ON THE NON-EXISTENCE OF THE FIELD $E^{(3)}$

ABSTRACT

It is demonstrated, using symmetry, that the Evans-Vigier field $B^{(3)}$ is not accompanied by a physical electric field $E^{(3)}$ and that this is compatible with the Lorentz transformation.

INTRODUCTION

It has become well known that the Evans-Vigier field $B^{(3)}$ is physical and gauge invariant within an O(3) symmetry gauge theory {1-5}. The $B^{(3)}$ field does not exist in Maxwell-Heaviside theory and yet, is a measurable entity which fits into an O(3) gauge theoretical structure for electrodynamics. In this short paper, we prove that the concomitant $E^{(3)}$ field does not exist. It has not been observed. In contrast, there is copious empirical evidence {1-5} for the existence of $B^{(3)}$ if we accept gauge theory, a major part of contemporary physics. This paper demonstrates straightforwardly that $B^{(3)}$ is unchanged under Lorentz transformation and that, formally, the same is true of $E^{(3)}$. However, the latter vanishes by symmetry. The dual of $B^{(3)}$ is therefore $B^{(3)}$. This does not mean that the Lagrangian associated with $B^{(3)}$ vanishes, and therefore $B^{(3)}$ is a classical, physically measurable and fundamental property of the electromagnetic field. It is also invariant under an O(3) gauge transformation. After quantization, it becomes a fundamental property of the photon. The field equations of O(3) electrodynamics are homomorphic with those of Barrett {6}, and under certain circumstances, indicate a non-zero photon mass. These O(3) field equations are also similar to those of Lehnert and Roy {7}.

LORENTZ TRANSFORMATION OF $B^{(3)}$

The field tensor of O(3) electrodynamics {1-5} is defined as a vector in the internal gauge space ((1, (2), (3)):

$$\boldsymbol{G}^{\mu\nu} \equiv G^{\mu\nu(1)}\boldsymbol{e}^{(1)} + G^{\mu\nu(2)}\boldsymbol{e}^{(2)} + G^{\mu\nu(3)}\boldsymbol{e}^{(3)} \tag{1}$$

The $G^{\mu\nu(3)}$ component can be written formally as:

$$G_{\mu\nu}^{(3)} = \begin{bmatrix} 0 & 0 & 0 & E_Z \\ 0 & 0 & -B_Z & 0 \\ 0 & B_Z & 0 & 0 \\ -E_Z & 0 & 0 & 0 \end{bmatrix} \tag{2}$$

where both B_Z and E_Z are directed in the axis of propagation, the (3) axis, which is the Z axis. Using Jackson's notation and his eqn. (11.113) of the first edition,{8} the Lorentz transformation of the field tensor $G_{\mu\nu}^{(3)}$ is given by:

$$G_{\mu\nu}^{(3)\prime} = a_{\mu\lambda} a_{\nu\sigma} G_{\lambda\sigma}^{(3)} \tag{3}$$

where

$$A_{\mu\nu} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \gamma & i\gamma\beta \\ 0 & 0 & -i\gamma\beta & \gamma \end{bmatrix}. \tag{4}$$

Here

$$\beta = \frac{v}{c}; \qquad \gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \tag{5}$$

where $\boldsymbol{v}$ is the velocity of one frame relative to the other in a Lorentz boost, and where c is the speed of light in vacuo. The $\boldsymbol{B}^{(3)}$ component transforms as:

$$\boldsymbol{B}^{(3)'} = \boldsymbol{B}^{(3)} \tag{6}$$

and similarly the $\boldsymbol{E}^{(3)}$ component transforms as $\boldsymbol{E}^{(3)}$. Therefore both components are unaffected by a simple Lorentz boost.

NON-EXISTENCE OF $\boldsymbol{E}^{(3)}$

The putative $\boldsymbol{E}^{(3)}$ field of radiation has never been observed, while $\boldsymbol{B}^{(3)}$ is an observable in many ways {1-5}. In the ((1), (2), (3)) frame, the $\boldsymbol{B}^{(3)}$ field is defined {1-5} as:

$$\boldsymbol{B}^{(3)} \equiv -ig\boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)} \tag{7}$$

where $\boldsymbol{A}^{(1)} = \boldsymbol{A}^{(2)*}$ is a transverse propagating vector potential. Therefore $\boldsymbol{B}^{(3)}$ is radiated and propagates in vacuo at the speed of light if we assume a massless photon, for the sake of argument. If the incorrect assertion is made that $\boldsymbol{B}^{(3)}$ is dual to a physical $\boldsymbol{E}^{(3)}$, then we obtain:

$$\boldsymbol{E}^{(3)} = ?\ -ig\boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)} \tag{8}$$

which is an equation which breaks parity symmetry on the classical level, indicating that $\boldsymbol{E}^{(3)}$ is identically zero. In consequence, only $\boldsymbol{B}^{(3)}$ appears in the Lagrangian and Hamiltonian, which lead to the equations of motion governing $\boldsymbol{B}^{(3)}$ through the Euler-Lagrange equation on the classical level. Under the dual tranformation:

$$\tilde{G}^{\mu\nu(3)} = \frac{1}{2}\varepsilon^{\mu\nu\sigma\rho} G^{(3)}_{\sigma\rho} \tag{9}$$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -B_Z & 0 \\ 0 & B_Z & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & -B_Z \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ B_Z & 0 & 0 & 0 \end{bmatrix}. \tag{10}$$

$B^{(3)}$ is dual to itself, and this concept does not occur in Maxwell-Heaviside electrodynamics. The Lagrangian due to $B^{(3)}$ is:

$$\mathcal{L} = -\frac{1}{4} G^{(3)}_{\mu\nu} G^{\mu\nu(3)} \tag{11}$$

and the Euler-Lagrange equation is:

$$\frac{\partial \mathcal{L}}{\partial \left(A^{(3)}_{\mu} \right)} = \partial_{\nu} \left(\frac{\partial \mathcal{L}}{\partial \left(\partial_{\nu} A^{(3)}_{\mu} \right)} \right) \tag{12}$$

giving

$$D_{\mu} \tilde{G}^{\mu\nu(3)} \equiv 0 \tag{13}$$

$$D_{\mu} G^{\mu\nu(3)} = J^{\nu(3)} \tag{14}$$

in the vacuum. The current $J^{\nu(3)}$ is a conserved Noether current that exists in vacuo as well as in field-matter interaction. Finally, eqn. (14) can take the particular vacuum solution:

$$\partial_{\mu} G^{\mu\nu(3)} = 0; \qquad J^{\nu(3)} = g \varepsilon_0 \varepsilon_{(1)(2)(3)} A^{(1)}_{\mu} G^{\mu\nu(2)} \quad \text{et cyclicum.} \tag{15}$$

which can be shown to lead to {5}:

$$En^{(3)} = \frac{1}{\mu_0} \int \boldsymbol{B}^{(3)} \cdot \boldsymbol{B}^{(3)} dV \tag{16}$$

The vacuum equations of motion governing $B^{(3)}$ are therefore:

$$\frac{\partial \boldsymbol{B}^{(3)}}{\partial t} = 0; \qquad \nabla \cdot \boldsymbol{B}^{(3)} = 0; \qquad \nabla \times \boldsymbol{B}^{(3)} = \boldsymbol{0} \tag{17}$$

and its contribution to the energy of the electromagnetic field in a volume V is:

$$En = \frac{1}{\mu_0} \int \boldsymbol{B}^{(3)} \cdot \boldsymbol{B}^{(3)} dV \tag{18}$$

All of this is straightforward in an O(3) gauge theory.

DISCUSSION

These equations have been shown to be successful in describing many phenomena inaccessible to U(1) electrodynamics, notably the Sagnac effect {9}, and Michelson interferometry {10}. There are no phenomena in nature due to $E^{(3)}$. The correct Stokes theorem to the use of $B^{(3)}$ is the integral form of eqns. (13) and (14). This is a non-Abelian Stokes theorem {11}, which is by no means a trivial construct. It is far easier to deal with the differential form. The hypothesis of the Evans-Vigier field is clear and simple, and has been verified to one part in 10^{22}. In the Sagnac effect {9}, this paper gives some of the fundamentals and the correct way of dealing with the theory which is self-consistent {1-5} on the classical level. The theory allows for the existence of magnetic monopoles but does not rely on them. Similarly, the theory allows for the existence of photon mass. It may be proven rigorously {1-5} that the indices (1), (2), (3) used over the Minkowski indices constitute a rigorously correct fiber bundle theory and extended Lie algebra. The mathematical foundations of the theory are therefore rigorous to the state of the art, and the theory is a major advance in classical electrodynamics from the U(1) Yang-Mills level (Maxwell-Heaviside theory) to the O(3) Yang-Mills level, which greatly enriches the whole subject {1-5}.

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