case of electro-statics, there is no photon or dynamic field, and g is zero. Therefore, in general, g has units of $e/\hbar$, or $\kappa/A^{(0)}$.

Recall that these novel conclusions are based on empirical data as reviewed earlier in this paper, data which are not accessible to the classical Maxwell-Heaviside point of view. These data, when interpreted in terms of the Barrett, Harmuth, Lehnert or Evans et alia equations, represent a major philosophical advance in thought in the area of classical electrodynamics. Similar advances are expected in quantum electrodynamics {24}, based on the empirical evidence which led to a modification of classical electrodynamics as just described. This represents therefore a logical progression. The presence of g in the theory (q in Barrett's equations) does not mean that the gauge bosons are charged after quantization. The role of g in this context is a dynamical coupling constant in a covariant derivative (SU(2) in the Barrett equations; O(3) in the Evans et al. Equations). It measures the "strength" with which the dynamical electromagnetic field (or photon) couples to its source. It has the **units** of $e/\hbar$, or of $\kappa/A^{(0)}$, i.e. C/(J-s). As described by Ryder {26}, this dynamical aspect of charge is a consequence of the gauge principle. It means that there is a conserved charge which couples to the gauge field (the electromagnetic field) in units of $e/\hbar$, or of $\kappa/A^{(0)}$. The concept of g originates in parallel transport and in special relativity and, in general, is simply a coefficient {26} needed to make sure units are balanced. Similarly, quantities such as magnetic flux density have units of J s C^{-1} m^{-2}; electric displacement (D) is C m^{-2} and so on, but neither $\boldsymbol{B}$ nor $\boldsymbol{D}$ is charged. The coupling constant g should be regarded analogously. It also appears as q in the Barrett field equations, and similarly, is not charged in those equations. Therefore neither the classical Barrett nor the classical Evans et alia equations should lead to charged gauge bosons (photons) in quantum electrodynamics. To ensure this requires the use of topological concepts {24} beyond the scope of this paper. These are likely to be of key importance in any advance in knowledge in quantum electrodynamics, and are based on empirical data.

The equivalent of the four Barrett equations in the O(3) internal gauge symmetry of eqns. (22) and (23) are as follows:

The O(3) Symmetry Gauss law (cf. Eqn. (11))

$$\nabla \cdot \boldsymbol{B}^{(1)*} \equiv ig\left(\boldsymbol{A}^{(2)} \cdot \boldsymbol{B}^{(3)} - \boldsymbol{B}^{(2)} \cdot \boldsymbol{A}^{(3)}\right) \tag{25}$$

$$\nabla \cdot \boldsymbol{B}^{(2)*} \equiv ig\left(\boldsymbol{A}^{(3)} \cdot \boldsymbol{B}^{(1)} - \boldsymbol{B}^{(3)} \cdot \boldsymbol{A}^{(1)}\right) \tag{26}$$

$$\nabla \cdot \boldsymbol{B}^{(3)*} \equiv ig\left(\boldsymbol{A}^{(1)} \cdot \boldsymbol{B}^{(2)} - \boldsymbol{B}^{(1)} \cdot \boldsymbol{A}^{(2)}\right) \tag{27}$$

The O(3) Symmetry Faraday Induction law (cf. Eqn. (12))

$$\nabla \times \boldsymbol{E}^{(1)*} + \frac{\partial \boldsymbol{B}^{(1)*}}{\partial t} \equiv -ig\left(cA_0^{(3)}\boldsymbol{B}^{(2)} - cA_0^{(2)}\boldsymbol{B}^{(3)} + \boldsymbol{A}^{(2)} \times \boldsymbol{E}^{(3)} - \boldsymbol{A}^{(3)} \times \boldsymbol{E}^{(2)}\right) \tag{28}$$

$$\nabla \times \boldsymbol{E}^{(2)*} + \frac{\partial \boldsymbol{B}^{(2)*}}{\partial t} \equiv -ig\left(cA_0^{(1)}\boldsymbol{B}^{(3)} - cA_0^{(3)}\boldsymbol{B}^{(1)} + \boldsymbol{A}^{(3)} \times \boldsymbol{E}^{(1)} - \boldsymbol{A}^{(1)} \times \boldsymbol{E}^{(3)}\right) \tag{29}$$

$$\nabla \times \boldsymbol{E}^{(3)*} + \frac{\partial \boldsymbol{B}^{(3)*}}{\partial t} \equiv -ig\left(cA_0^{(2)}\boldsymbol{B}^{(1)} - cA_0^{(1)}\boldsymbol{B}^{(2)} + \boldsymbol{A}^{(1)} \times \boldsymbol{E}^{(2)} - \boldsymbol{A}^{(2)} \times \boldsymbol{E}^{(1)}\right) \tag{30}$$

The O(3) Symmetry Coulomb law (cf. Eqn. (9))

$$\nabla \cdot \boldsymbol{E}^{(1)*} - \frac{\rho^{(1)*}}{\varepsilon_0} = ig\left(\boldsymbol{A}^{(2)} \cdot \boldsymbol{E}^{(3)} - \boldsymbol{E}^{(2)} \cdot \boldsymbol{A}^{(3)}\right) \tag{31}$$

$$\nabla \cdot \boldsymbol{E}^{(2)*} - \frac{\rho^{(2)*}}{\varepsilon_0} = ig\left(\boldsymbol{A}^{(3)} \cdot \boldsymbol{E}^{(1)} - \boldsymbol{E}^{(3)} \cdot \boldsymbol{A}^{(1)}\right) \tag{32}$$

$$\nabla \cdot \boldsymbol{E}^{(3)*} - \frac{\rho^{(3)*}}{\varepsilon_0} = ig\left(\boldsymbol{A}^{(1)} \cdot \boldsymbol{E}^{(2)} - \boldsymbol{E}^{(1)} \cdot \boldsymbol{A}^{(2)}\right) \tag{33}$$

The O(3) Symmetry Ampère-Maxwell law (cf. Eqn. (10))

$$\nabla \times \boldsymbol{B}^{(1)*} - \frac{1}{c^2}\frac{\partial \boldsymbol{E}^{(1)*}}{\partial t} - \frac{1}{c^2}\frac{\boldsymbol{J}^{(1)*}}{\varepsilon_0} = -\frac{ig}{c}\left(A_0^{(2)}\boldsymbol{E}^{(3)} - A_0^{(3)}\boldsymbol{E}^{(2)} + c\boldsymbol{A}^{(2)} \times \boldsymbol{B}^{(3)} - c\boldsymbol{A}^{(3)} \times \boldsymbol{B}^{(2)}\right) \tag{34}$$

$$\nabla \times \boldsymbol{B}^{(2)*} - \frac{1}{c^2}\frac{\partial \boldsymbol{E}^{(2)*}}{\partial t} - \frac{1}{c^2}\frac{\boldsymbol{J}^{(2)*}}{\varepsilon_0} = -\frac{ig}{c}\left(A_0^{(3)}\boldsymbol{E}^{(1)} - A_0^{(1)}\boldsymbol{E}^{(3)} + c\boldsymbol{A}^{(3)} \times \boldsymbol{B}^{(1)} - c\boldsymbol{A}^{(1)} \times \boldsymbol{B}^{(3)}\right) \tag{35}$$

$$\nabla \times \boldsymbol{B}^{(3)*} - \frac{1}{c^2}\frac{\partial \boldsymbol{E}^{(3)*}}{\partial t} - \frac{1}{c^2}\frac{\boldsymbol{J}^{(3)*}}{\varepsilon_0} = -\frac{ig}{c}\left(A_0^{(1)}\boldsymbol{E}^{(2)} - A_0^{(2)}\boldsymbol{E}^{(1)} + c\boldsymbol{A}^{(1)} \times \boldsymbol{B}^{(2)} - c\boldsymbol{A}^{(2)} \times \boldsymbol{B}^{(1)}\right). \tag{36}$$

In the limit of electrostatics, the coupling constant g disappears because the wave-number κ disappears, and the fields $\boldsymbol{E}^{(3)}$ and $\boldsymbol{B}^{(3)}$ disappear for the same reason. With the additional realization, $\boldsymbol{E}^{(1)} = \boldsymbol{E}^{(2)}$ and $\boldsymbol{B}^{(1)} = \boldsymbol{B}^{(3)}$

in the static limit, we recover the empirically well verified Gauss, Coulomb and Ampère laws. So in this view, the magnetic monopole and Lehnert space charge are topological or dynamical objects.

In electrodynamics, the coupling constant g is always non-zero under all conditions, and each of the four laws is a set of three simultaneous, non-linear, equations which must be solved numerically. The presence of g (or Barrett's q) means that the dynamical electromagnetic field is always coupled to its source, an electron, even if that electron be quasi-infinitely distant from the field. The terms premultiplied by ig in each equation are conserved Noether currents {16, 17} with the same meaning as the corresponding terms in Barrett's equations.

There may be particular solutions for which the terms in the brackets following ig are zero {21-24}. In this case, the equations reduce to

$$\nabla \cdot \boldsymbol{B}^{(i)*} \equiv 0; \qquad i = 1,2,3 \tag{37}$$

$$\nabla \times \boldsymbol{E}^{(i)*} + \frac{\partial \boldsymbol{B}^{(i)*}}{\partial t} = \boldsymbol{0}; \qquad i = 1,2,3 \tag{38}$$

$$\nabla \cdot \boldsymbol{E}^{(i)*} - \frac{\rho^{(i)*}}{\varepsilon_0} = 0; \qquad i = 1,2,3 \tag{37a}$$

$$\nabla \times \boldsymbol{B}^{(i)*} - \frac{1}{c^2}\frac{\partial \boldsymbol{E}^{(i)*}}{\partial t} - \frac{1}{c^2}\frac{\boldsymbol{J}^{(i)*}}{\varepsilon_0} = \boldsymbol{0}; \qquad i = 1,2,3 \tag{38a}$$

which are the Gauss, Faraday, Coulomb and Ampère-Maxwell laws for the complex conjugate plane waves $\boldsymbol{B}^{(1)} = \boldsymbol{B}^{(2)*}$ and $\boldsymbol{E}^{(1)} = \boldsymbol{E}^{(2)*}$, together with electrodynamical equations for $\boldsymbol{B}^{(3)}$ and $\boldsymbol{E}^{(3)}$. If $\boldsymbol{A}^{(1)} = \boldsymbol{A}^{(2)*}$ is a plane wave, then $\boldsymbol{B}^{(3)}$ is phaseless and a constant of motion {21-24}. It has been argued elsewhere {24} that $\boldsymbol{B}^{(3)}$ is self-dual, and so the electrodynamical equations for index (3) reduce to:

$$\nabla \cdot \boldsymbol{B}^{(3)} = 0 \tag{39}$$

$$\nabla \times \boldsymbol{B}^{(3)} = \boldsymbol{0} \tag{40}$$

$$\frac{\partial \boldsymbol{B}^{(3)}}{\partial t} = \boldsymbol{0}. \tag{41}$$

There is empirical evidence for $\boldsymbol{B}^{(3)}$ of several different kinds {21-24}, notably, it is directly proportional to the third Stokes parameter and is an observable {21-24} of the inverse Faraday effect. Its interaction with matter is determined by the interaction of $\boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)}$ with matter {21-24} and $\boldsymbol{B}^{(3)}$ is also an observable in the topological phase {17, 24}.

The field $\boldsymbol{E}^{(3)}$ also has a physical significance in O(3) electrodynamics but appears not to be a straightforward first order observable as is its counterpart $\boldsymbol{B}^{(3)}$. For example, there is no electric equivalent of the inverse Faraday effect. An imaginary $i\boldsymbol{E}^{(3)}$ however, may have physical significance because its square modulus would contribute to the field energy as $\boldsymbol{E}^{(3)2}$. The minimum action associated with both the Barrett and Evans et alia equations give physical instantons. In this view, the magnetic monopole and the Lehnert space charge are physical instantons, or pseudo-particles.

This inference shows that the new electrodynamics has rich topological structure which may have many physical consequences.

If $\boldsymbol{B}^{(1)} = \boldsymbol{B}^{(2)*}$ is a plane wave and if $g = \kappa/A^{(0)}$, we recover the B Cyclic theorem {21-24}:

$$\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)} - iB^{(0)}\boldsymbol{B}^{(3)*} \quad \text{et cyclicum} \tag{42}$$

through which the existence of $\boldsymbol{B}^{(3)}$ was inferred originally {21-24} from the empirical data available in the third Stokes parameter and inverse Faraday effect.

The isomorphism between the Barrett field equations (9-12) and those of Evans et al. (25-36) can be illustrated as follows (in work by L.B. Crowell) through the introduction of the Higgs field. The non-Abelian field tensor is in general {21-24}:

$$F_{ij}^{a} = \partial_j A_i^a - \partial_i A_j^a + \varepsilon\left[A_i^b, A_j^c\right] \tag{43}$$

whose magnetic component is

$$B_i^a = \varepsilon_i^{\ jk} F_{jk}^a \tag{44}$$

where i, j, and k are restricted to spatial indices. We introduce the Higgs field:

$$H = \left(H^1, H^2, H^3\right) \tag{45}$$

with covariant derivative:

$$D_i H^a = \partial_i H^a + i\varepsilon^{abc} e A_i^b H^c . \tag{46}$$

The Lagrangian for the gauge field is:

$$\mathcal{L}_G = -\frac{1}{4} F_{ij}^a F^{aij} \tag{47}$$

The action is the function dual to the 4-form F $\vee$ F that determines the instanton number or topological charge{26} k:

$$8\pi^2 k = \int F^a \vee F^a . \tag{48}$$

The Lagrangian for the Higgs field is:

$$\mathcal{L}_H = \frac{1}{2} D_i H^a D^i H^a + \frac{1}{2}\mathcal{L}\left(|H|^2 - m\right)^2 \tag{49}$$

and the sum is:

$$\mathcal{L} = \mathcal{L}_G + \mathcal{L}_H . \tag{50}$$

We investigate the breaking of the SU(2) field (Barrett equations) into the O(3) field (equations of Evans et alia). The classical action is the minimum of the Lagrangian, and fluctuations about this minimum represent quantum fluctuations. The minimum can occur at

$$A^a = 0 \tag{51}$$

where the quartic potential term vanishes. If the Higgs potential vanishes:

$$H^3 = \sqrt{m}; \qquad H^1 = H^2 = 0 \tag{52}$$

This defines the ground state of the system where the massive H^3 restricts the vacuum from the symmetries of SU(2). The Higgs field is then a map from $SU(2) \sim S^3 \to S^3$. Its action on the gauge fields is to return functions of the gauge fields. We impose finite energy conditions on $|H|$ as $r \to \infty$. Further, the cross terms in the Lagrangian are bounded above by the minimization condition:

$$\left(B_i^a \pm D_i H^a\right)\left(B_i^a \pm D_i H^a\right) \geq 0. \tag{53}$$

The corresponding Hamiltonian to the Lagrangian then satisfies the condition:

$$H \geq \frac{1}{2}\mathcal{L}\left(|H|^2 - m\right)^2 \pm B^{ai} D_i H^a. \tag{54}$$

Invoking the Taubes condition:

$$\frac{1}{2}\int B^{ai} D_i H^a = 4\pi n. \tag{55}$$

The total energy in the integral over the space-like manifold of the magnetic field is defined by:

$$En \geq \frac{1}{2}\mathcal{L}\left(|H|^2 - m\right)^2 \pm 8\pi n \tag{56}$$

where equality occurs when:

$$B_i^a = \mp D_i H^a. \tag{57}$$

This is the Bogomolny condition. Now define the quantity:

$$Q = \int B_i^a H^a d^2 x^i \tag{58}$$

which is the Gauss law in integral form for the calculation of charge within a region. Using the Divergence theorem:

$$\begin{aligned} Q &= \int B_i^a H^a d^2 x^i \\ &= \int \nabla_i \left(B_i^a H^a\right) d^3 x \\ &= \int B_i^a D^i H^a d^3 x = 8\pi n \end{aligned} \tag{59}$$

$$B_i^a = \varepsilon_i^{jk}\left(\partial_i A_k^a + i\varepsilon^{abc} A_i^b A_j^c\right). \tag{60}$$

For $a = 1$ and 2, the commutator implied by the Levi-Civita symbol involves the A^3 field which is very massive {27, 28}, and vanishes. This leaves the standard curl terms in the integral, that sum to zero. For $a = 3$:

$$Q = -\iint \left[A^1, A^2 \right]^2 d^3x \tag{61}$$

which implies that the commutator of the A^1 and A^2 fields are associated with a magnetic monopole field. At first order, a similar commutator defines the topological $B^{(3)}$ field, (eqn. (21)). Further, the Bogomolny condition implies that the magnetic fields are the result of two scalar fields as inferred by Whittaker {29, 30}.

The derivation of eqn. (61) by Crowell is consistent with the definition {24} of the magnetic monopole as an area integral over $\boldsymbol{B}^{(3)}$:

$$Q = \frac{1}{V} \int B^{(3)} dAr. \tag{62}$$

To demonstrate the link between the two equations (61) and (62), we use the free space definition (from eqn. (21)):

$$B^{(0)} = \frac{e}{\hbar} A^{(0)2} \tag{63}$$

and the additional relations:

$$B^{(0)} = \frac{\omega}{c} A^{(0)}; \qquad \hbar\omega = ecA^{(0)} = \frac{1}{\mu_0} B^{(0)2} V \tag{64}$$

where V is an integration volume {21-24}. It follows straightforwardly that:

$$Q = \frac{1}{V} \int B^{(3)} dAr = \frac{e\kappa^2}{\mu_0 c\hbar^2} \int A^{(0)4} dV \tag{65}$$

so the magnetic charge or monopole is a volume integral over a quartic in the potential, as in eqn. (61), as well as being the area integral over $\boldsymbol{B}^{(3)}$ and observable {24} in the topological phase {17}.

DISCUSSION

It has been shown that the forms of the Harmuth and Lehnert equations, when combined, form the Barrett field equations, which are isomorphic with the equations of Evans et alia. In this view, the magnetic monopole and the Lehnert space charge are regarded as low energy physical instantons {10}. Therefore, the fundamental equations of electrodynamics in a higher symmetry, of the Yang-Mills form, have been derived independently by four authors: Harmuth, Lehnert, Barrett and Evans. In the electro-static limit, the coupling constant g vanishes and the equations reduce to the Ampère, Coulomb, and Gauss laws, without instantons, or pseudo-particles, or solitons localized in space and time. The O(3) equations (25) to (36) can be reduced to U(1) form for the indices (1) and (2) plus equations for the $\boldsymbol{B}^{(3)}$ field, eqns. (39) to (41). In general, however, the equations (25) to (36) must be solved without approximation. This has been carried out for the inverse Faraday effect, for example, in ref. (24). Therefore the equations developed in this paper allow for the existence of magnetic charge and current as topological instantons, not as Dirac monopoles and currents {10, 16, 17}. Under certain circumstances, the instantons disappear, for example if

$$\boldsymbol{A}^{(2)} \cdot \boldsymbol{B}^{(3)} = \boldsymbol{B}^{(2)} \cdot \boldsymbol{A}^{(3)} \tag{66}$$

$$\boldsymbol{A}^{(3)} \cdot \boldsymbol{B}^{(1)} = \boldsymbol{B}^{(3)} \cdot \boldsymbol{A}^{(1)} \tag{67}$$

$$A^{(1)} \bullet B^{(2)} = B^{(1)} \bullet A^{(2)} \tag{68}$$

in eqns. (25) to (27). Note that this condition still leaves a non-zero:

$$B^{(3)} = -igA^{(1)} \times A^{(2)} \tag{69}$$

so the $B^{(3)}$ field is a fundamental field of the Barrett and Evans et alia equations under all circumstances {21-24}, being proportional to the conjugate product $A^{(1)} \times A^{(2)}$ through the coupling constant g. Therefore, if we "symmetry break" equations (25) to (27) from O(3) using eqns. (66) to (68), the $B^{(3)}$ field is still non-zero, and the B cyclic theorem (42) still exists.

In this view, the Faraday induction law can be regained using the conditions:

$$cA_0^{(3)}B^{(2)} - cA_0^{(2)}B^{(3)} + A^{(2)} \times E^{(3)} - A^{(3)} \times E^{(2)} = 0 \tag{70}$$

$$cA_0^{(1)}B^{(3)} - cA_0^{(3)}B^{(1)} + A^{(3)} \times E^{(1)} - A^{(1)} \times E^{(3)} = 0 \tag{71}$$

$$cA_0^{(2)}B^{(1)} - cA_0^{(1)}B^{(2)} + A^{(1)} \times E^{(2)} - A^{(2)} \times E^{(1)} = 0 \tag{72}$$

with an additional law for $B^{(3)}$, eqn.(41). The Lehnert space charge disappears under the condition:

$$A^{(2)} \bullet E^{(3)} = E^{(2)} \bullet A^{(3)} \tag{73}$$

$$A^{(3)} \bullet E^{(1)} = E^{(3)} \bullet A^{(1)} \tag{74}$$

$$A^{(1)} \bullet E^{(2)} = E^{(1)} \bullet A^{(2)} \tag{75}$$

and a similar condition:

$$A_0^{(2)}E^{(3)} - A_0^{(3)}E^{(2)} + cA^{(2)} \times B^{(3)} - cA^{(3)} \times B^{(2)} = 0 \tag{76}$$

$$A_0^{(3)}E^{(1)} - A_0^{(1)}E^{(3)} + cA^{(3)} \times B^{(1)} - cA^{(1)} \times B^{(3)} = 0 \tag{77}$$

$$A_0^{(1)}E^{(2)} - A_0^{(2)}E^{(1)} + cA^{(1)} \times B^{(2)} - cA^{(2)} \times B^{(1)} = 0 \tag{78}$$

can be used to regain the Ampère-Maxwell Law, but with an additional equation for $B^{(3)}$ {21-24}. Therefore these cyclic equations can be regarded as a kind of "symmetry breaking" of the more general O(3) equations to U(1) equations. Note that eqn. (38) is in U(1) form, but there is reason to believe {24} that $B^{(3)}$ is dual to a pure imaginary $-iE^{(3)}$, or is self-dual {24}. Self-consistently, the B Cyclic theorem (42) is valid when $B^{(1)}$ and $B^{(2)}$ are plane waves in U(1) form, and the B Cyclic theorem is consistent with the special case (37) to (38). However, the most general form of electrodynamics requires a Lehnert space charge for reasons developed in refs {18-20}; and requires a Harmuth magnetic monopole and current {16, 17}. Electromagnetic fields can be conditioned by polarization into SU(2) form as shown by Barrett {16, 17}, and the $B^{(3)}$ field is the result of an O(3) symmetry electrodynamics isomorphic with the Barrett equations.

We conclude that there is beginning to be an appreciation that there is a more general form of classical electrodynamics than the Maxwell-Heaviside equations, and a more general form of quantum electrodynamics than the U(1) form. Under certain circumstances, the consequences of these more general equations can be observed empirically, showing clearly the Maxwell-Heaviside equations are incomplete. This type of generalization of electrodynamics was first suggested by Yang and Mills in 1954 {34}.

ACKNOWLEDGMENTS

Internet discussions with many colleagues are gratefully acknowledged; together with funding for individual members of AIAS (Alpha Foundation's Institute for Advanced Study).

REFERENCES

{1} V.F. Mikhailov, Phys. Lett., **130B**, 331 (1983).
{2} V.F. Mikhailov, J. Phys. A, **18**, L903 (1985).
{3} V.F. Mikhailov, Annales de la Fond L. de Broglie, **12**, 491 (1987).
{4} V.F. Mikhailov, J. Phys. A, **24**, 53 (1991).
{5} V.F. Mikhailov and L.I. Mikhailova, J. Phys. A, **23**, 53 (1990).
{6} V.F. Mikhailov and L.I. Mikhailova, J. Phys. Cond. Matt., **5**, 351 (1993).
{7} V.F. Mikhailov and L.I. Mikhailova, Acad. Sci. Kazakh SSR, Alma Ata, HEPI-91-04 (1991).
{8} V.F. Mikhailov and L.I. Mikhailova, ibid., HEPI-90-07 (1990).
{9} V.F. Mikhailov in T.W. Barrett and D.M. Grimes (eds.), "Advanced Electromagnetism" (World Scientific, Singapore, 1995), pp. 593 ff.
{10} T.W. Barrett, Annales de la Fond. L. de Broglie, **19**, 291 (1994).
{11} H.F. Harmuth, in ref. (9), pp. 506 ff.
{12} H.F. Harmuth, "Information Theory applied to Space-Time Physics" (World Scientific, Singapore, 1993).
{13} H.F. Harmuth and M.G.M. Hussain, "Propagation of Electromagnetic Signals" (World Scientific, Singapore, 1994),
{14} H.F. Harmuth, IEEE Trans. Electromagn. Compat., EMC-**28**(4), 250, 259, 267 (1986).
{15} H.F. Harmuth, R.N. Boles and M.G.M. Hussain, ibid., EMC-**29**(4), 318 (1987).
{16} T.W. Barrett in A. Lakhtakia (ed.), "Essays on the Formal Aspects of Electromagnetic Theory" (World Scientific, Singapore, 1993), pp. 6-86.
{17} T.W. Barrett, in ref. (9), pp. 278 ff.
{18} B. Lehnert, Phys. Scripta, **19**, 204 (1996).
{19} B. Lehnert, Optik, **99**, 113 (1995).
{20} B. Lehnert and S. Roy, "Extended Electromagnetic Theory" (World Scientific, Singapore, 1998).
{21} M.W. Evans and A.A. Hasanein, "The Photomagneton in Quantum Field Theory" (World Scientific, Singapore, 1994).
{22} M.W. Evans and S. Kielich (eds.), "Modern Nonlinear Optics" (Wiley, New York, 1992, reprinted1993, 1997 (paperback)), part 2.
{23} M.W. Evans, J.P. Vigier, S. Roy and S. Jeffers, "The Enigmatic Photon" (Kluwer, Dordrecht, 1994 to 1999) in five volumes.
{24} M.W. Evans and L.B. Crowell, "Classical and Quantum Electrodynamics and the $\boldsymbol{B}^{(3)}$ Field." (World Scientific, Singapore, 1999 / 2000, in prep.)
{25} L.B. Crowell, work in progress and in ref. (24).
{26} L.H. Ryder, "Quantum Field Theory" (Cambridge, 1987, 2nd. Ed.).
{27} J.D. Jackson, "Classical Electrodynamics" (Wiley, New York, 1962).
{28} M.W. Evans, Physica B, **182**, 227, 237 (1992).
{29} F. Ehrenhaft, Phys. Z., **31**, 47 (1930).
{30} L.B. Crowell and M.W. Evans, Found. Phys. Lett., submitted.
{31} L.B. Crowell and M.W. Evans, Found. Phys. Lett., submitted.
{32} E.T. Whittaker, Math, Ann., **57**, 333 (1903).
{33} E.T. Whittaker, Proc. London Math. Soc., **1(2)**, 367 (1904).
{34} C.N. Yang and R.L. Mills, Phys. Rev., **96**, 191 (1954).